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Clock Jitter Estimation based on PM Noise Measurements*

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ABSTRACT - "Jitter" is the noise modulation due to random time shifts on an otherwise ideal, or perfectly on-time, signal transition. In the absence of ultra-high-speed jitter analyzers, spectrum analysis is an alternate noise measurement for timing jitter. Conventionally, jitter has been defined as a the integral of the phase noise. This paper presents a modified way of calculating timing jitter using phase-modulation (PM) noise measurements of high-speed digital clocks, which considers the frequency response of the jitter analyzer, providing a more accurate map. Measurements of phase noise are typically much more sensitive to phase (or time) fluctuations than a jitter analyzer. A summary table is provided for mapping the results of these measurements in the Fourier frequency domain to jitter in the τ domain for various random (specifically, power-law) noise types, spurs, vibration, and power-supply ripple. In general, one cannot unambiguously map back, that is, translate from jitter measurements to phase noise.

1. INTRODUCTION AND SUMMARY

A widely used method of characterizing jitter is histogram statistics associated with a photograph of an "eye" pattern. While histograms are useful, near-instantaneous sampling of a high-rate reference clock operating at, say, 100 GHz as an example, implies the need for many hundreds of gigahertz of bandwidth in a jitter analyzer. Therefore, this rate is prone to several pitfalls associated with high-speed digital sampling: trigger errors, resolution, and time base distortions [1–4]. Second, a histogram misses an important piece of information for nonstationary kinds of noise, namely, how its width varies with delay, called τ in this writing. At what rate does width get larger *vs.* τ? These questions arise from the fact that most jitter measurements assume that a mean value of τ exists when in most cases one does not. The primary motivation for this writing is that nonstationary noise will occur at some level and that a proper statistic must be used. In particular, this paper suggests the use of measurements of phase noise that provide clues into the origin of clock jitter in general. Two definitions of jitter are explored. The fundamental measurement performed by most jitter analyzers is based on a first difference of time errors. However, when this first difference operates on time errors that are not white, there is an unreliable functional dependence on the τ. In these cases, a second-difference operator is employed.

We explain the methods and reasons for calculating clock jitter *vs.* τ using measurements of the clock's phase-modulation (PM) spectral noise based upon common definitions of the two. This discussion derives from comprehensive work done in frequency standards, characterization of noise, and state-of-the-art methods of measuring time errors [5–10]. This paper is a quick guide to estimating clock jitter from PM noise measurements. In section 2, generic definitions of jitter are introduced. Section 3 introduces the concept of spectral density of phase fluctuations, defines L(f), and categorizes the five noise types. Section 4 presents tables that map PM noise measurements to two definitions of jitter.

2. JITTER DEFINITION

Clock jitter from a reference clock sets the baseline performance for those digital components using that clock. A jitter analyzer is an oscilloscope that displays time-error noise after an arbitrary trigger time t at time $t + \tau$. The horizontal axis is running time (the "sweep" signal). Generically, the "transition" is that portion of an oscillating signal in the neighborhood of its zero-crossings or other defined crossings. The transition's timing error is subjectively measured with what is called an "eye" diagram that shows integer "half-period" transition errors. Here, a major goal of a good statistic is prediction of some parameter of interest based on past statistics. A jitter analyzer quantifies the statistical noise on a predic-

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tion that

$$\overbrace{\hat{x}(t+\tau)}^{\text{delay error}} = \overbrace{x(t)}^{\text{trig. error}} + \overbrace{\varepsilon(\tau)}^{\text{timing error}}, \quad (1)$$

where $x(t)$ is a timing error at trigger-point t (often assumed to be 0), $\hat{x}(t+\tau)$ is the prediction of a future timing error viewed at time τ later, and $\varepsilon(\tau)$ is the observed difference between a zero-crossing or transition and a delay (subject to an instrument-related delay error) from the trigger-point claimed by the analyzer. Note that τ can be only a multiple of a minimum sample interval $\tau_0 = \frac{T}{2}$, the half-period of the clock signal itself. Figure 1 illustrates the transition errors with respect to t and τ. For example, if the trigger-point timing error $x(t) = 0$, and transitions occur every $\tau_0 = 1$ ns in the clock signal (a half-period corresponding to a 500 MHz clock), and τ= 10 ns, we expect zero error at $\hat{x}(t$+10 ns), but it is in fact perturbed by a noise burst, say, ε(10 ns)= +2 ps. Rewriting, we obtain

