Proceedings of the 2003 IEEE International Frequency Control Symposium and PDA Exhibition
Jointly with the 17th European Frequency and Time Forum

LOW INSTABILITY, LOW PHASE-NOISE FEMTOSECOND OPTICAL FREQUENCY COMB MICROWAVE SYNTHESIZER*

T. M. Ramond[1], A. Bartels[1], S. A. Diddams[1], L. Hollberg[1], H. Kurz[2]

[1]National Institute of Standards and Technology, Boulder, CO 80305
[2]Institut für Halbleitertechnik, RWTH, Aachen, Germany

Abstract – **The ability of a frequency comb from a femtosecond mode-locked laser to faithfully reproduce the properties of optical oscillators at microwave frequencies is examined. The fractional frequency instability of the femtosecond comb microwave synthesizer is 2 x $10^{-14}/\tau$, and could improve by at least a factor of 10 upon elimination of excess photodetection noise. Phase-noise levels are also examined. The femtosecond comb frequency synthesizer is compared to other high-performance microwave oscillators and synthesizers and is found to be among the best available. With the realization of predicted improvements in phase-noise and stability for the femtosecond combs and optical frequency standards, the femtosecond comb could eventually synthesize the highest quality microwaves.**

Keywords – **Allan deviation, femtosecond comb, microwave synthesizer, optical clocks, phase-noise**

INTRODUCTION

Although the field of microwave atomic frequency standards is more established and mature, many research efforts have turned towards the development of optical atomic frequency standards. Optical oscillators offer important advantages over microwave oscillators. For example, because instability scales as $\Delta\nu/\nu$ where $\Delta\nu$ is the linewidth and ν is the oscillator frequency, the ~10^5 jump from microwave to optical frequencies should result in significantly lower instability. Also, very low noise optical oscillators have been developed for the purpose of probing the optical atomic transitions of the optical frequency standards. If the properties of the optical oscillators could be coherently and reliably transferred to other frequencies, these oscillators could prove valuable for a variety of scientific and technological applications.

The recent development of ultrabroad femtosecond optical combs based on mode-locked lasers has provided a relatively simple and straightforward way to translate optical frequency standards to other optical and to microwave frequencies. The Fourier transform of a femtosecond pulse train is a comb of evenly spaced frequencies, typically spanning some 10 % of the optical range. The frequency of individual femtosecond comb teeth, f_n, can be uniquely described by the relation $f_n = n f_r + f_o$, where n is a large integer (~ 450,000), f_r is the repetition rate of the laser, and f_o is the offset of the entire comb from harmonics of f_r [1]. Controlling two degrees of freedom of the comb precisely determines the frequency of each comb tooth f_n. Both f_r and f_o are microwave frequencies, whereas the comb teeth f_n are optical frequencies; therefore the femtosecond comb architecture provides a phase-coherent translation between the two frequency ranges (see Fig. 1). There are many different schemes possible to achieve stabilization of the comb, each allowing examination of different quantities. One specific use of the femtosecond comb synthesizer is that of a microwave synthesizer, which is expressed as the solid line in Fig. 1. In this case, the comb is stabilized to an optical oscillator and the microwave output of the comb appears as the repetition rate of the laser. Ideally, the optical oscillator input will be an optical frequency standard, and these have shown exemplary levels of stability at short averaging times (4 x 10^{-15} $\tau^{-1/2}$) [2], [3] and are probed by lasers with extremely low optical linewidths ($\Delta\nu \leq 0.2$ Hz at 532 THz) [4]. This paper addresses how well the femtosecond comb frequency synthesizer translates the desirable optical oscillator qualities to radio frequencies.

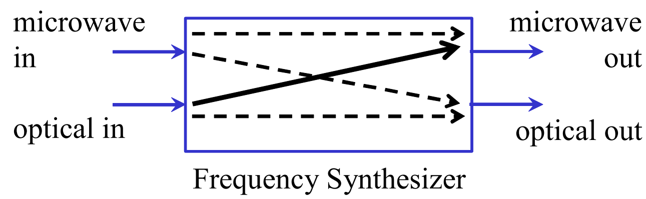

Fig. 1. Femtosecond optical frequency comb as a multi-purpose frequency synthesizer. Using optical and/or microwave inputs, the synthesizer generates both optical and microwave frequency combs as outputs.

