Measuring Clock Jitter at 100 GHz from PM Noise Measurements*

by

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1. Abstract

"Jitter" is the noise modulation due to random time shifts on an otherwise ideal, or perfectly on-time, signal transition. This paper presents ways of calculating timing jitter using phase-modulation (PM) and amplitude-modulation (AM) noise measurements of high-speed digital clocks. A 100 GHz case is used for illustration and based on actual measurements. In the absence of ultra-high speed jitter analyzers, spectrum analysis is an alternate noise measurement for timing jitter. A summary table is provided for mapping the results of these measurements in the Fourier frequency domain to jitter in the τ domain for various random (specifically, power-law) noise types, spurs, vibration, and power-supply ripple. In general, one cannot unambiguously map back, that is, translate from jitter measurements to phase noise. Measurements of phase noise are typically much more sensitive to phase (or time) fluctuations than a jitter analyzer.

2. Introduction and Summary

A widely used method of characterizing jitter is histogram statistics associated with a photograph of an "eye" pattern. While histograms are useful, near-instantaneous sampling of a 100 GHz clock transition implies the need for many hundreds of gigahertz of bandwidth in a jitter analyzer which are prone to several pitfalls associated with high-speed digital sampling, trigger errors, resolution, and time base distortions generally [1–4]. Secondly, a histogram misses an important piece of information for nonstationary kinds of noise, namely, how its width varies with delay, called τ in this writing. Does width get larger, smaller or stay the same *vs.* τ-delay? The primary motivation for this writing is that nonstationary noise is very likely and that a proper statistic must be used. For example, the fundamental measurement performed by most jitter analyzers presumes statistics based on a first-difference of time errors, which is not suitable for a full range of common noise types. In particular, when the first-difference operator yields a noise that is not white, for example, when devices convert one noise to another, there is a functional dependence on τ-delay that is unreliable but may be taken as truth. On the other hand, a second-difference operator shows a consistent τ-dependence that can actually help determine the cause of random noise.

I explain reasons, methods, and examples for calculating clock jitter *vs.* τ using measurements of the clock's phase-modulation (PM) and amplitude-modulation (AM) spectral noise which provide many more clues to the origin of clock jitter in general. This discussion derives from comprehensive work done in frequency standards, characterization of noise, and state-of-the-art methods of measuring time errors [5–9]. I describe the identification and possible cause of noise based on models of common power-law noises, spurs, vibration sensitivity, etc. I show how to map from PM noise to jitter and how to calculate jitter for a 100 GHz amplifier.

3. Jitter Definition

Clock jitter from a 100 GHz reference clock sets the baseline performance for those digital applications using that clock. A jitter analyzer is an oscilloscope that displays time-error noise after an arbitrary trigger time t at time $t+\tau$. The horizontal axis is running time (the "sweep" signal). Generically, the "transition" is that portion of an oscillating signal in the neighborhood of its zero-crossings or other defined crossings. Transitions are alternately positive-going and negative-going since the basic signal repeats like a sine wave. Negative-going transitions should occur in between two positive-going transitions, and vice-versa. The transition's timing error is subjectively measured with what is called an "eye" diagram that shows integer "half-period" transition errors. Here, a major goal of a good statistic is prediction of some parameter of interest based on past statistics. A jitter analyzer quantifies the statistical noise on a predic-

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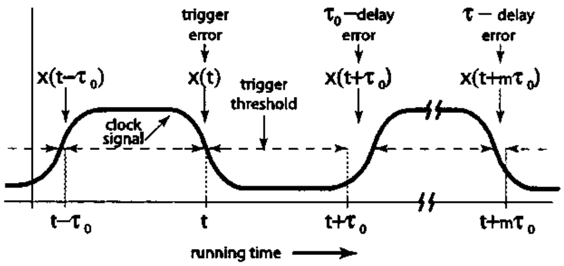

Figure 1: Sampling 'scope display. Jitter is the rms of time-delay errors, each given by $x(t+\tau)$, relative to a trigger-point whose single-shot error is $x(t)$. See eqn. (2) and (5) in text. To suppress the effects of phase drift and/or to measure "jitter walk," a second definition (eqn. (6) and (7)) is recommended.

tion that

$$\overbrace{\hat{x}(t+\tau)}^{\text{delay error}} = \overbrace{x(t)}^{\text{trig. error}} + \overbrace{\varepsilon(\tau)}^{\text{timing error}}, \qquad (1)$$

where $x(t)$ is a time-error at trigger-point t (often assumed to be 0), $\hat{x}(t+\tau)$ is the prediction of a future time-error viewed at time τ later, and $\varepsilon(\tau)$ is the observed difference between a zero-crossing or transition and a τ delay (subject to an instrument-related delay error) from the trigger-point claimed by the analyzer. Note that τ can only be a multiple of a minimum sample interval $\tau_0 = \frac{T}{2}$, the half-period of the clock signal itself. Figure 1 illustrates the transition errors with respect to t and τ. For example, if the trigger-point time-error $x(t) = 0$, and transitions occur every $\tau_0 =$ 1 ns in the clock signal (a half-period corresponding to a 500 MHz clock), and τ= 10 ns delay, we expect 0-error at $\hat{x}(t$+10 ns), but it is in fact perturbed by a noise burst, say, ε(10 ns)= +2 ps. Rewriting, we obtain