$$\underbrace{\varepsilon(\tau)}_{\text{timing error}} = \hat{x}(t+\tau) - x(t), \quad (2)$$

whose form is called a "first difference" in terms of $x(t)$ because $\varepsilon(\tau)$ is the difference of two time errors as shown in (2). Let $< \cdot >$ designate an average (more precisely, "expectation," if an infinite ensemble average is calculated). Then the mean square (or variance) of $\varepsilon(\tau)$ is

$$< \varepsilon^2(\tau) >=< (\hat{x}(t+\tau) - x(t))^2 >, \quad (3)$$

$$\text{after squaring} \quad =< \hat{x}^2(t+\tau) > + < x^2(t) >, \quad (4)$$

or the sum of the delay-error variance plus trigger-error variance. (4) assumes values of $x(t)$ are independent for given discrete or sampling intervals of t, which is the case for white PM noise only.

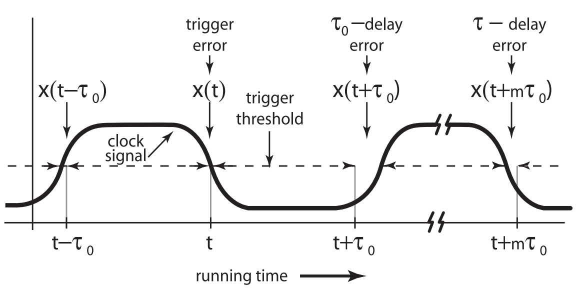

Figure 1: Sampling 'scope display. Jitter is the rms of time-delay errors, each given by $x(t+\tau)$, relative to a trigger-point whose single-shot error is $x(t)$. See eqn. (2) and (5) in text. To suppress the effects of phase drift and/or to measure "jitter walk," a second definition (eqn. (6) and (7)) is recommended.

There are a variety of ways that jitter has been defined based on the sampling oscilloscope and its limitations [11–14]. For this discussion, many analyzers provide at least a root-mean-square (rms) of timing errors based on (2), or

$$\text{Jitter } (vs.\ \tau) \quad \equiv \quad < \varepsilon^2(\tau) >^{\frac{1}{2}}. \quad (5)$$

This does not encompass a full model, but is sufficient for the goal of this writing of converting from PM noise to jitter as defined above and later in (7).

Because of the fact that real clocks seldom satisfy the stationary criteria, it is preferable to define jitter as the "second difference" of time-error measurements $x(t)$. The preferred definition is based on

$$\underbrace{\varepsilon_{2^{nd}}(\tau)}_{\text{timing error}} = -\hat{x}(t+\tau) + 2x(t) - x(t-\tau). \quad (6)$$

$$\text{Jitter-2 } (vs.\ \tau) \quad \equiv \quad < \varepsilon^2_{2^{nd}}(\tau) >^{\frac{1}{2}}. \quad (7)$$

This second-difference definition differs from (5) by the factor $\frac{1}{\sqrt{3}}$ for white PM as will be shown later, but handles the problem of a nonstationary, moving average such as a phase drift or random-walk behavior in $x(t+\tau) - x(t)$, appropriately dubbed a "jitter-walk" behavior. As first pointed out by Barnes in [8] and revisited by Walls in [15], jitter-2 can be used as a measure of time dispersion and permits models of noise (mainly power-law noises) that extend to virtually any device or signal under test, such as free-running oscillators, filters, multipliers and dividers, rf synthesizers, amplifiers, flip-flops, and logic gates.

At this point, an important clarification needs to be made. A clock reference is a repeating signal with period T that precisely defines the timing in synchronous digital systems by <u>when</u> clock transitions occur. This paper addresses the fundamental noise limit given by the clock reference noise and translates this to clock jitter. Jitter on data transitions (those that are <u>not</u> reference-clock transitions) are not the subject of this paper. <u>Data</u> jitter contains the cumulative effects of noise from digital logic, digital modulation schemes, filters, component linearity, amplifiers, additive and multiplicative noise, crosstalk, etc., and cannot be easily calculated from the PM noise measurement methods described here.

