Analysis of Noise Mechanisms Limiting the Frequency Stability of Microwave Signals Generated With a Femtosecond Laser

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***Abstract*—Excess phase noise is observed in the spectrum of the microwave signal extracted from a photodetector illuminated by a train of ultrashort light pulses from the femtosecond laser. This noise affects the stability of frequency transfer from optical to microwave domains with the femtosecond laser. Some contributions to the excess phase noise are related to intrinsic beam-pointing fluctuations of the femtosecond laser and optical power fluctuations of the detected light. These factors contribute to excess phase noise at the harmonics of the pulse repetition rate due to power-to-phase conversion in the photodetector, spatially dependent time delays, and photodiode nonlinearities that distort the pulse shape. With spatial filtering of the laser beam and active control of its power, the additional fractional frequency fluctuations of pulse repetition rate associated with the excess noise of the photodetection process were reduced from $6 \cdot 10^{-14}$ to approximately $3 \cdot 10^{-15}$ over 1 s of averaging. The effects of other noise mechanisms, such as laser shot noise and phase noise introduced by a microwave amplifier, were also examined but were found to be at a less significant level.**

***Index Terms*—Femtosecond lasers, frequency stability, optical clocks, optical frequency metrology, phase noise.**

I. INTRODUCTION

HIGH repetition rate passively mode-locked femtosecond lasers are a key element of a coherent link between optical and microwave frequency domains [1]–[3]. Such a link has already enabled measurements of optical frequencies with statistical uncertainty near the limit of the current microwave standard—the Caesium fountain clock [4]. Using a mode-locked laser, it is also possible to transfer the frequency stability of an optical frequency standard to radio and microwave frequencies. These developments are an enabling technology for frequency standards based on optical transitions, which promise greatly improved stability and accuracy for atomic clocks of the future [5], [6].

The phase-coherent connection from optical to microwave frequencies is realized by referencing a femtosecond laser to an optical standard and extracting a microwave signal at one of the harmonics of the pulse repetition rate. Recent experiments at National Institute of Standards and Technology (NIST, Boulder, Colorado) indicate that the residual fractional frequency instability on the optical comb from a phase-locked femtosecond laser can be close to $6 \cdot 10^{-16}$ over 1 s of averaging [7], [8] and thus such lasers are able to support the best present optical standards. On the other hand, the direct measurements of the frequency stability of microwave signals synthesised with femtosecond lasers have failed to confirm the above results, indicating more than an order of magnitude worse noise performance [8]. The main reason for such a discrepancy is believed to be related to excess phase noise that arises with the photodetection of optical pulse trains and, thus, the conversion of a highly stable optical signal into an electrical signal. In this work, we discuss our present understanding of some of the basic noise mechanisms affecting the frequency stability of a 1-GHz microwave signal produced with a femtosecond laser. These include the laser shot noise, phase noise of the microwave amplifier, power-to-phase conversion in photodetectors, and laser beam-pointing fluctuations.

II. SYNTHESIS OF MICROWAVE SIGNALS WITH FEMTOSECOND LASERS

As was experimentally established [1], [3], [7], [9], the spectrum of a femtosecond laser consists of equidistant spectral lines: $f_n = nf_R + f_o$, where n is an integer on the order of 10^6, f_R and f_o are the pulse repetition rate and offset frequency, respectively [1]. A phase-coherent connection between optical and microwave domains is achieved by stabilizing frequencies f_R and f_o either with respect to a microwave standard, for example a hydrogen maser, or an optical frequency standard, based on cold atoms or ions. The latter approach was chosen in the construction of the optical frequency synthesiser at NIST [10].

A schematic diagram of the NIST optical frequency synthesiser is shown in Fig. 1. It is based on the Ti:sapphire mode-locked laser that emits a train of optical pulses of ~25-fs duration at the repetition rate $f_R \approx 1$ GHz. The spectrum of the optical pulse train is broadened to ~300 THz in a photonic microstructure fiber. Such a broadening occurs due to the self-phase modulation phenomena in a dispersion compensated microstructure fiber when optical pulses with the peak power density of hundreds of gigawatts per cm^2 propagate through its core. By broadening the optical comb to a whole octave, a signal at the offset frequency f_o is extracted. This is done by mixing a frequency-doubled infrared part of the optical comb spectrum