$$\underbrace{\varepsilon(\tau)}_{\text{timing error}} = \hat{x}(t+\tau) - x(t), \qquad (2)$$

whose form is called a "first-difference" in terms of $x(t)$ because $\varepsilon(\tau)$ is the difference of two time errors as shown in (2). Let $< \cdot >$ designate an average (more precisely, "expectation" if an infinite ensemble average is calculated). Then the mean-square (or variance) of $\varepsilon(\tau)$ is

$$<\varepsilon^2(\tau)> = <(\hat{x}(t+\tau) - x(t))^2>, \qquad (3)$$

$$\text{after squaring} \quad = <\hat{x}^2(t+\tau)> + <x^2(t)>, \quad (4)$$

or the sum of the delay-error variance plus trigger-error variance. (4) assumes $x(t)$'s are independent

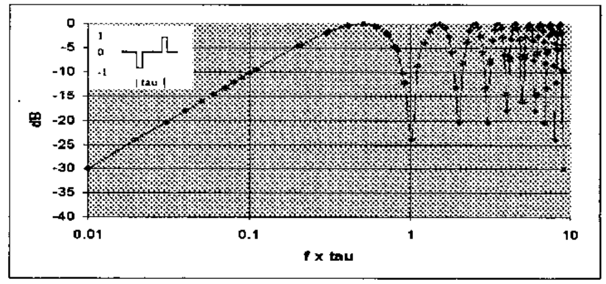

Figure 2: Frequency response $H(f)$ of a jitter analyzer with -1,+1 τ-spaced sampling coefficients, shown in the upper left. The transfer characteristic corresponds to a high-pass filter with a 20 dB/decade low-frequency skirt, which is only sufficient to measure jitter that does not drift or "walk" with a moving, non-stationary mean value. (This condition is seldom satisfied in real clocks.)

for given discrete or sampling intervals of t, which is the case for white PM noise only.

There are a variety of ways that jitter has been defined based on the sampling oscilloscope and its limitations [10–13]. For this discussion, many analyzers provide at least a root-mean-square (rms) of timing errors based on (2), or

$$\text{Jitter } (vs.\ \tau) \equiv <\varepsilon^2(\tau)>^{\frac{1}{2}}. \qquad (5)$$

This does not encompass a full model, but is sufficient for the goal of this writing of converting from PM noise to jitter as defined above and later in (7). In general, any given jitter measurement is essentially a broadband phase noise measurement, so it is possible to calculate jitter from a conventional narrowband phase noise measurement passed through an equivalent jitter analyzer "filter." A jitter analyzer can be regarded as measuring first differences of time deviations as a function of time-delay-from-trigger (function of τ). The equivalent filter in this case, the Fourier transform of the jitter analyzer's first-difference sampling function (given by -1 and +1 separated by τ as shown in (2)), turns out to be a high-pass filter, so high-cutoff frequency f_h directly affects the level of jitter for common white phase noise. The sampling function and equivalent frequency-response, or real part of the filter transfer function $H(f)$, are shown in figure 2.

For given τ interval, $\varepsilon^2(\tau)$ is inversely sensitive (inversely proportional) to low Fourier frequency PM

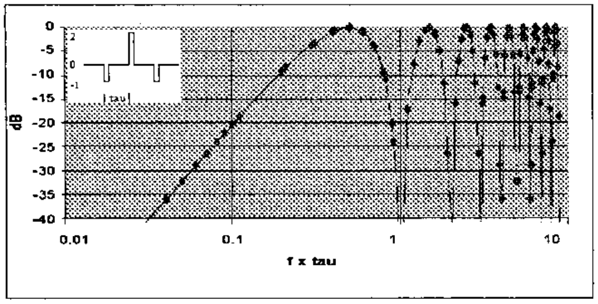

Figure 3: Frequency response $H(f)$ of jitter definition 2 of eqn. (6) and (7). This transfer characteristic corresponds to -1,+2,-1 τ-spaced sampling coefficients of eqn. (6), shown in the upper left. The steep 40 dB/decade low-frequency skirt is sufficient to measure jitter with a moving average carrier frequency (dubbed "jitter-walk") or phase drift.

noise starting at $f = \frac{1}{2\tau}$, and has sensitivity to high frequency noise regardless of f. The PM noise spectrum at high f is usually white (independent of offset frequency f), thus jitter depends on an upper cutoff frequency or bandwidth (BW) limit f_h.

Because of these problems and the fact that real clocks seldom satisfy the stationary criteria, it is preferable to define jitter as the "second-difference" of time-error measurements $x(t)$. The preferred definition is based on

$$\underbrace{\varepsilon_{2^{nd}}(\tau)}_{\text{timing error}} = -\hat{x}(t+\tau) + 2x(t) - x(t-\tau). \qquad (6)$$

$$\text{Jitter-2 } (vs.\ \tau) \quad \equiv \quad < \varepsilon^2_{2^{nd}}(\tau) >^{\frac{1}{2}} . \qquad (7)$$

This second-difference definition differs from (5) by the factor $\frac{1}{\sqrt{3}}$ for white PM but handles the problem of a non-stationary, moving average such as a phase drift or random walk behavior in $x(t+\tau)-x(t)$, appropriately dubbed a "jitter walk" behavior (see figure 3). As first pointed out by Barnes in [7] and revisited by Walls in [14], jitter-2 can be used as a measure of time dispersion and permits models of noise (mainly power-law noises) that extend to virtually any device or signal under test, that is, free-running oscillators, filters, multipliers and dividers, rf synthesizers, amplifiers, flip-flops, logic gates, etc. This is discussed in Section 9. Figure 3 shows the equivalent frequency response $H(f)$ of the -1,+2,-1 τ-spaced sampling coefficients of (6), shown in the upper left.