The measurement of jitter given by (5) or (7) will estimate the statistical standard deviation of clock-timing errors. The standard deviation is the 1σ histogram width regarded as the jitter level measured at integer half-period increments $m\frac{T}{2}, m = 1, 2, 3, \ldots$.

3. PHASE NOISE MEASUREMENTS

We want to measure the time when a transition occurs in the neighborhood of an idealized on-time point. Noise occurs on an ideal fundamental frequency of $\frac{1}{T}$. The rf power spectrum is regarded as the ideal carrier plus "total baseband modulation power noise" due to PM + AM noise on an otherwise perfect carrier. An rf power spectrum measurement cannot distinguish PM from AM noise, so we often measure both independently, or just PM noise in cases where the AM noise contribution is considerably lower or not of concern. This noise spectrum appears above and below the frequency of the fundamental. The single-sideband power of the noise relative to carrier power is comprised of phase and amplitude spectral densities $S_\phi(f)$ and $S_{AM}(f)$. In particular, $S_\phi(f)$ is the power spectral density of phase fluctuations measured in a bandwidth of 1 Hz at a Fourier separation of f Hz. The units are $radians^2$/Hz. However, single-sideband PM noise $L(f)$ is *defined* as $\frac{1}{2}S_\phi(f)$. Typically, its expression in a logarithmic form is

$$L(f) = 10\,log\left(\frac{1}{2}S_\phi(f)\right), \text{ in units of dBc/Hz.} \quad (8)$$

Time fluctuations on zero-crossings or transitions are phase fluctuations, or phase noise, on a sine-wave signal generator. Phase fluctuation spectral density is measured by passing such a signal through a phase comparator and measuring the detector's output power spectrum. A common technique is to use a loose phase-locked loop (PLL) as described in [6,7]. The measurement of $\phi(t)$ uses a phase-locked loop and makes use of the relation that for small deviations ($\delta\phi \ll 1$ radian) between the oscillator under test and a reference locked oscillator,

$$L(f) = \frac{1}{2}S_\phi(f) = \frac{1}{2}\left[\frac{V_{rms}(f)}{K_d}\right]^2, \quad (9)$$

where $V_{rms}(f)$ is measured on a spectrum analyzer as the root-mean-square noise voltage per $\sqrt{Hz}$ at Fourier frequency "f", and K_d is the sensitivity (volts per radian) at the phase quadrature output of a phase detector that is comparing the test to a reference oscillator.

Jitter-2 *vs.* τ can be used for a range of five common power-law, or $b_\beta f^\beta$, types of noise, where β is an integer exponent corresponding to five different slopes as shown in the log-log plots of figure 2. For commonly encountered high-speed digital clocks and oscillators, $S_\phi(f)$ is modeled by

$$S_\phi(f) = b_{-4}\frac{1}{f^4} + b_{-3}\frac{1}{f^3} + b_{-2}\frac{1}{f^2} + b_{-1}\frac{1}{f} + b_0 = \sum_{\beta=-4}^{0} b_\beta f^\beta, \quad (10)$$

where b_β are the levels of the noise types for slopes

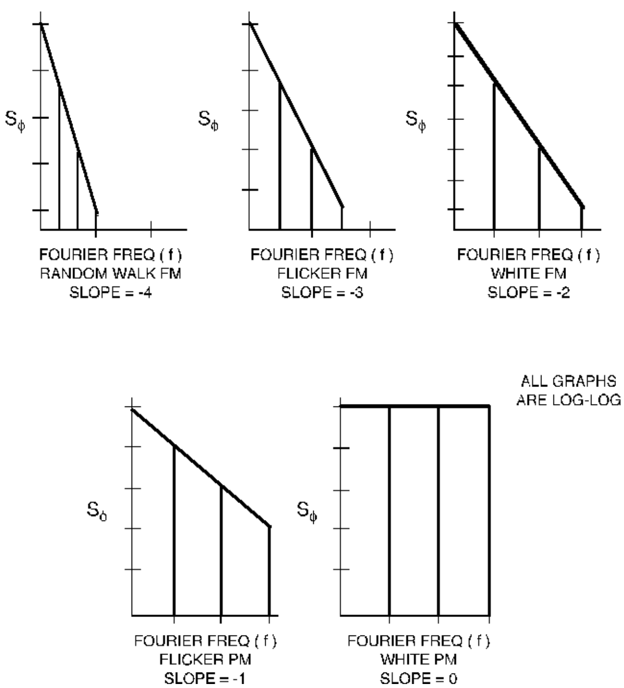

Figure 2: The five power-law noise processes create five different slopes on a phase noise plot (a log-log scale). β is the integer value of the slope corresponding to a specific model of noise.