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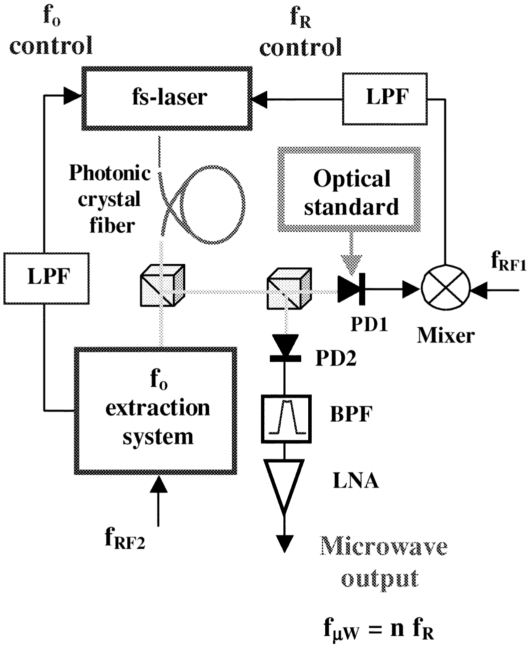

Fig. 1. NIST optical frequency synthesiser.

with its original green part. The extracted beat note f_o is phase locked to a radio-frequency oscillator (RF oscillator 2 in Fig. 1) by controlling the pump power of the femtosecond laser.

Another beat note is produced by the photodetector PD1 (Fig. 1). It results from the interference between the optical frequency standard and the closest spectral line of the femtosecond comb. The phase of the beat note is compared to that of the stable RF oscillator 1, and the resulting error signal is filtered, amplified, and fed back to PZT mirror mounts of the laser resonator to control the pulse repetition rate [11]. This closes the phase-locked loop (PLL) stabilizing the beat note frequency: $f_{\rm beat} = nf_R + f_o - f_{\rm opt} = f_{\rm RF1}$, where $f_{\rm opt}$ and $f_{\rm RF1}$ are frequencies of the optical standard and first RF oscillator, respectively.

With both frequencies ($f_{\rm beat}$ and f_o) stabilized, the optical frequency synthesiser can serve as a source of microwave signals with potentially the same fractional frequency stability as that of an optical standard. Such signals are produced at the harmonics of the pulse repetition rate and can be extracted either from the “internal” (PD1) or “external” (PD2) photodetectors. Below, we analyze the noise mechanisms affecting the frequency stability of the extracted microwave signals.

III. NOISE MECHANISMS AFFECTING THE STABILITY OF THE PULSE REPETITION RATE

In general, fluctuations of the pulse repetition rate of the optical frequency synthesizer in Fig. 1 can be expressed as

$$\delta f_R = \frac{\delta f_{\rm opt}}{n} + \delta f_R^{\rm error} \tag{1}$$

where $\delta f_{\rm opt}$ corresponds to frequency fluctuations of the optical standard and $\delta f_R^{\rm error}$ characterizes the combined effect of all other noise mechanisms. By assuming that both frequencies $f_{\rm beat}$ and f_o are tightly phase-locked to the respective RF oscillators, the error term is reduced to

$$\delta f_R^{\rm error} \approx \frac{1}{n}\left\{\delta f_{\rm RF1} + \delta f_{\rm beat}^{\rm PLL} + \delta f_{RF2} + \delta f_o^{\rm PLL}\right\} \tag{2}$$

where $\delta f_{\rm RF1,RF2}$ correspond to frequency fluctuations of the first and second RF oscillators, and $\delta f_{\rm beat}^{\rm PLL}$ and $\delta f_o^{\rm PLL}$ are the noise floors of the PLLs controlling $f_{\rm beat}$ and f_o. By utilizing (2), one can estimate the effect of phase fluctuations of each of the RF oscillators on the stability of the pulse repetition rate in the following manner:

$$\sigma_{RR} = \sigma_{\rm RF1}\frac{f_{\rm RF1}}{f_{\rm opt}} \tag{3}$$

where σ_{RR} and $\sigma_{\rm RF1}$ are Allan deviations of fractional fluctuations of the pulse repetition rate and RF oscillator 1, respectively. Assuming that $f_{\rm RF1} \approx 100$ MHz and $\sigma_{\rm RF1}(1s) \sim 10^{-13}$ (which is the instability of a good quality synthesizer referenced to a hydrogen maser as used in these experiments) results in a very low number: $\sigma_{RR}(1s) \sim 2\cdot 10^{-20}$. Thus, the RF oscillators and phase locks of Fig. 1 do not present a significant limit to the stability of the repetition rate. And, in fact, a quartz oscillator with $\sigma_{\rm RF1}\,(1s) \sim 10^{-10}$ would be more than adequate.