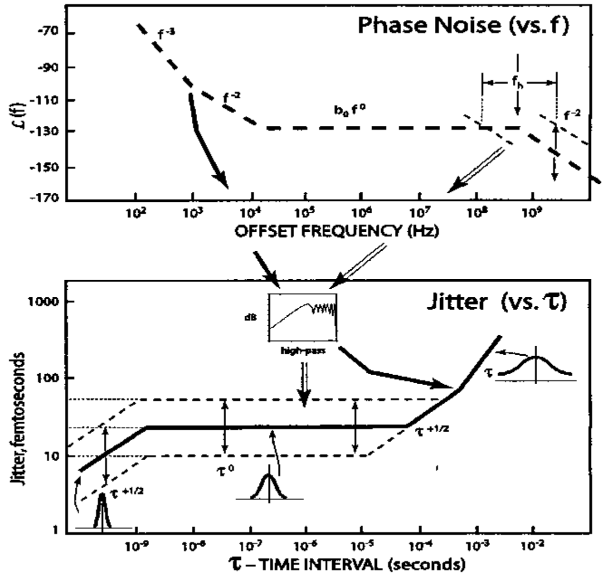

Figure 4: Mapping from phase noise to jitter. $L(f)$ passes through a high-pass filter (figure 3) to create a jitter *vs.* τ plot. The plot transposes $L(f)$, that is, high offset frequencies map to short-τ time intervals, and *vice versa*. Jitter histogram 1σ-width equals jitter level at a given delay, or τ-value.

4. Meaning of Clock Jitter

A clock reference is a repeating signal with period T that precisely defines the timing in synchronous digital systems by when clock transitions occur. This paper addresses the fundamental noise limit given by the clock reference noise and translates this to clock jitter. Jitter on data transitions (those that are not reference clock transitions) are not the subject of this paper. Data jitter contains the cumulative effects of noise from digital logic, digital modulation schemes, filters, component linearity, amplifiers, additive and multiplicative noise, crosstalk, etc., and cannot be easily calculated from PM noise measurement methods described here. For illustration, we will consider a 100 GHz clock that is used as a reference for data transitions. If the data jitter goal is, say, 100 fs, then I will show how a PM and AM noise measurement on the clock can be used to determine the 100 GHz jitter floor as if a jitter analyzer had measured the clock.

The measurement of jitter given by (5) or (7) will estimate the statistical standard deviation of clock-timing errors. The standard deviation is the 1σ histogram width regarded as the jitter level measured at integer half-period increments $m\frac{T}{2}, m = 1, 2, 3, \ldots$

5. Jitter as it relates to Spectral Purity

Note that we want to measure when a transition occurs in the neighborhood of an idealized on-time point. Noise occurs on an ideal fundamental frequency of $\frac{1}{T}$. The rf power spectrum is regarded as the ideal carrier plus "total baseband modulation power noise" due to PM + AM noise on an otherwise perfect carrier. An rf power spectrum measurement cannot distinguish PM from AM noise, so we often measure both independently, or just PM noise in cases where the AM noise contribution is considerably lower or not a concern. This noise spectrum appears above and below the frequency of the fundamental. The single-sideband power of the noise relative to carrier power is comprised of phase and amplitude spectral densities $S_\phi(f)$ and $S_{AM}(f)$. In particular, $S_\phi(f)$ is the power spectral density of phase fluctuations measured in a bandwidth of 1 Hz at a Fourier separation of f Hz. The units are $radians^2$/Hz. However, single-sideband PM noise $L(f)$ is *defined* as $\frac{1}{2}S_\phi(f)$. Typically, its expression in a log form is

$$L(f) = 10\,log\left(\frac{1}{2}S_\phi(f)\right),\ \text{in units of dBc/Hz.} \quad (8)$$

The shape of near-to-carrier (low-frequency) PM noise is what determines clock jitter level for long averaging times. Likewise, the shape of far-from-carrier (high-frequency) PM noise is what determines clock jitter level for short averaging times. The averaging time is denoted by τ and the mapping from phase noise to jitter is illustrated by figure 4, to be explained. AM noise is often significantly below PM noise in simple oscillators, while AM noise in amplitude-leveled synthesizers is often larger than the PM noise.

