Interpreting AM and PM Noise Measurements in Two-channel Interferometric Measurement Systems

E. N. Ivanov* and F. L. Walls**

*Frequency Standards and Metrology Group, Physics Department, The University of Western Australia, Nedlands, 6907, WA

**Time and Frequency Division, National Institute of Standards and Technology, 325 Broadway, Boulder, CO 80303

ABSTRACT

In this work we show experimentally and theoretically that cross-correlation analysis of the output of a two-channel noise measurement system (NMS) yields the differential PM and AM noise of one input relative to the reference port. We also show that the statistical uncertainty, which sets the ultimate spectral resolution in the thermal-noise-limited regime, is approximately the same for both single- and two-channel NMS.

1. INTRODUCTION

In this paper we examine two-channel interferometer noise measurement systems (NMS) that use cross-correlation to measure PM or AM noise in a device under test (DUT). We use a simplification of the entire measurement system to facilitate the measurements at X-band. By changing the noise temperature of the two inputs we show that

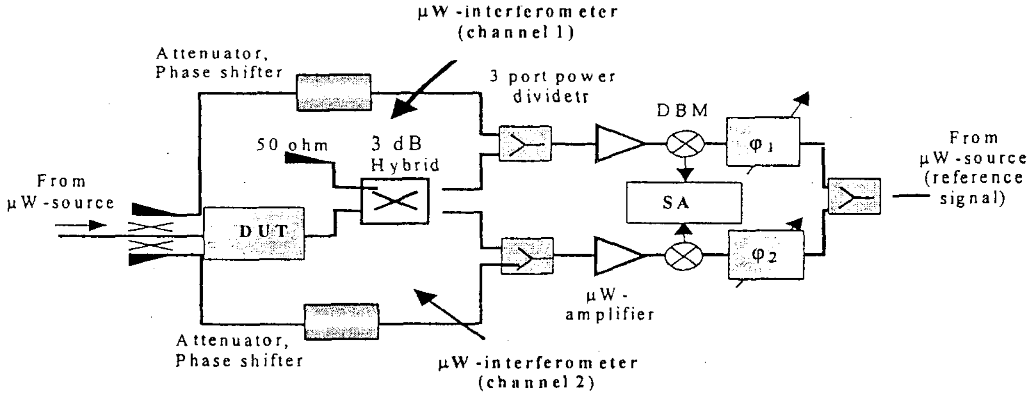

Figure 1a. Two-channel interferometric noise measurement system (NMS). DUT is the device under test, SA is a spectrum analyzer, DBM is a double-balanced mixer, and $\varphi_{1,2}$ are phase shifters.

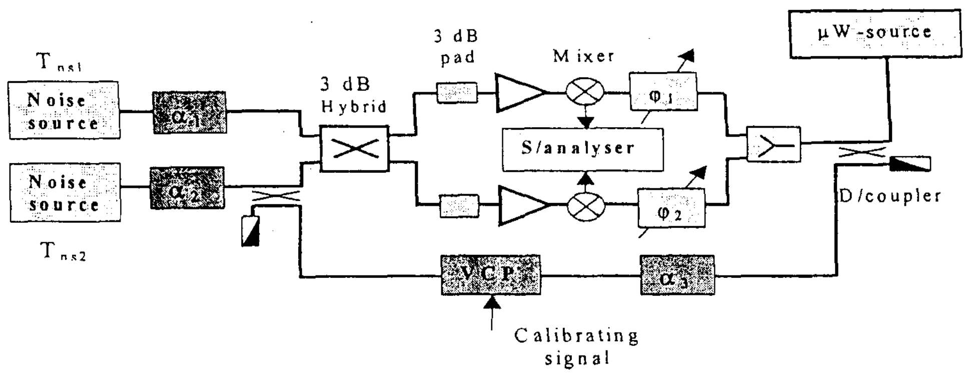

Figure 1b. Experimental setup for studying the basic properties of cross-correlation noise measurement systems. VCP is a voltage controlled phase shifter.

this approach measures only the difference in input noise between the two inputs. We show that the statistical uncertainty is approximately the same for single- and two-channel systems. Practical considerations, however, indicate that the two-channel systems will provide better confidence for the measurement when the noise added by the DUT is near or below the ambient thermal noise in the reference arm of the interferometer.