Secondly, we estimate the effect of laser shot noise on the frequency stability of the extracted signal. Assuming the shot noise contributes to the instability as white phase noise, and expressing it in terms of the Allan variance of fractional frequency fluctuations of pulse repetition rate, one can derive the following:

$$\sigma_y^{\rm shot}\,(\tau) \approx \frac{1}{2\pi\tau N f_R}\sqrt{\frac{3P_{\rm shot}}{P_{\rm signal}}} \tag{4}$$

where τ is the integration time and $P_{\rm signal}$ is the power of the extracted microwave signal at frequency Nf_R. Parameter $P_{\rm shot} = 2e\bar{I}\Delta f R$ is the shot-noise power, where e is the electron charge, $\bar{I}$ is an average photocurrent, Δf is the bandwidth of a filter used for the signal extraction, and R is the load impedance of a photodetector. For a typical high-speed photodetector (GaAs PIN photodiode Ortel PD050-OM[1]) we find $P_{\rm signal} \approx 0.3$ mW at $\bar{I} \approx 3$ mA and $f_R \approx 1$ GHz. By assuming that $R \approx 50\Omega$ and $\Delta f \approx 100$ kHz, we obtain $\sigma_y^{\rm shot}(1s) \approx 10^{-15}$, which can be further improved by decreasing the filter bandwidth or improving the photodiode performance.

Phase fluctuations introduced by the microwave amplifier (LNA in Fig. 1) in the path of the extracted signal were also studied. The resulting noise spectra measured at different levels of amplifier input power $P_{\rm inp}$ are shown in Fig. 2. They show the increase in the magnitude of phase fluctuations from the amplifier with input power resulting from the saturation of the amplifier. At $P_{\rm inp} \approx -15$ dBm the limit imposed by the amplifier phase fluctuations on the frequency stability of the extracted microwave signal was evaluated to be $\sigma_y\,(1s) \approx 3\cdot 10^{-16}$. This limit is almost two orders of magnitude less than that associated with the photodetection process (see below).

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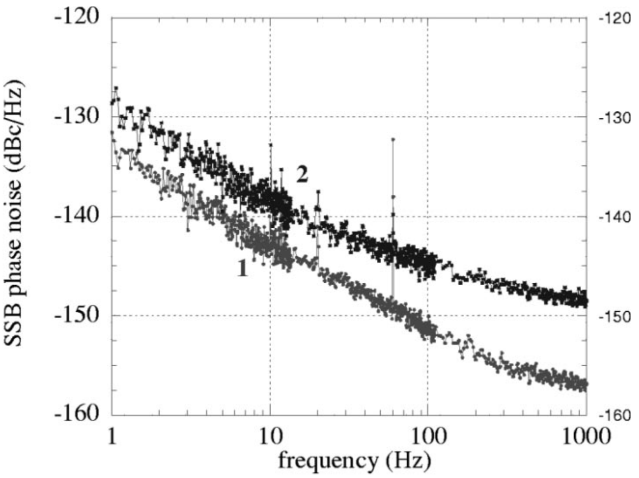

Fig. 2. Spectra of phase fluctuations of a 1-GHz bipolar transistor amplifier for two different input power levels: $P_{\rm inp} = -25$ dBm (curve 1), $P_{\rm inp} = -15$ dBm (curve 2). Noise spectra were measured at frequency 1 GHz a for a bipolar amplifier AM-1551 manufactured by Miteq.

The study of the noise properties of the photodetection system was initially performed in the time domain with the experimental setup shown in Fig. 3. Here, one of the photodetectors (PD1) is used for phase locking of the pulse repetition rate, while another (PD2) enables measurements of the excess noise associated with the photodetection process. Assuming that the pulse repetition rate is controlled by a high-gain PLL, phase fluctuations of signals extracted from PD1 and PD2 ($\delta\varphi_R^{(1)}$ and $\delta\varphi_R^{(2)}$, respectively) are given

$$\delta\varphi_R^{(1)} \approx \delta\varphi_{\rm osc} + \delta\varphi_{\rm PLL} \tag{5.1}$$

$$\delta\varphi_R^{(2)} \approx \delta\varphi_{\rm osc} + \delta\varphi_{\rm PLL} + \left(\delta\varphi_{\rm add}^{(2)} - \delta\varphi_{\rm add}^{(1)}\right) \tag{5.2}$$

where $\delta\varphi_{\rm osc}$ and $\delta\varphi_{\rm PLL}$ denote the phase fluctuations of the RF oscillator and intrinsic phase noise of the PLL. The last two terms in brackets correspond to additional fluctuations of pulse repetition rate introduced by the photodetectors. It should be noted that pulse repetition rate fluctuations of a free-running laser are not present in (5.1) and (5.2) due to the assumption of a high loop gain.

These measurements involve: offsetting the frequency of the extracted signal from photodiode 2, recombining it with the signal from the RF synthesiser in a mixer, and counting the frequency of the beat note. Following the above procedure, one can also measure the differential phase fluctuations between two extracted signals as shown in Fig. 3. In such a case, there is no need to stabilize the pulse repetition rate of the laser; its fluctuations being common for both arms of the measurement system do not contribute to the instability of the beat note, as long as the time delay between the tow arms is reasonably well balanced.