EXPERIMENT

In this work we test both the stability and phase-noise of the repetition rate of the femtosecond comb frequency synthesizer when stabilized to an optical oscillator. Measurement of these quantities can be limited by the performance of the device against which they are compared (for example, a hydrogen maser) [5]. For this experiment, therefore, we tested the microwave output of our optical clock (i.e., the repetition rate) against the most stable and lowest phase-noise source readily available to us – the repetition rate of a second optical clock.

The details of the experiment are given elsewhere [6]; a brief summary is given here. The experiments compare the repetition rate output from two different optical clocks. The first is based on a mode-locked Ti: sapphire oscillator with repetition rate of 1 GHz whose output is spectrally broadened in a microstructure fiber [3], [7]. The second is another Ti: sapphire oscillator at 1 GHz that produces an ultrabroadband continuum directly, without any external broadening mechanism [5], [8]. In both cases, the offset frequency f_o was measured and phase-locked. To control the second degree of freedom of the comb, one of the comb teeth from each laser was heterodyned against the same stable optical continuous wave oscillator, a 657 nm single-frequency laser diode stabilized to a temperature-controlled high-finesse Fabry-Perot cavity. In this way, any fluctuation due to this optical oscillator will ideally be common-mode and therefore cancel in the final result.

We compared the repetition rate output of each of the optical clocks in two different ways. The first was an optical cross-correlation method where the pulse trains are overlapped in a 0.5 mm thick $LiIO_3$ crystal. The resulting sum frequency generated signal was then incident on a photomultiplier tube, filtered and amplified. In the second method, each of the individual pulse trains was incident on separate p-i-n photodiodes and then mixed electronically. In either method, the pulse trains were mixed to base band to record the phase-noise on a spectrum analyzer. To measure the fractional frequency instability, or Allan deviation, the two signals were mixed to $\Delta f_r \sim 10$ kHz and then sent to a counter.

OPTICAL VERSUS ELECTRONIC MIXING

The Allan deviation of the overlapped pulse trains examined through the two methods is displayed in Fig. 2. The data shown have been divided by √2 based on the assumption that both optical combs make the same contribution to the instability. The optical cross correlation yields an instability of pulse trains at 2×10^{-15} at 1 second averaging time τ, and averages down as τ^{-1} for $\tau > 1$ s. Furthermore, for $\tau \geq 1$ sec, the measurement is limited by the resolution of the counter (the dashed line in Fig. 2). In contrast, the Allan deviation of the electronic signal is 2×10^{-14} in 1 second. Although these data exhibit the same τ^{-1} slope as the optical cross-correlation experiment, the instability values show an order of magnitude increase. There is clearly some excess noise introduced into our electronic measurement, which will be discussed further below.

Fig. 3 displays the single side band phase-noise spectrum on the 1 GHz carrier from both the optical cross correlation and electronic mixing experiments. A prominent feature is a region of pronounced phase-noise in both plots, between roughly 0.1 to 1 kHz. This noise can be attributed to

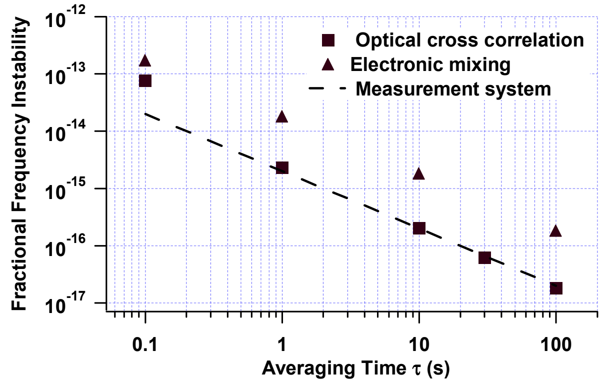

Fig. 2. Allan deviation of results from comparison of two femtosecond pulse trains in optical and electronic formats.

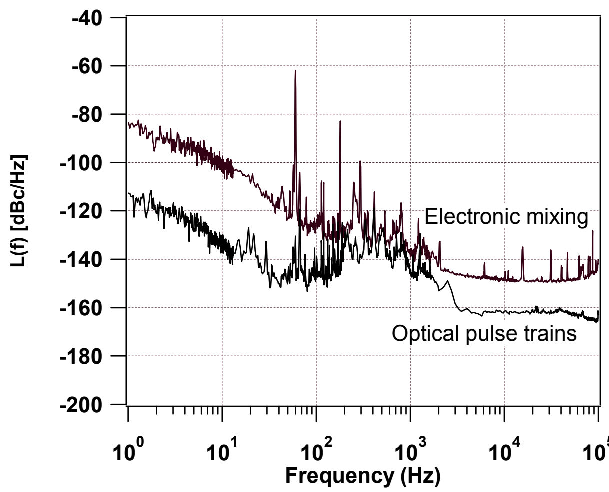

Fig. 3. Single side band phase-noise plot of optical cross correlation and electronic mixing results at 1 GHz.