6. Basic Relationships

We start with the following basic relationships that are built from simple differentials. A white PM noise type becomes a differentiable function of time (a random-walk PM, explained in Section 9) when we consider the eventual high-frequency cutoff. This is illustrated by $f_h \approx 1$ GHz (or 10^9 Hz) at the top of figure 4. With this in mind, we can write:

$$\underbrace{\overbrace{\frac{x(t)}{T}}^{\text{period dev}}}_{\text{ideal period in sec}} = \underbrace{\overbrace{-\frac{\Delta\nu}{\nu_0}}^{\text{freq dev}}}_{\text{ideal freq in Hz}} = \underbrace{\overbrace{\frac{\Delta\phi}{2\pi\nu_0\tau}}^{\text{phase dev}}}_{\text{cycles in radians}} \quad (9)$$

For example, if $\frac{x(t)}{T}$ is $\frac{1}{100}$ or 1%, then $\frac{\Delta\nu}{\nu_0}$ is too, with a sign reversal, that is, if ν_0 is 100 GHz, then $\Delta\nu$ is -1 GHz. $\Delta\nu$ is the offset (also called Fourier) frequency designated by symbol f. In this example, say an offset of -1 GHz occurs at every half-period of an otherwise perfect 100 GHz clock. The output frequency becomes 99 GHz. Equivalently, a 50 fs constant time deviation occurs every 5 ps (half-period) interval. Now imagine that the 50 fs time deviation occurs only during one half period and reverts back to 0 error after the next half period. The average output frequency is 100 GHz with near-instantaneous power at offsets of $\pm$ 1 GHz. One point to be made is that small time deviations can spread power over a large range of offset frequencies.

An equally important point is that the smallest interval at which $\frac{\Delta\nu}{\nu_0}$ is measureable is an "instantaneous" frequency error. The smallest interval (called τ_0) corresponds to a high-cutoff frequency of $f_h = \frac{1}{2\tau_0}$, assuming proper filtering above f_h to avoid aliasing issues. If $f_h = 1$ GHz, then we cannot measure any time deviations in a shorter span than 500 ps. In other words, we cannot actually measure a half-period time deviation, given these parameters. However, we often can extrapolate jitter level to very small intervals based on the shape of associated PM noise. This kind of extrapolation is difficult to do with a jitter analyzer and motivates measurements of clock PM noise.

7. Phase Noise Measurements

Time fluctuations on zero-crossings or transitions are phase fluctuations, or phase noise, on a sine wave signal generator. Phase fluctuation spectral density is measured by passing such a signal through a phase comparator and measuring the detector's output power spectrum. A common technique is to use a loose phase-locked loop (PLL) as described in [5,6]. The measurement of $\phi(t)$ uses a phase-locked loop and makes use of the relation that for small deviations ($\delta\phi \ll 1$ radian) between the oscillator under test and a reference locked oscillator,

$$L(f) = \frac{1}{2}S_\phi(f) = \frac{1}{2}\left[\frac{V_{rms}(f)}{K_d}\right]^2 \quad (10)$$

where $V_{rms}(f)$ is measured on a spectrum analyzer as the root-mean-square noise voltage per $\sqrt{Hz}$ at Fourier Frequency "f" and K_d is the sensitivity (volts per radian) at the phase quadrature output of a phase detector which is comparing the test to a reference oscillator. Using Parceval's equality (a conservation principle) stating that total power over all time must equal total power over all frequencies, we can write:

$$\sigma_\xi{}^2(t) = \int_{-\infty}^{\infty} S_\xi(f)df, \tag{11}$$

and we can derive a useful formula for an actual data run of running-time phase deviations $\Delta\phi(t)$ as

$$\sigma^2_{\Delta\phi}(t)|_{t_0}^{\tau} = 2\int_{\frac{1}{2\tau}}^{f_h} L(f)[H(f)]^2 df, \tag{12}$$

where τ is a time interval in spacings of $m\frac{T}{2}, m = 1,2,3,...,$ $[H(f)]$ is our analysis filter transfer function discussed in Section 3, and $S_{\Delta\phi}(f) \cong 2L(f)$ for $\Delta\phi \ll 1$ [6]. Converting $\Delta\phi(t)$ to $x(t)$ by the basic relationships, the mean squared error $\sigma^2_{\Delta\phi}$ with respect to $x(t)$ is the finite-time variance version of (3) given by:

$$\sigma_x{}^2(t)|_{t_o}^{\tau} = \frac{1}{2(\pi\nu_0)^2}\int_{\frac{1}{2\tau}}^{f_h} L(f)[H(f)]^2 df = <\varepsilon^2(\tau)>, \tag{13}$$

and jitter is the usual square root, by (5).

To illustrate concepts, the examples to follow are white PM noise only; precise jitter-2 calculations from $L(f)$ in Table 1 differ slightly (see discussion starting with Section 9). If $L(f)$ is constant $= b_0$, a white pm (WHPM) process, and $H(f)$ is the high-pass of figure 2 or 3, then from (13):

$$\sigma_x{}^2(t)|_{t_0}^{\tau} = <\varepsilon^2(\tau)> \doteq \frac{(f_h - \frac{1}{2\tau})b_0}{2(\pi\nu_0)^2}. \tag{14}$$

$$\text{Hence, jitter} = <\varepsilon^2(\tau)>^{\frac{1}{2}} = \sqrt{\frac{(f_h - \frac{1}{2\tau})b_0}{2(\pi\nu_0)^2}}, \tag{15}$$

and is essentially determined by the square-root of b_0 times upper cutoff frequency f_h (lower cutoff frequency $\frac{1}{2\tau}$ is a small contribution to the final value of jitter).