The fractional frequency resolution of the measurement system in Fig. 3 was measured to be near 10^{-15} over 1 s of integration time. This was achieved due to the use of a low-noise frequency shifter, as well as the application of the direct digital synthesiser (DDS) referenced to the hydrogen maser for generation of the offset frequency. The latter was chosen to be 10 kHz to minimize the triggering error of the frequency counter.

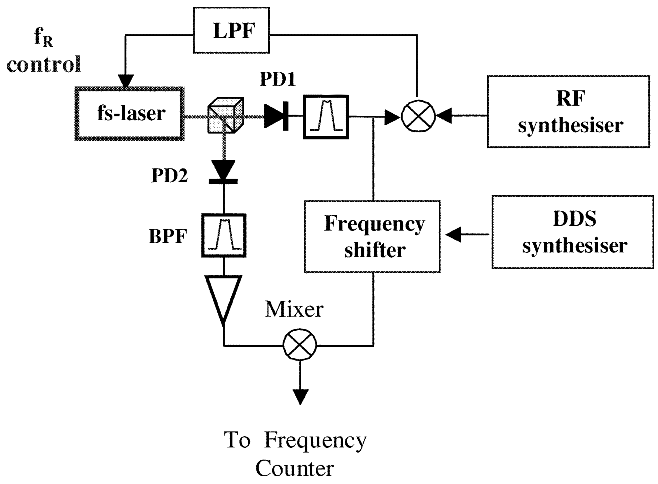

Fig. 3. Experimental setup for measurement of differential frequency fluctuations in time domain.

First, the differential frequency fluctuations between the 1-GHz signal extracted from the "internal" photodetector PD1 and the RF synthesizer were measured. These experiments did not detect any noise above the measurement floor for a range of integration times we employed (0.1–30 s). At the same time, comparison of the 1-GHz signals from PD2 and the RF synthesizer revealed a frequency fluctuation of 60 μHz in 1 s of averaging, indicating an additional noise associated with the photodetectors. Furthermore, differential frequency fluctuations between signals directly extracted from PD1 and PD2 were also measured (see Fig. 3), resulting in a frequency fluctuation close to 60 μHz in 1 s of averaging. In such a case, there was no need to stabilize the pulse repetition rate of the femtosecond laser. Its fluctuations being common for both arms of the measurement system did not contribute to the instability of the beat note. Stated another way, these experiments show that two photodiodes detecting what is apparently the same pulse train emitted by a femtosecond laser could show fluctuations in the measured frequencies of about six parts in 10^{14} in 1 s of averaging.

Measurements in the time domain were complemented by the measurements in frequency domain carried out with the experimental setup shown in Fig. 4. It enabled simultaneous measurements of spectral densities of residual phase fluctuations of two extracted signals relative to the master oscillator. During these measurements, the phase sensitive tuning of the readout system based on mixer 1 was maintained by the PLL. The phase sensitivity of the second measurement system based on mixer 2 was optimized with a variable phase shifter φ. In tuning the second measurement system one should remember that power fluctuations of the femtosecond laser may represent a problem when measuring the phase noise. For this reason, attention must be paid to reduction of the residual AM-sensitivity of the second measurement system. There are various approaches to this problem. First, we connected the photodetector PD2 to the local oscillator (LO) port of mixer 2. This takes advantage of the reduced sensitivity of the double-balanced mixer to amplitude fluctuations at its LO port. Secondly, an AM-modulated signal was generated with the RF synthesiser and an amplitude response of the second mixer was

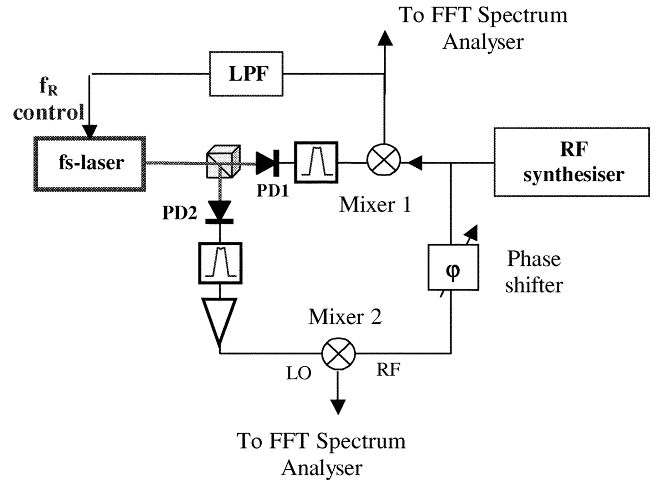

Fig. 4. Experimental setup for measurement of phase fluctuations of extracted microwave signals.