acoustic sources, e.g. vibrations of mirror mounts and stands, that are expected to be eliminated with the appropriate optomechanical improvements. The Allan deviation data presented in Fig. 2 for an averaging time of 0.1 seconds show that the quality of the optical cross-correlation result is degraded from the τ^{-1} behavior followed for $\tau \geq 1$ s. Acoustic noise causes the optical pulse train phase-noise to rival the level recorded for the electronic mixing. It is possible that this effect is starting to degrade the optical cross-correlation instability results at shorter gate times. Furthermore, it is likely that the phase-noise of the optical pulse trains at frequencies above ~ 5 kHz is limited by the measurement system. The most notable feature of Fig. 3 is the increase in phase-noise by up to ~ 30 dB when going from optical to electronic mixing of the pulses. As was seen in the Allan deviation plot, detecting the optical clock signals before mixing adds significant noise to the result. Separate studies indicate that the chief source of noise in this experiment is the photodetectors themselves [9], including conversion of amplitude noise to phase-noise in the photodetector, and pointing instability combined with

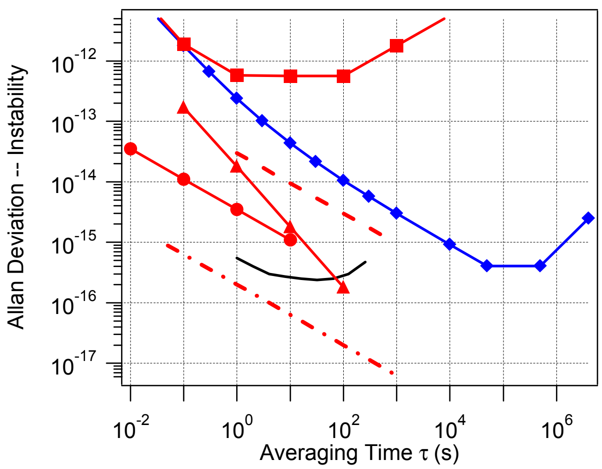

Fig. 4. Fractional frequency instability for the following sources: quartz oscillator (squares); hydrogen maser (diamonds); cesium fountain clock (dashed line); Ca optical standard (circles); cryogenically cooled sapphire oscillator cavity (solid line); Ca predicted performance (dashed dotted line); electronic output of femtosecond laser synthesizer (triangles).

limited temporal response and nonlinearity of the photodiodes. However, our attempts to control these factors were not sufficient to recover the difference in the outcomes shown in Fig. 3. Work is currently underway to understand and overcome this photodection noise.

COMPARISON TO OTHER MICROWAVE SOURCES

Although excess noise exists in the photodetection process, referencing one of our femtosecond lasers to an optical standard does nevertheless result in a 1 GHz electronic signal with an instability of 2×10^{-14} in 1 second. The following section is devoted to a discussion of how well this result compares to the performance of other microwave oscillators and synthesizers. In addition, we discuss to what extent the femtosecond comb limits stability and phase-noise transfer between the initial optical oscillator and the microwave frequency domain.

Fig. 4 shows the Allan deviation of our microwave signal mixing results—the best electronic stability results from our femtosecond comb frequency synthesizer—and present and predicted levels from the NIST ^{40}Ca 657 nm optical frequency standard [10]. The instability of the Ca optical standard of 4×10^{-15} has been measured [2], and we expect this value to average down as $\tau^{-1/2}$ [3]. The $^{199}Hg^{+}$ 532 nm optical frequency standard [4], [11] is based on a laser oscillator locked to a very high stability optical cavity that is steered to the ion resonance at τ = 10-30 s with a potential instability of $\sim 1 \times 10^{-15} \tau^{-1/2}$. With the Allan deviation of the instability of the femtosecond comb behaving as $2 \times 10^{-14} \tau^{-1}$, it will cross the Ca (red dots in Fig. 4) and Hg^{+} (not shown) instability curves at values of τ between 10 and 100 s. Therefore, our data confirm that the femtosecond comb