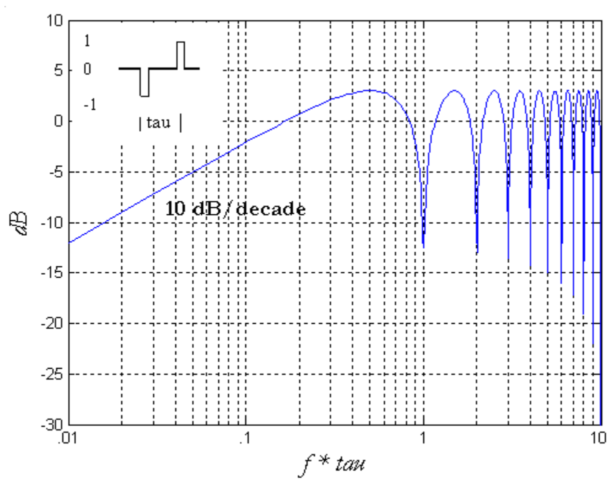

Figure 3: Frequency response $H(f)$ of a jitter analyzer with -1,+1 τ-spaced sampling coefficients, shown in the upper left. The transfer characteristic corresponds to a high-pass filter with a 10 dB/decade low-frequency skirt, which is sufficient to measure only jitter that does not drift or "walk" with a moving, non-stationary mean value. (This condition is seldom satisfied in real clocks.)

$\beta = 0, -1, -2, -3, -4$, identified respectively as White PM (WHPM), Flicker PM (FLPM), Random-Walk PM (RWPM, also known as White FM (WHFM)), Random-Run PM (RRPM, also known as Flicker FM (FLFM)), and Random Walk FM (RWFM).

Jitter measurements do not readily distinguish the effects of spurs and sensitivity to vibration and power-supply ripple. For these, conventional narrow-band measurements of phase noise that use a phase detection scheme and spectrum analyzer are superior for quickly identifying these noise sources. In general, measurements of phase noise reveal substantially more than measurements of jitter.

4. MAPPING PHASE NOISE TO JITTER

In general, any given jitter measurement is essentially a broadband phase noise measurement, so it is possible to calculate jitter from a conventional narrow-band phase-noise measurement passed through an equivalent jitter analyzer "filter." A jitter analyzer can be regarded as measuring first differences of time deviations as a function of time-delay-from-trigger (function of τ). The equivalent filter in this case, the Fourier transform of the jitter analyzer's first-difference sampling function (given by -1 and +1 separated by τ as shown in (2)), turns out to be a high-pass filter, so the high-cutoff frequency f_h directly affects the level of jitter for common white phase noise. The sampling function and equivalent frequency-response is shown in figure 3. Figure 4 shows the equivalent frequency response $H(f)$ of the -1,+2,-1 τ-spaced sampling coefficients of (6), shown in the upper left.

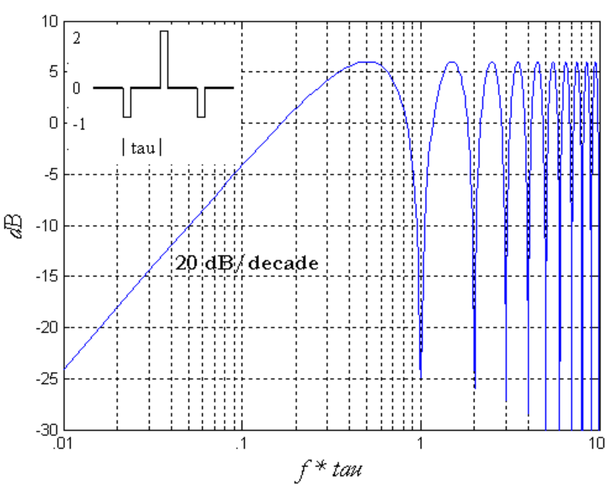

Figure 4: Frequency response $H(f)$ of jitter definition 2 of eqn. (6) and (7). This transfer characteristic corresponds to -1,+2,-1 τ-spaced sampling coefficients of eqn. (6), shown in the upper left. The steeper 20 dB/decade low-frequency skirt is sufficient to measure jitter with a moving average carrier frequency (dubbed "jitter-walk") or phase drift.