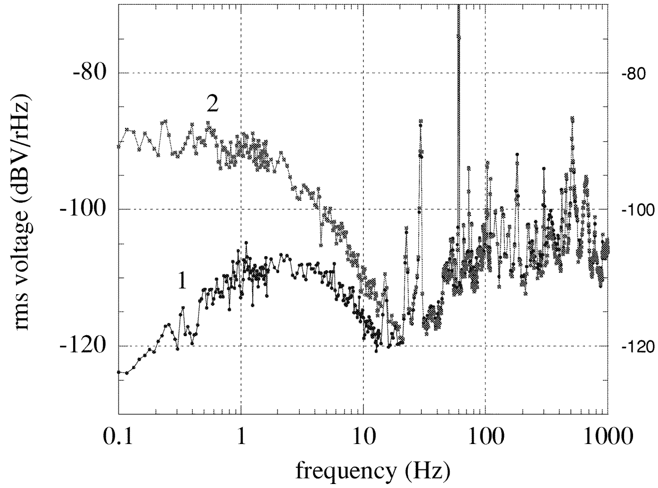

Fig. 5. Spectra of voltage fluctuations at the output of “internal” (curve 1) and “external” (curve 2) phase detectors.

cancelled by the fine tuning of the variable phase shifter φ. Typically, such tuning results in a slight degradation ($\sim$10%) of the phase sensitivity of the second measurement system.

Spectra of voltage fluctuations at the output of the phase noise measurement system are shown in Fig. 5. They indicate that the intensity of the low-frequency voltage noise at the output of the “external” mixer 2 (curve 2) is much higher than that at the output of the “internal” mixer 1 (curve 1). This confirms the time-domain observations regarding the excess noise associated with the photodetection process. In relation to the noise spectra in Fig. 5, it is worth noting that the divergence of two noise spectra at Fourier frequencies below 20 Hz does not mean that the bandwidth of the PLL is that low. At frequencies $f < 20$ Hz fluctuations of the pulse repetition rate are strongly suppressed, and the excess noise of the photodetectors becomes easily measurable. The bandwidth of the PLL used in these experiments was close to a few kilohertz, which was primarily limited by the frequency of the lowest mechanical resonance in the PZT mirror mount of the femtosecond laser [11].

To understand the origin of the additional phase noise we varied the magnitude of power fluctuations of the femtosecond laser. This was accomplished by stabilizing the laser power with an acousto-optical modulator (AOM) and altering the gain of the power servo. A schematic diagram of this experiment is shown in Fig. 6. To our initial surprise, we did not see any effect of the power control system on the additional phase noise. Also, we found that under some circumstances stabilizing the optical power in one arm of the optical readout system could raise the intensity of power fluctuations in another arm by a factor of two.

IV. DISCUSSION

A. Light Power Fluctuations

The “anomalous” behavior of the power control system described in the preceding paragraph suggests that power fluctuations of the extracted signals:

1) originate from the same source;
2) have similar magnitudes;
3) vary in opposite phases.

Power fluctuations alone do not satisfy the above description; they cause synchronous fluctuations of power in both arms of the optical readout system. Polarization fluctuations of the laser light have also been ruled out as a possible cause of the induced intensity noise. Insertion of a polarizer between the AOM and beamsplitter did not have any effect on the intensity of the output voltage noise (Fig. 6).

There are two reasons to consider the intrinsic beam pointing fluctuations of the femtosecond laser as being primarily responsible for the “anomalous” behavior of the power control system. First, due to the angular dependence of the reflection coefficient of the dielectric, angular fluctuations of the laser light incident on the beamsplitter are converted to power fluctuations. Secondly, such power fluctuations have opposite signs for the reflected and transmitted beams, as illustrated by Fig. 7.

In the general case, when both the power and direction of the light beam incident on the beam-splitter fluctuate, power fluctuations of the reflected and transmitted beams δP_1 and δP_2, respectively, are given by

$$\delta P_1 = R\delta P_{\text{inc}} + P_{\text{inc}}\dot{R}_\theta \delta\theta \tag{6.1}$$

$$\delta P_2 = (1-R)\,\delta P_{\text{inc}} - P_{\text{inc}}\dot{R}_\theta \delta\theta \tag{6.2}$$

where δP_{inc} denotes the power fluctuations of the laser light incident on the beam-splitter, $\delta\theta$ is the beam-pointing fluctuations, and R and $\dot{R}_\theta$ are the reflection coefficient and its derivative with respect to the angle of incidence.