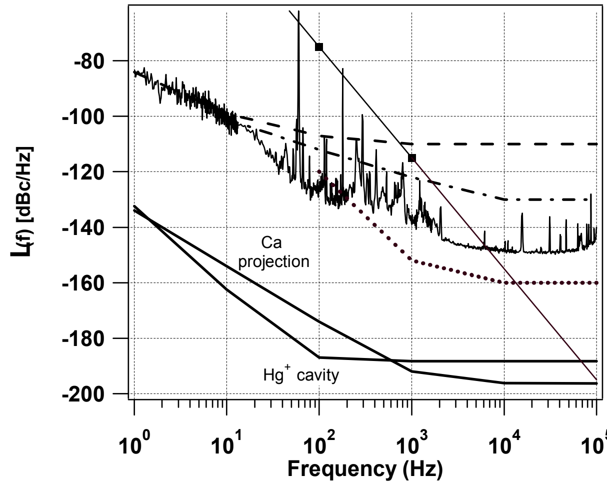

Fig. 5. Single side band phase-noise for synthesizers and oscillators at 1 GHz: low-noise quartz oscillator (dashed line); low-noise RF/microwave synthesizer residual noise (dashed dotted line); sapphire loaded cavity microwave oscillator (dotted line); femtosecond comb synthesizer (solid line). Also shown are the projection of the limit of the 657 nm Ca optical standard noise and that of the cavity of the 532 nm Hg^{+} optical standard. The phase-noise of a free-running femtosecond laser is shown by the squares plus solid line.

will not limit the stability of the present optical clocks after these relatively short averaging times.

Fig. 4 also shows the fractional frequency instability of several microwave oscillators and frequency standards. Here we see how the performance of optical frequency standards compared to that of most microwave oscillators is consistently better, particularly for short times. Extremely high stability microwave oscillators do exist in the form of cryogenically cooled sapphire oscillators (Fig. 4). Referencing a cesium fountain clock to this oscillator can reap some of the benefits of this low instability, and should allow operation of cesium fountain clocks with a large number of atoms at about $3 \times 10^{-14} \tau^{-1/2}$ [12].

It is worth noting that although Fig. 4 shows that the Cs fountain and H maser can reach the desirable instability levels in the 10^{-16} range, one must average for over 1000 seconds to get there. In contrast, instability levels of 10^{-16} for the Ca optical clock should be attainable well before a 100 second averaging time is reached, a remarkable savings in signal averaging.

In light of these results, the motivation to eliminate the excess noise in our photodetectors is clear. Moving the femtosecond comb instability down by an order of magnitude would give a Ca optical clock stability limited by the atoms for times of ~ 1 s and longer. Even when the performance of the ^{40}Ca standard improves and offers instability levels into the 10^{-16} range at 1 s (dashed dotted line in Fig. 4 for Ca, for example) [2], averaging times of only 100 seconds or better should be necessary to have the optical clock microwave output stability limited by the atoms instead of by the femtosecond laser: the femtosecond comb still would not prove a real limitation to the experiment.

Fig. 5 shows the single-side-band phase-noise spectrum L(f) of the femtosecond comb electronic mixing experiment from Fig. 3. Included for comparison are several high-quality microwave oscillators and synthesizers at 1 GHz carrier. The figure illustrates that the femtosecond comb synthesizer outperforms both the electronic synthesizer and the quartz oscillator for frequencies greater than 10 Hz, with a difference of up to 50 dB at the higher frequencies. In addition, the phase-noise of a free-running femtosecond laser is plotted in Fig. 5 [13]. This was measured down to approximately -130 dBc/Hz relative to the RF/microwave synthesizer given in Fig. 5. It varies as f^{-4}, indicating random walk FM noise. If we assume that the f^{-4} behavior continues past ~ 1 kHz, the laser does not have significant phase noise above a few kilohertz. This implies that residual noise in our femtosecond comb synthesizer originates from other sources such as measurement system noise.

As in the case of the Allan deviation, there is great incentive to eliminate the source of excess noise in the photodetection process. Reducing the phase-noise curve by 10 to 30 dB would make the femtosecond comb synthesizer competitive with or better than the sapphire-based oscillator systems for all frequencies shown in Fig. 5. Furthermore, the femtosecond comb synthesizer should improve such that the optical clock performance becomes limited by the projected optical standard noise shown in the lowest curves in Fig. 5, which is significantly better than the sapphire oscillator over the entire frequency range. If this is so, synthesis of microwave frequencies from optical atomic standards would provide the lowest phase noise microwave signals available.