2. EXPERIMENTAL STUDY OF THERMAL NOISE CANCELLATION EFFECTS IN TWO-CHANNEL NOISE MEASUREMENT SYSTEMS

Figure 1a shows a two-channel NMS. Figure 1b shows the readout portion (low-noise homodyne down-converter) of this system that was used to study the noise processes in a two-channel NMS [1-5]. Two 3 dB pads simulate the loss in the power combiners used in the double interferometer for carrier suppression. The experimental setup also contains two microwave amplifiers and two double balanced mixers. Phase shifters φ_1 and φ_2 are used for optimizing the sensitivities of both channels with respect to either phase or amplitude variations of the incoming signal. The latter is derived from the same source, which drives the mixer's local oscillator (LO) ports and enables the calibration of the readout system. Two noise sources with effective temperatures T_{ns1} and T_{ns2} are attached to the inputs of the measurement system (ports 1 and 2 of 3 dB hybrid, respectively). Variable attenuators α_1 and α_2 allow the effective temperatures of the input noise, T_{inp1} and T_{inp2}, to be varied from the highest values of T_{ns1} and T_{ns2} to the level of ambient temperature T_o. For example,

$$T_{inp1} = T_{ns1}\,\alpha_1 + T_o(1-\alpha_1). \qquad (1)$$

Variable attenuator α_3 in the path of the calibration signal was used to set the level of the carrier at the inputs of the microwave amplifiers to approximately -47 dB below 1 mW (-47 dBm), which was a typical operating condition. To simplify the data analysis, the sensitivities of both channels of the NMS were made approximately equal. This was achieved by adjusting the gain of low noise amplifier G_{LNA}, in one of the channels.

The spectral densities of various voltage fluctuations measured with the above setup as a function of the intensity of the input noise are shown in Fig. 2. These data were collected at a Fourier frequency of 35 kHz, which was chosen to minimize the effect of microwave amplifier flicker noise on the voltage noise floor of the NMS (see noise spectra in Fig. 3). The error bars in Fig. 2 show the statistical uncertainty in the three measurements, (see discussion below).

The left-hand vertical scale shows the output in dB below 1 V in a 1 Hz bandwidth (dBV/Hz). The right-hand vertical scale shows the equivalent temperature.

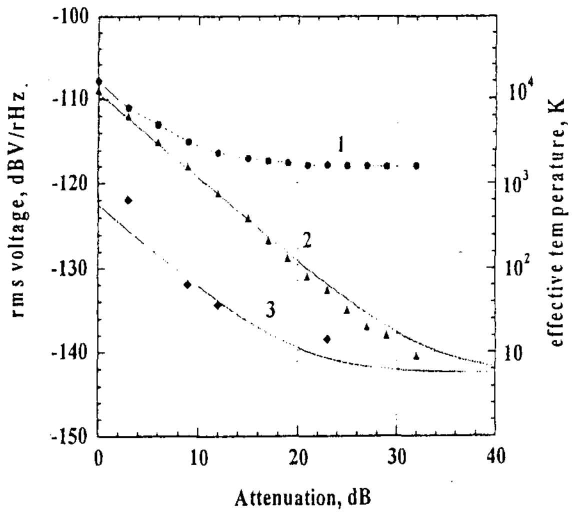

Figure 2. Voltage fluctuations (or equivalent temperature) at the output of a PM or AM NMS as a function of intensity of the input noise T_{inp1} at a carrier frequency of 9 GHz and a Fourier frequency of 35 kHz. Curve 1 is the single-channel voltage noise spectral density for $T_{inp2} = T_o$. Curve 2 is the cross-channel voltage noise spectral density for $T_{inp2} = T_o$. Curve 3 is the cross-channel voltage noise spectral density when T_{inp2} is *increased* from T_o to T_{inp1}.