Example: Consider a phase noise measurement that is used to compute jitter in which $\tau = 1$ ns and $b_0 = 10^{-13}$ corresponding to $L(f)$= -130 dBc/Hz. Using $f_h - \frac{1}{2\tau}$ =1 GHz, ν_0 =100 GHz,

$$\sigma_x^2|^{1ns} = \frac{10^9}{2(\pi 10^{11})^2}10^{-13} = \frac{10^{-13}10^{-13}}{2\pi^2} = \frac{10^{-26}}{2\pi^2}, \tag{16}$$

$$\text{and} \quad \sigma_x|^{1ns} = \frac{1}{\pi\sqrt{2}}\sqrt{10^{-26}} = \frac{10^{-13}}{\pi\sqrt{2}}, \tag{17}$$

or the clock jitter is 22.5 fs. If $f_h - \frac{1}{2\tau}$ quadrupled to 4 GHz, the jitter would double to 45 fs. For $L(f)$= -150 dBc/Hz, a 20dB lower level, the jitter would compute to ten times less, or 2.25 fs.

Remarks:

1. Jitter is an rms measurement (representing statistical standard deviation) of time deviation errors around an ideal, on-time signal transition assumed to have "zero" error. The sample mean is often used as the zero-error value.
2. Jitter doesn't depend on sample time interval τ for stationary noise processes such as WHPM. If noise is not stationary, we see that the value of jitter will vary depending on τ and when and how long a data run occurs. Most noise is not strictly stationary (see section 9 on Power-law Noise Processes).
3. Jitter is directly proportional to the square-root of phase noise level.
4. A constant jitter level maps from a constant phase noise level (a white PM noise) and is essentially proportional to square-root of high-cutoff f_h. A $\frac{1}{f}$ PM noise (flicker PM) also maps to a constant jitter level, but has a much smaller dependence on high-cutoff f_h, since $\frac{1}{f}$ can be regarded as a low-pass filter on a white PM noise source [15] .

8. Duty-cycle Amplitude Noise Measurements

The duty cycle of an equispaced train of pulses is the ratio of time-high to time-low. If a basic clocking rate is constant $= \frac{1}{T}$ and τ_{hi} is the desired "time high" interval, then duty cycle $= \frac{\tau_{hi}}{T}$. For a half-period jitter measurement, duty cycle is 0.5, or 50%. The rms departure from 50% is a measure of half-period jitter. From the basic equations above, this is equivalent to measuring rms phase instability and converting the answer to rms time error.

A precise way to measure duty cycle is by making an AM noise measurement. The ratio of fluctuations in the amplitude relative to the sample mean (average) amplitude is proportional to half-period jitter, assuming the CW (always-high) AM noise is small by comparison. $\Delta V(f)$ is proportional to $x(t)$ and V_{ave} = average duty cycle, so for $\frac{\tau_{hi}}{T} = 50\%$, half-period measurements, we can write:

$$\overbrace{\frac{\Delta V}{V_{ave}}}^{\text{volt dev}} = \overbrace{\frac{\Delta \phi}{\pi}}^{\text{phase dev}} = \overbrace{\frac{x(t)}{\tau_{ave}}}^{\text{time dev}} = 2\frac{x(t)}{T} = 2\nu_0 x(t), \tag{18}$$

$$\text{therefore,} \quad x(t) = \frac{1}{2\nu_0}\frac{\Delta V}{V_{ave}}, \tag{19}$$

for half-period jitter. Likewise, the AM noise process is stationary if the mean is constant (meaning that it is independent of when we measure).

An AM detector converts the amplitude of a signal to a voltage. The measurement of AM noise in a 1 Hz BW *via* a spectrum analyzer is the measurement of these voltage fluctations as a function of Fourier f ($\pm\frac{1}{2}$ hertz), that is, $S_{AM}(f) = \left(\frac{\Delta V(f)}{V_{ave}}\right)^2_{rms}$. So we can sum the contributions to variance of $x(t)$ from (19) to obtain

$$\int_{\frac{1}{2\tau}}^{f_h} S_{AM}(f)[H(f)]^2 df \doteq$$

$$(f_h - \frac{1}{2\tau})\left(\frac{\Delta V(f)}{V_{ave}}\right)^2_{rms} = (2\nu_0)^2 < \varepsilon^2(\tau) >, \tag{20}$$

where f_h, selected as a measurement-system high-cutoff frequency, is due to at least an analog filter that gets rid of the spurs caused by the transition or pulse repetition frequency (PRF), taken to be ν_0. A notch filter at the PRF is required with additional filtering for higher harmonics. Jitter measurements from duty-cycle AM measurements are simple when $\frac{1}{PRF} \ll \tau$, as is often the case, but they are not possible when $\frac{1}{PRF} \geq \tau$ because of the measurement's post-filter loss of sensitivity. As in the previous section, jitter is the square root of terms in (20).

Example: Consider a duty-cycle jitter calculation where ν_0 =100 GHz (T = 10 ps), τ = 1 ns and $BW = f_h - \frac{1}{2\tau}$ =1 GHz. If all the AM noise is due to duty-cycle fluctuations, say that we measure $S_{AM}(f = 1GHz) = \left(\frac{\Delta V(f)}{V_{ave}}\right)^2_{rms} = (10^{-15})$ corresponding to $L_{AM}(f = 1GHz)$ = -150dBc/Hz. One obtains jitter = 5 fs. If $L_{AM}(f = 1GHz)$ = -130dBc/Hz, the jitter calculates to 50 fs.