Using Parceval's equality (a conservation principle) stating that total power over all time must equal total power over all frequencies, we can write

$$\sigma_\xi{}^2(t) = \int_{-\infty}^{\infty} S_\xi(f)df, \tag{11}$$

and we derive a useful formula for an actual data run of running-time phase deviations $\Delta\phi(t)$ as

$$\sigma^2_{\Delta\phi}(t)|^{\tau}_{t_0} = 2\int_{\frac{1}{2\tau}}^{f_h} S_\phi(f)[H(f)]^2 df, \tag{12}$$

where τ is a time interval in spacings of $m\frac{T}{2}, m = 1, 2, 3, ...$ and $[H(f)]$ is our analysis filter transfer function. The factor of 2 comes in because the limits of integration consider only a one-sided spectrum. Converting $\Delta\phi(t)$ to $x(t)$ by the basic relationships, the mean squared error $\sigma^2_{\Delta\phi}$ with respect to $x(t)$ is the finite-time variance version of (3) given by

$$\sigma_x{}^2(t)|^{\tau}_{t_o} = \frac{1}{2(\pi\nu_0)^2}\int_{\frac{1}{2\tau}}^{f_h} S_\phi(f)[H(f)]^2 df = <\varepsilon^2(\tau)>, \tag{13}$$

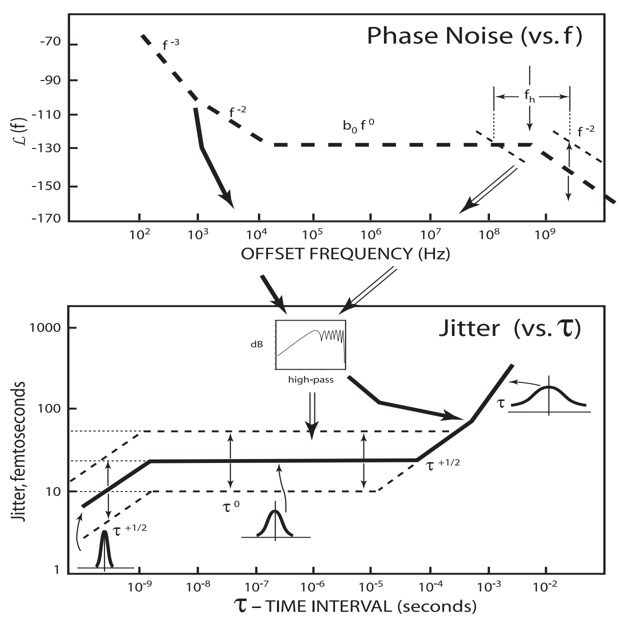

Figure 5: Mapping from phase noise to jitter. $L(f)$ passes through a high-pass filter (figure 4) to create a jitter *vs.* τ plot. The plot transposes $L(f)$, that is, high offset frequencies map to short-τ time intervals, and *vice versa*. Jitter histogram 1σ-width equals jitter level at a given delay, or τ-value.

Table 1: Jitter is calculated from $S_\phi(f)$ noise type and level, which are determined by a slope β and amplitude b_β. $\frac{V_{rms}}{K_d}$ of the last row is spur level divided by the PM measurement sensitivity K_d.

$S_\phi(f) = b_\beta f^\beta$	Jitter *vs.* τ
b_0 (WHPM)	$\sqrt{\frac{b_0}{(\pi\nu_0)^2}\left(f_h - \frac{1}{2\tau}\right)}$
$b_{-1}f^{-1}$ (FLPM)	$\sqrt{\frac{b_{-1}}{(\pi\nu_0)^2}}\times \sqrt{ln(2\pi\tau f_h) - 1.0711 + \frac{1}{(2\pi f_h\tau)^2}}$
$b_{-2}f^{-2}$ (RWPM)	$\sqrt{\frac{2b_{-2}}{(\pi\nu_0)^2}\left(1.1168\tau - \frac{1}{2f_h}\right)}$
Spur or *Sinusoid* with level V_{rms}	$\sqrt{\frac{2V_{rms}sin^2(\pi f_m\tau)}{K_d(\pi\nu_0)^2}}$
	$^*f_h$ is a high-freq. cutoff

and jitter is the usual square root, given by (5).