Curve 1 in Fig. 2 shows the dependence of the rms voltage noise at the output of a single-channel NMS on attenuation α_1 with the temperature of port 2 $T_{inp2} = T_o$ (attenuation α_2 is set to a maximum). From these data we see that the noise floor of the single-channel NMS expressed in temperature units was approximately 1300 K.

Curve 2 shows the cross-channel rms voltage as a function of α_1. It was acquired when both channels were identically tuned (phase sensitivities were maximized) and $T_{inp2} = T_o$. These data indicate that, to within the experimental error, the cross-spectral density of output voltage

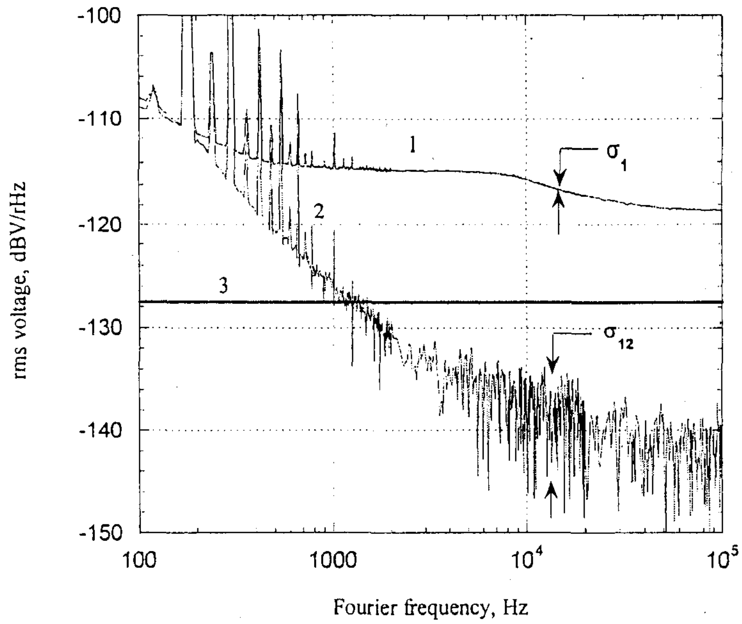

Figure 3. Curve 1 is the measured rms voltage noise floor of a single-channel NMS, curve 2 is the measured rms voltage noise floor of a two-channel NMS, and horizontal line 3 is the calculated rms voltage noise of a single-channel measurement system due to ambient temperature fluctuations.

fluctuations $S_{u1,2}$ is proportional to the difference of effective noise temperatures:

$$S_{u1,2} \propto T_{inp1} - T_{imp2}. \qquad (2)$$

Curve 3 was acquired under the same conditions as above, except that the noise temperature at port 2 was *increased* to match that of the port 1. These data are also consistent with Eq. (2).

The data of Fig.2 provide conclusive experimental evidence that using cross-correlation analysis of the two identically tuned (either phase or amplitude sensitive) outputs of a two-channel NMS yield the difference of the spectral density of the noise between the two inputs.

Fig. 3 shows the dependence of the data from curve 1 and 2 from Fig.2 on Fourier frequency offset f. The heavy horizontal line corresponds to the thermal noise floor of a single-channel NMS at T = 300K. For f above a few kHz the mean value of cross-spectral voltage noise is 12 to 15 dB below the thermal noise floor of a single-channel system.