When the power control loop from PD 2 in Fig. 6 is closed, light power fluctuations of the reflected and transmitted beams δP_1^{lock} and δP_2^{lock} become

$$\begin{aligned}\delta P_1^{\text{lock}} &= \frac{R\delta P_{\text{inc}}}{1+\gamma} \\ &\quad + P_{\text{inc}}\dot{R}_\theta \delta\theta \left(1 + \frac{R}{1-R}\frac{\gamma}{1+\gamma}\right)\end{aligned} \tag{7.1}$$

$$\begin{aligned}\delta P_2^{\text{lock}} &= \frac{(1-R)\,\delta P_{\text{inc}}}{1+\gamma} \\ &\quad + \frac{P_{\text{inc}}\dot{R}_\theta \delta\theta}{1+\gamma}\end{aligned} \tag{7.2}$$

where γ is the gain of the power control loop.

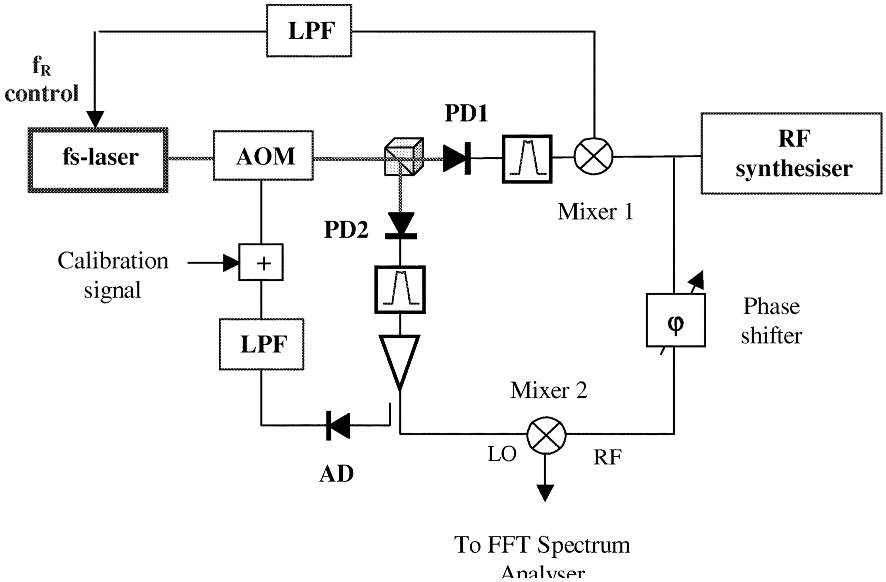

Fig. 6. Phase noise measurement system with an additional control system stabilizing average power of the optical comb.

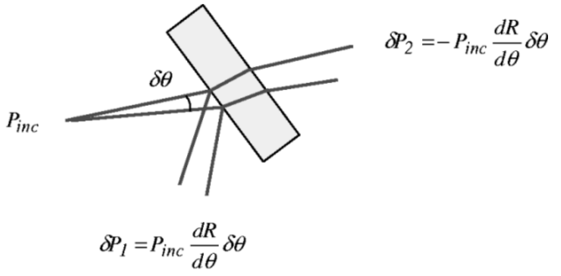

Fig. 7. Conversion of angle fluctuations into intensity fluctuations.

Assuming that $R \approx 1/2$ and $|\gamma| >> 1$, (7.1) and (7.2) are simplified

$$\delta P_1^{\text{lock}} \approx 2P_{\text{inc}}\dot{R}_\theta\delta\theta \quad (8.1)$$

$$\delta P_2^{\text{lock}} \approx 0. \quad (8.2)$$

On the other hand, from experimental observations it follows:

$$\delta P_1^{\text{lock}} \approx 2\delta P_1 \quad (9)$$

which means that

$$P_{\text{inc}}\dot{R}_\theta\,|\delta\theta| >> R\delta P_{\text{inc}} \quad (10)$$

and, therefore, (6.1) and (6.2) can be rewritten as

$$\delta P_1 = P_{\text{inc}}\dot{R}_\theta\delta\theta \quad (11.1)$$

$$\delta P_2 = -P_{\text{inc}}\dot{R}_\theta\delta\theta. \quad (11.2)$$

This result is consistent with the model that the intrinsic beam-pointing fluctuations of the femtosecond laser can be connected with the optical power fluctuations observed in our experiment.

If we assume that the beam-pointing fluctuations are the dominant source of the total power noise, one can calculate the standard deviation of the beam angle of the femtosecond laser σ_θ. Considering a beam splitter with the refractive index $n \approx 1.5$ and angle of incidence $\theta = 45\,\text{deg}$, this results in: $\sigma_\theta(1s) \approx 5\cdot 10^{-3}\,\text{deg}$. While this estimated level of beam pointing fluctuations is rather large for such solid state lasers, we note that this analysis does not account for phase noise that arises from the spatially dependent temporal response of the photodetectors, which can be a large effect. As might be expected, we have observed that phase delays in the extracted microwave signal depend on where the laser beam actually hits the photodetector. This is another way in which beam pointing fluctuations can again be coupled to phase fluctuations.