Remarks:

1. Jitter level is directly proportional to amplitude noise level if the amplitude noise is proportional only to duty cycle. However, AM noise that is not due strictly to duty cycle will cause the computation of jitter to be arbitrarily too high. The measurement can be used to set an upper limit on the true underlying jitter level.
2. Jitter is independent of averaging time τ if the noise processes is stationary. However, most noise is not strictly stationary (see next section).
3. As with WHPM noise, white AM (WHAM) noise is determined by upper cutoff frequency f_h; lower cutoff frequency $\frac{1}{2\tau}$ is a small contribution to total jitter. Therefore, jitter is essentially proportional to $\sqrt{f_h}$ for WHAM.

9. Power-law Noise Processes

Jitter-2 *vs.* τ can be used to distinguish a range of five common power-law, or $b_\beta f^\beta$, types of noise, where β is an integer exponent corresponding to five different slopes as shown in the log-log plots of figure 5. For commonly encountered high-speed digital clocks and oscillators, $S_\phi(f)$ is modeled by

$$S_\phi(f) = b_{-4}\frac{1}{f^4} + b_{-3}\frac{1}{f^3} + b_{-2}\frac{1}{f^2} + b_{-1}\frac{1}{f} + b_0 = \sum_{\beta=-4}^{0} b_\beta f^\beta, \tag{21}$$

where b_β are the levels of the noise types for slopes $\beta = 0, -1, -2, -3, -4$ identified respectively as White PM (WHPM), Flicker PM (FLPM), Random Walk PM (RWPM, also known as White FM (WHFM)), Random Run PM (RRPM, also known as Flicker FM (FLFM)), and Random Walk FM (RWFM). If phase variations become too large and $\Delta\phi \ll 1$ radian is violated, we can analyze the frequency variations associated with these large phase variations using the Allan or preferably the Total deviation [16]. It is possible to use Total deviation to derive an expression for

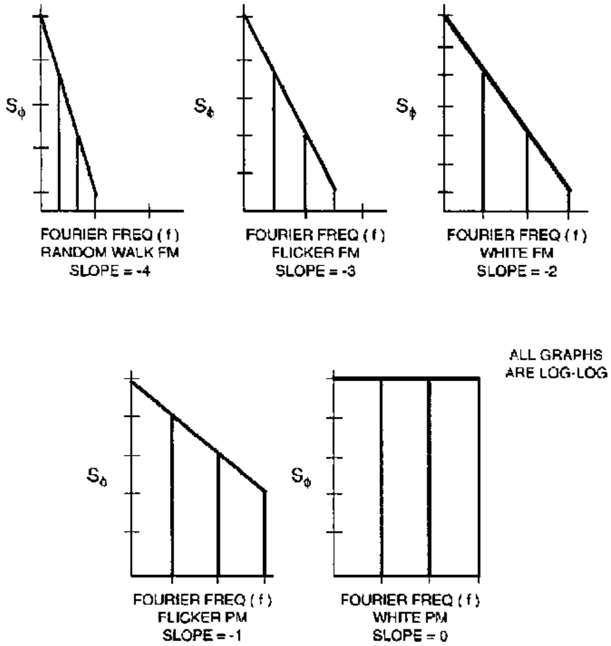

Figure 5: The five power-law noise processes create five different slopes on a phase noise plot (a log-log scale). β is the integer value of the slope corresponding to a specific model of noise.

jitter and jitter-2, which removes the deep nulls in the high-frequency passbands of figures 2 and 3.

Power-law noise originates primarily from the following causes [17]:

$\beta = 0$: White PM (f^0, constant *vs.* f) noise is broadband phase noise whose level is independent of f. It is basically the ratio of signal power to kT, or thermal, noise. This type of noise is common, even in signal sources of the highest quality, because in order to bring the signal amplitude up to a usable level, amplifiers must be used. STAGES OF AMPLIFICATION are often responsible for White PM noise. It is also found at the output of DIGITAL DIVIDERS and MULTIPLIERS and so affects the output of PHASE LOCKED LOOPS and FREQUENCY SYNTHESIZERS. White PM noise can be kept at a very low value with good amplifier design, hand-selected components, the addition of narrowband filtering at the output, or increasing, if feasible, the power of the frequency source itself.

$\beta = -1$: Flicker PM ($1/f$, -3 dB/oct., -10 dB/dec.) or "pink" phase noise may relate to a physical resonance mechanism in an oscillator, but it usually is added simply by NOISY ELECTRONICS or COMPONENTS, sometimes but not always in PHASE LOCKED LOOPS. Flicker PM noise exists in virtually all amplifiers and may be introduced in amplifier stages, frequency multipliers or synthesizers. Flicker PM can be reduced with good low-noise amplifier design (e.g., using carefully designed rf negative feedback) and hand-selecting transistors and other electronic components. The addition of NARROWBAND FILTERING at an output may reduce White PM, but increase Flicker PM. JITTER ANALYZERS CANNOT NORMALLY DISTINGUISH FLICKER PM FROM WHITE FM, WHILE A PHASE NOISE MEASUREMENT CAN.