To illustrate, if $L(f)$ is constant $= b_0$, a white pm (WHPM) process, and $H(f)$ is the high-pass of figure 3, then from (13),

$$\sigma_x{}^2(t)|_{t_0}^{\tau} = <\varepsilon^2(\tau)> = \frac{1}{2(\pi\nu_0)^2}\int_{\frac{1}{2\tau}}^{f_h} b_0 4\sin^2(\pi f\tau)df$$

$$\doteq \frac{(f_h - \frac{1}{2\tau})b_0}{(\pi\nu_0)^2}. \quad (14)$$

$$\text{Hence, jitter} = <\varepsilon^2(\tau)>^{\frac{1}{2}} = \sqrt{\frac{(f_h - \frac{1}{2\tau})b_0}{(\pi\nu_0)^2}}, \quad (15)$$

and is determined essentially by the square root of b_0 times upper cutoff frequency f_h (lower cutoff frequency $\frac{1}{2\tau}$ is a small contribution to the final value of jitter).

A high-frequency cutoff must be specified; however, it plays a significant contribution only for the first two rows, corresponding to WHPM and FLPM. With a spur or sine-wave modulation at frequency f_m, Table 1 includes its conversion to jitter. $\frac{V_{rms}}{K_d}$ of the last row is measured spur level divided by the PM measurement sensitivity. $\frac{V_{rms}}{K_d}$ is the usual peak height of the spur in dBc.

If the usual jitter analyzer is assumed using a first difference (see (2) and (5)), then only the first three noise types (WHPM, FLPM, and RWPM) can be used to calculate jitter. Table 1 shows the conversion to jitter of these three power-law noises. FLFM and RWFM do not converge under this definition of jitter, and jitter level is not calculable.

Jitter-2 using a second-difference approach (see (6) and (7)) can be calculated for all five noise types, including FLFM and RWFM. For spectra containing FLFM and RWFM, you must use Table 2 and must note the use of a different definition of jitter suggested in this writing, namely (6) and (7).

Table 2: Jitter-2 can be calculated for all five noises. If FLFM and RWFM are predominant noise types over any range of f, conventional jitter cannot be calculated. Use this table in these cases.

$S_\phi(f) = b_\beta f^\beta$	Jitter-2 *vs.* τ
b_0 (WHPM)	$\sqrt{\frac{3b_0}{(\pi\nu_0)^2}\left(f_h - \frac{1}{2\tau}\right)}$
$b_{-1}f^{-1}$ (FLPM)	$\sqrt{\frac{b_{-1}}{(\pi\nu_0)^2}}\times \sqrt{3ln(2\pi\tau f_h) - 3.11696 + \frac{3.4288}{(2\pi f_h\tau)^2}}$
$b_{-2}f^{-2}$ (RWPM)	$\sqrt{\frac{2b_{-2}}{(\pi\nu_0)^2}\left(3.50808\tau - \frac{3}{2f_h}\right)}$
$b_{-3}f^{-3}$ (FLFM)	$\sqrt{\frac{2b_{-3}}{\nu_0^2}\left(0.4256\tau^2 - \frac{3}{(2\pi f_h)^2}\right)}$
$b_{-4}f^{-4}$ (RWFM)	$\sqrt{\frac{13.034b_{-4}}{(\pi\nu_0)^2}\left(\tau^3 - \frac{1}{2f_h^3}\right)}$
Spur or *Sinusoid* with level V_{rms}	$2\sqrt{\frac{2V_{rms}}{K_d}}\cdot\left(\frac{sin^2(\pi f_m\tau)}{\pi\nu_0}\right)$
	$^*f_h$ is a high-freq. cutoff

The shape of near-carrier (low-frequency) PM noise is what determines clock jitter level for long averaging times. Likewise, the shape of far-from-carrier (high-frequency) PM noise is what determines clock jitter level for short averaging times. The delay is denoted by τ and the mapping from phase noise to jitter is illustrated by figure 5.

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