B. *Additional Fluctuations of the Pulse Repetition Rate*

Power fluctuations of the detected light induce additional phase noise in the spectra of the extracted microwave signals at the harmonics of pulse repetition rate. This happens due to the power-to-phase conversion in photodetectors. We first observed the power-to-phase conversion in time domain when studying the demodulation of femtosecond light pulses with an ultrafast oscilloscope. It manifests itself as a power-dependent time shift between the optical and the demodulated electrical pulses, as well as the power-dependent broadening of the electrical pulses. While the exact physical mechanisms remain to be identified, this may be due in part to saturation in the photodetector, where a buildup of a space charge in the depleted region affects the velocity of photogenerated carriers [13].

By introducing the power-to-phase conversions of the photodetectors PD1 and PD2 as $d\Phi_1/dP$ and $d\Phi_2/dP$, the additional phase noise of a signal extracted from the "external" photodetector PD2 (Fig. 4) can be obtained from (5.2)

$$\delta\varphi_{\text{add}} = \delta\varphi_{\text{add}}^{(2)} - \delta\varphi_{\text{add}}^{(1)} = \frac{d\Phi_2}{dP}\delta P_2 - \frac{d\Phi_1}{dP}\delta P_1. \quad (12)$$

This equation provides an explanation for the relative independence of the additional phase noise on the operation of the power stabilization system. Indeed, assuming that the optical power stabilization loop is disabled and substituting (11.1) in (12) yields

$$\delta\varphi_{\text{add}} = \left(\frac{d\Phi_1}{dP} + \frac{d\Phi_2}{dP}\right)P_{\text{inc}}\dot{R}_\theta\delta\theta. \quad (13)$$

On the other hand, when the power control loop is closed

$$\delta\varphi_{\text{add}} = 2\frac{d\Phi_1}{dP}P_{\text{inc}}\dot{R}_\theta\delta\theta. \quad (14)$$

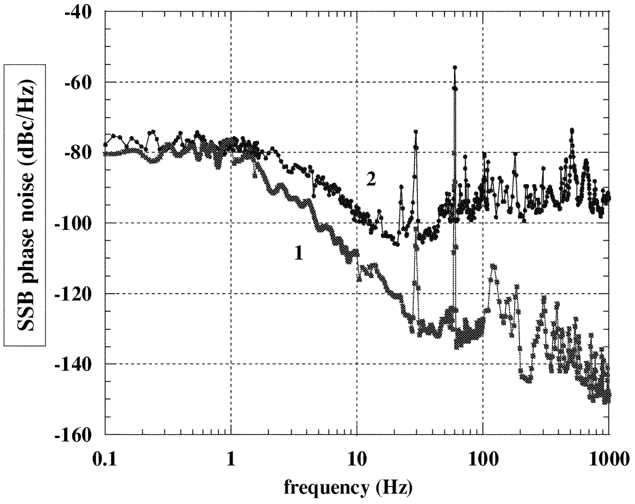

Fig. 8. Spectral density of the additional phase noise of a signal extracted from the "external" photodetector: inferred from the laser power fluctuations (curve 1) and measured directly (curve 2).

Therefore, the spectral density of additional phase noise is not affected by the power control system provided that photodetectors PD1 and PD2 have similar power-to-phase conversion efficiencies.

We measured the power-to-phase conversion for both GaAs and Si photodetectors with the experimental setup shown in Fig. 6. During these experiments, the power of the laser beam was modulated with the AOM and amplitude of the ac response of each mixer was measured. The modulation frequency was chosen to be higher than the bandwidth of the PLL stabilizing f_R in order to avoid the cancellation of the useful signal. Power-to-phase conversion was found to be a diminishing function of optical power. Typical values of power-to-phase conversion of 1.5 GHz bandwidth silicon PIN photodetectors (S-5973 from Hamamatsu[2]) were close to 35 and 15 rad/W at $P_{\rm inc} = 1$ mW and $P_{\rm inc} = 3$ mW, respectively, on the 1-GHz carrier frequency. At $P_{\rm inc} = 3$ mW, the broad-band GaAs PIN photodetectors (PD050-OM from Ortel[3]) had a power-to-phase conversion coefficient that was approximately ten times smaller.