CONCLUSION

The phase-noise and Allan deviation of a femtosecond comb microwave frequency synthesizer were measured. Fractional frequency instability of $2 \times 10^{-14} \tau^{-1}$ was attained. While an unresolved source of excess noise in the photodetection process exists, measurements suggest that this result can be improved by an order of magnitude or more. Comparison to other high-quality oscillators and synthesizers shows that the femtosecond comb is among the best microwave synthesizers. Furthermore, optical frequency standards have not yet realized their full potential with respect to phase-noise and stability. It is likely, therefore, that continued pursuit of this goal will eventually yield a femtosecond frequency comb referenced to an optical atomic standard as a microwave source of the highest quality available.

REFERENCES

[1] D. J. Jones, S. A. Diddams, J. K. Ranka, A. Stentz, R. S. Windeler, J. L. Hall, and S. T. Cundiff, "Carrier-envelope phase control of femtosecond mode-locked lasers and direct optical frequency synthesis," Science, vol. 288, pp. 635 – 639, April 2000.

[2] C. W. Oates, E. A. Curtis, and L. Hollberg, "Improved short-term stability of optical frequency standards: approaching 1 Hz in 1 s with the Ca standard at 657 nm," Opt. Lett., vol. 25, pp. 1603 – 1605, November 2000.

[3] S. A. Diddams, Th. Udem, J. C. Bergquist, E. A. Curtis, R. E. Drullinger, L. Hollberg, W. M. Itano, W. D. Lee, C. W. Oates, K. R. Vogel, and D. J. Wineland, "An Optical Clock Based on a Single Trapped $^{199}Hg^{+}$ Ion," Science, vol. 293, pp. 825 – 828, August 2001.

[4] B. C. Young, F. C. Cruz, W. M. Itano, and J. C. Bergquist, "Visible Lasers with Subhertz Linewidths," Phys. Rev. Lett., vol. 82, pp. 3799-3802, May 1999.

[5] T. M. Ramond, S. A. Diddams, L. Hollberg, and A. Bartels, "Phase-coherent link from optical to microwave frequencies by means of the broadband continuum from a 1-GHz Ti:sapphire femtosecond oscillator," Opt. Lett., vol. 27, pp. 1842 – 1844, October 2002.

[6] A. Bartels, S. A. Diddams, T. M. Ramond, and L. Hollberg, "Mode-locked laser pulse trains with subfemtosecond timing jitter synchronized to an optical reference oscillator," Opt. Lett., vol. 28, pp. 663-665, April 2003.

[7] J. K. Ranka, R. S. Windeler, and A. J. Stentz, "Visible continuum generation in air-silica microstructure optical fibers with anomalous dispersion at 800 nm," Opt. Lett., vol. 25, pp. 25 – 27, January, 2000.

[8] A. Bartels and H. Kurz, "Generation of a broadband continuum by a Ti:sapphire femtosecond oscillator with a 1-GHz repetition rate," Opt. Lett., vol. 27, pp. 1839 – 1841, October 2002.

[9] E. N. Ivanov, L. Hollberg, and S. A. Diddams, in *Proceedings of the 2002 IEEE International Frequency Control Symposium*, Institute of Electrical and Electronics Engineers, Piscataway, N.J., 2002.

[10] C.W. Oates, F. Bondu, R. W. Fox, and L. Hollberg, "A diode-laser optical frequency standard based on laser-cooled Ca atoms: Sub-kilohertz spectroscopy by optical shelving detection," Eur. Phys. J. D, vol. 7, pp. 449 – 460, 1999.

[11] R. J. Rafac, B. C. Young, J. A. Beall, W. M. Itano, D. J. Wineland, and J. C. Bergquist, "Sub-dekahertz ultraviolet spectroscopy of $^{199}Hg^{+}$," Phys. Rev. Lett., vol. 85, pp. 2462 – 2465, September 2000.

[12] F. Pereira Dos Santos, H. Marion, S. Bize, Y. Sortais, A. Clairon, and C. Salomon, "Controlling the cold collision shift in high precision atomic interferometry," Phys. Rev. Lett., vol. 89, pp. 233004-1 – 4, December 2002.

[13] E. N. Ivanov, S. A. Diddams, and L. Hollberg, "Experimental study of noise properties of a Ti:Sapphire femtosecond laser," IEEE T. Ultrason. Ferr., vol. 50, pp. 355 – 360, April 2003.