$\beta = -2$: Random Walk PM (or equivalently called White FM) ($1/f^2$, -6 dB/oct., -20 dB/dec.) or "red" phase noise is very common, usually present as one approaches both the shortest possible averaging times and the longest. This is because the highest possible Fourier frequencies often exhibit a white PM noise and are filtered by at least the equivalent of a single-pole RC low-pass filter. A low-pass filter such as a coax cable, scope front end, rolloff in an amplifier, etc., converts a white PM process to a random walk PM process. Most oscillators do not exhibit this noise in the long term as it is masked by Random run PM $1/f^3$ and Random walk FM $1/f^4$. Random Walk PM is most often found in FREQUENCY LOCKED LOOPS such as in PASSIVE-RESONATOR FREQUENCY STANDARDS and is more commonly called White FM in this case. These standards contain a slave oscillator, often quartz, which is locked to a resonance feature of another device that behaves much like a high-Q filter. Cesium and rubidium standards exhibit Random Walk PM (or White FM) noise characteristics. For non-atomic oscillators at intermediate τ-values, white PM is integrated in the oscillating loop to make $1/f^2$ noise [18].

$\beta = -3$: Random Run PM (or Flicker FM) ($1/f^3$, -9 dB/oct., -30 dB/dec.) is a noise found in virtually all oscillators. This is because in an ACTIVE OSCILLATOR, an amplifer having Flicker PM creates the oscillating condition inside the BW of a high-Q resonator that converts the amplifier's PM noise to FM noise at the signal output through an integration. Physical cause of Random Run PM (or Flicker FM) is related to the PHYSICAL RESONANCE MECHANISM of an ACTIVE OSCILLATOR or the DESIGN OR CHOICE OF PARTS USED FOR THE ELECTRONICS, or ENVIRON-

MENTAL SENSITIVITY. Random Run PM is the fundamental close in PM noise of a crystal resonator oscillator. Although $1/f^3$ noise has not been found in cavity oscillators and bulk dielectric resonators, virtually all microwave oscillators have this noise due to basic $1/f$ PM noise in the sustaining amplifier and the action of the oscillating loop [19, 20]. Random Run PM, while common in high-quality oscillators, may be masked by Random Walk PM ($1/f^2$) or Flicker PM ($1/f$) in lower-quality oscillators.

$\beta = -4$: Random Walk FM ($1/f^4$, -12 dB/oct., -40 dB/dec.) noise is difficult to measure since it is usually very close to the carrier. Random walk FM is usually observed very close to the carrier and linked to an OSCILLATOR'S PHYSICAL ENVIRONMENT. It is convenient to measure Random Walk FM using time-series analysis [5, 6]. Sensitivity to MECHANICAL SHOCK, VIBRATION, TEMPERATURE, HUMIDITY, or other environmental effects may be causing "random" shifts in the frequency of an oscillating signal. It is common practice to remove FREQUENCY DRIFT from oscillator data. Unfortunately, Random Walk FM is often mistaken for FREQUENCY DRIFT when data runs are not long enough to observe upper and lower frequency limits on Random Walk FM behavior.

10. Spurs and Sinusoidal Modulation

Jitter measurements *per se* do not readily distinguish the effects of spurs and sensitivity to vibration and power-supply ripple. For these, conventional narrow-band phase noise measurements using a phase detection scheme and spectrum analyzer are superior for quickly identifying these noise sources. In general, measurements of phase noise reveal substantially more than jitter measurements.

Spurs: A commonly encountered noise that is not the power-law type is given by the presence of 60 Hz AC line noise usually caused by induced AC getting into the measurement system or the source under test. In the plot of $L(f)$, one observes discrete line spectra at at least 60, 120, and 180 Hz. Since $L(f)$ is a measure of spectral density, one should use a straight measurement of amplitude, not normalized by an analyzer's resolution bandwidth, and obtain V_{rms} of the largest spectral line to map to jitter.

Vibration: One might see vibration and acoustic sensitivity in the signal source with the device under vibration. Again, one should obtain V_{rms} of the largest spectral line to map to jitter.

11. Mapping Phase Noise to Jitter

Table 1 shows the conversion to jitter of power-law noise measured by conventional narrow-band PM noise spectrum analysis, that is, $L(f)$. $L(f)$ noise type and level are determined by a slope β and amplitude b_β using the left column which map to a slope and amplitude for jitter *vs.* τ on the right. A high-frequency cutoff must be specified for the first two rows (corresponding to WHPM and FLPM).

Table 1: Jitter-2 is calculated from $L(f)$ noise type and level, which are determined by a slope β and amplitude b_β. $\frac{V_{rms}}{K_d}$ of the last row is spur level divided by the PM measurement sensitivity K_d.