To verify the above analysis, one can deduce the spectrum of the additional phase noise (from the measured intensity of light power fluctuations and power-to-phase conversion) and compare it with the results of direct measurements. We performed such a comparison with the experimental setup in Fig. 6. This time, the measurement system was tuned to be amplitude sensitive by swapping mixer ports (RF port of the mixer 2 was coupled to the output of the PD2) and adjusting the phase shift φ to maximize the mixer 2 dc voltage. The spectrum of the deduced phase noise is shown by curve 1 in Fig. 8. The directly measured spectrum of the additional phase fluctuations is given by curve 2. It is seen that two noise spectra are almost identical at

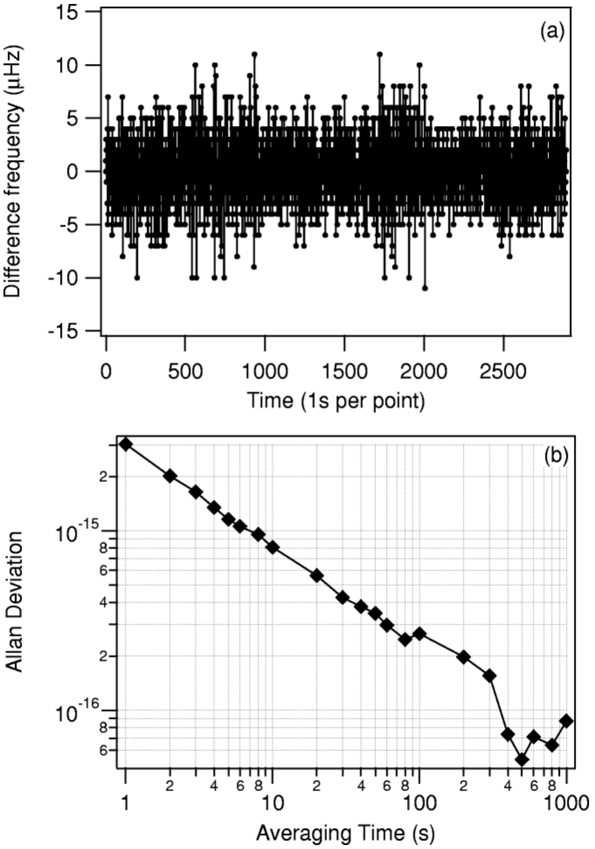

Fig. 9. (a) Difference in 1-GHz photodetector signals using the measurment system of Fig. 3 Spatial filtering and optical power control were employed to obtain these results. (b) Allan deviation of these measured fluctuations.

Fourier frequencies below a few hertz, which is consistent with the model regarding the origin of the additional phase noise. As mentioned above, the divergence of two noise spectra at higher frequencies is caused by the diminishing gain of the PLL and, therefore, increased contribution of pulse repetition rate fluctuations of a free-running femtosecond laser to the total phase noise.

Having identified some of the mechanisms of the additional phase noise, one can think of various methods of its minimization. One such method is a spatial filtering of the laser output. For this purpose, an AOM followed by a piece of a single-mode optical fiber were introduced in front of the beamsplitter (Fig. 3). In this way, beam pointing fluctuations are converted to power fluctuations at the output of the optical fiber, which are then cancelled by the power control system similar to that shown in Fig. 6. The results are given in Fig. 9, where it is seen that the fractional fluctuations of the difference frequency between the microwave signals extracted from two photodetectors can be as low as to be $3 \cdot 10^{-15}$ at $\tau = 1$ s [12], indicating more than an order of magnitude improvement in the stability of the extracted microwave signal. As can be seen by the discrete nature of the frequency excursions in Fig. 9(a), this result is near the resolution limit of the frequency counter.

V. CONCLUSION

Power and beam pointing fluctuations were found to be important sources of additional phase noise, observed when a mi-

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crowave signal is extracted from a phase-locked femtosecond laser. The effect of other noise mechanisms, such as laser shot noise, phase noise introduced by a microwave amplifier, and frequency fluctuations of the reference RF oscillator were also examined, but were found to be at a less significant level. Having introduced spatial filtering of the laser beam and active control of its power, we proved that two independent photodectors can measure the same repetition frequency of a femtosecond laser with uncertainty of $3 \cdot 10^{-15}$ over 1 s of averaging. While we view this as a necessary condition, it is not a sufficient condition to guarantee the extraction of a stable microwave signal from an optical clock at the level of $1 \cdot 10^{-15}$. A further investigation is required to demonstrate the above level of frequency stability in experiments with two separate femtosecond lasers referenced to either the same or different optical frequency standards. Taking into account the potential of high-performance optical clocks, it is critical to understand how fluctuations associated with the frequency transfer from optical to microwave domains can be reduced below the level of $1 \cdot 10^{-15}$.

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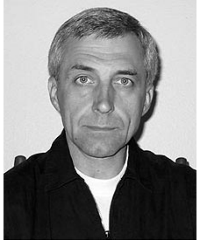

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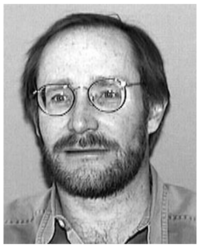

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