$L(f) = b_\beta f^\beta$	Jitter-2 *vs.* τ
b_0 (WHPM)	$\sqrt{\frac{3f_h}{(2\pi)^2\nu_0^2}\cdot b_0}$
$b_{-1}f^{-1}$ (FLPM)	$\sqrt{\frac{1.038+3ln(2\pi\tau f_h)}{(2\pi)^2\nu_0^2}\cdot b_{-1}}$
$b_{-2}f^{-2}$ (RWPM)	$\sqrt{\tau}\cdot\sqrt{\frac{1}{2\nu_0^2}\cdot b_{-2}}$
$b_{-3}f^{-3}$ (FLFM)	$\tau\cdot\sqrt{\frac{2ln2}{\nu_0^2}\cdot b_{-3}}$
$b_{-4}f^{-4}$ (RWFM)	$\tau^{\frac{3}{2}}\cdot\sqrt{\frac{(2\pi)^2}{6\nu_0^2}\cdot b_{-4}}$
Spur or *Sinusoid* with level V_{rms}	$\frac{V_{rms}}{K_d}\cdot\left(\frac{(sin\pi f_m\tau)^2}{\pi}\right)$

*f_h is a high-freq. cutoff, $2\pi f_h\tau \gg 1$.

If the usual jitter analyzer is assumed using a first-difference (see (2) and (5)), then only the first three rows (WHPM, FLPM, and RWPM) can be used to calculate jitter. FLFM and RWFM (the bottom two rows) do not converge under this definition of jitter, and jitter level is not calculable. Jitter-2 using a second-difference approach (see (6) and (7)) can be calculated for all five noise types. For spectra containing spurs or sine-wave modulation at frequency f_m, Table 1 includes its conversion to jitter. $\frac{V_{rms}}{K_d}$ of the last row is spur level divided by the PM measurement sensitivity.

12. Data for a 100 GHz Amplifier

$L(f)$ of a 100 GHz amplifier for use with a reference clock signal is shown in figure 6. The measurement was configured in such a way that the noise of the oscillator itself cancelled to a high degree, leaving the noise of a +10 dB gain InP amplifier which we wished to measure. This is done using a pair

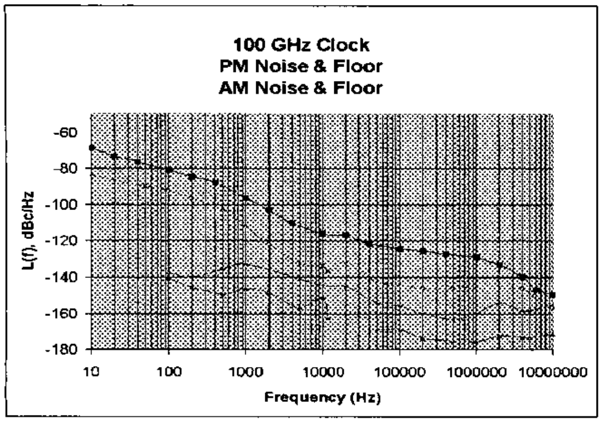

Figure 6: The top plot is the PM noise of a InP +10 dB gain amplifier operating at 100 GHz. AM noise and floor (lower plots) represent insignificant contributions to total noise. $\frac{1}{f^2}$ (RWPM) noise is observed from 10 to 10 000 Hz and again from 1 to 10 MHz. The middle region approximates a $\frac{1}{f}$ (FLPM) noise.

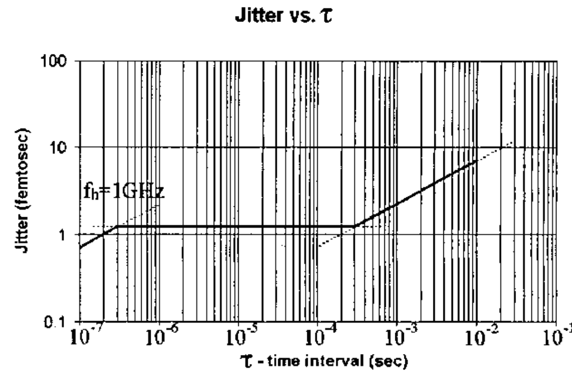

Figure 7: Using Table 1, we calculate jitter *vs.* τ as shown above for the three power-law noise regions in the noise measurement data of figure 6. An offset high-cutoff $f_h = 1$ GHz was used.

of phase-sensitive detectors operating simultaneously, one measuring the amplifier + source oscillator, and the other measuring just the source oscillator. Thus, $L(f)$ is the noise of the amplifier whose input would be regarded as from a perfect "noiseless" 100 GHz reference clock. The lower three plots of figure 6 in descending order are PM and AM measurements for (1) the amplifier removed, indicating the PM measurement noise floor, (2) AM noise of the amplifier, and (3) amplifier-removed AM measurement noise floor.

Figure 7 shows the conversion of $L(f)$ shown in figure 6 with center frequency $\nu_0 = 100$ GHz and with an assumed measurement-system (or maximum offset) high-cutoff frequency of $f_h = 1$ GHz to be consistent with earlier examples. There are three segments to $L(f)$ corresponding to f^{-2} from 10 to 10 000 Hz, f^{-1} from 10 kHz to 1 MHz, and again f^{-2} from 1 to 10 MHz. There are no regions of f^0, or WHPM noise. Therefore, we need to use only rows two and three from Table 1 to map these three ranges of offset frequency f to jitter *vs.* τ as plotted in figure 7.

Results show that the use of the amplifier with an ideal reference source would set a lower limit on clock jitter of between 1 and 10 fs for τ delays from 200 ns to 20 ms (see figure 7). This performance is commendable and better than the minimum jitter level needed for the signal-processing application involving this amplifier.

13. References

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