The fractional statistical uncertainty of the measurement in curve 1 of Fig. 3 is $\pm 1/\sqrt{N_{avg}}$, where N_{avg} is the number of averages [6]. When the DUT noise is much less than the single-channel noise, the average value of the cross-correlated output also falls as $1/\sqrt{N_{avg}}$ [6,7]. The result is that the fractional statistical uncertainty of the cross-correlated two-channel measurement in curve 2 of Fig. 3 is approximately 1 when the effective temperature of the input noise T_{inp} is close to the ambient temperature. The large uncertainty in voltage noise cross-spectral density at Fourier frequencies above 3 kHz indicates that the noise originates from the single-channel noise reduced by the cross-correlation signal processing. This residual noise sets an upper limit to the smallest variations in the measurement system noise floor (or DUT) that can be resolved. In Fig. 3 we compare the spectral resolutions of a single- and a two-channel NMS by considering their voltage noise floors in curves 1 and 2 respectively. Each noise floor looks like a 'fuzzy' trace due to the scatter of experimental data. By measuring the width of such a trace in the vicinity of a given Fourier frequency, one can empirically estimate the spectral resolution of the measurement system. Intervals σ_1 and σ_{12} approximately characterize the spectral resolutions of the single- and two-channel measurement systems, respectively. At Fourier frequencies above 35 kHz: $\sigma_1 \approx 7x10^{-8} V/\sqrt{Hz}$ and $\sigma_{12} \approx 1.2x10^{-7} V/\sqrt{Hz}$ for $N_{avg} = 10^5$. These results are consistent with the fluctuations in both noise measurements decreasing as $1/\sqrt{N_{avg}}$. See Section III for further discussion.

Similar effects of single-channel noise suppression were observed in the early experiments with conventional two-channel cross-correlation NMS that used double balanced mixers as phase detectors [8]. However, the differential noise floors did not drop below $k_B T_o/P_{imp}$ until much higher Fourier frequencies due to the high level of flicker noise exhibited by these mixers.

3. THEORETICAL ANALYSIS OF CROSS-SPECTRAL DENSITY OF VOLTAGE FLUCTUATIONS IN THE THERMAL-NOISE-SUPPRESSION REGIME

The complex amplitudes of the output signals of an ideal 3 dB hybrid coupler are given by

$$U_3 = \left(U_1 e^{j\varphi_H} + U_2\right)/\sqrt{2}$$
$$U_4 = \left(U_1 + U_2 e^{j\varphi_H}\right)/\sqrt{2}, \tag{3}$$

where φ_H is a hybrid differential phase shift and U_1 and U_2 are the complex amplitudes of the input signals (see Fig. 1). The differential phase shift in (3) is $\pi/2$ to satisfy the energy conservation conditions.

If there is a loss in the hybrid, equations (3) are no longer valid and a complete set of S-parameters are required to describe the relationship between the complex amplitudes of input and output signals [9]. Apart from that, the differential phase shift in a real hybrid coupler is frequency dependent. For example, for commonly used coplanar stripline couplers, φ_H varies by $\pm 7^o$ with respect to $\pi/2$ in the operating frequency range. In the following analysis we assume the above description of an ideal 3 dB coupler, remembering that $\varphi_H = \pi/2$.

Considering a noise source with a white power spectrum at the input of the measurement system (port 1 in Fig. 1), we can write analytical expressions for the cross-spectral densities of the output voltage fluctuations that are a result of both PM and AM components.

The effect of PM components is given by

$$S_{u1,2}^{(1)PM} = \chi^2 \frac{k_B T_{inp1}}{8} \left\{\cos(\varphi_H - \Delta_{21}) - \cos(\varphi_H - \Sigma_{21})\right\} \tag{4}$$

where T_{inp1} is the effective temperature of the noise entering port 1, χ^2 is the calibration factor, and Δ_{21} and Σ_{21} are phase angles calculated as $\Delta_{21} = \varphi_2 - \varphi_1$ and $\Sigma_{21} = \varphi_2 + \varphi_1$.[10]

The voltage noise cross-spectral density due to AM components of input noise is

$$S_{u1,2}^{(1)AM} = \chi^2 \frac{k_B T_{inp1}}{8} \left\{\cos(\varphi_H - \Delta_{21}) + \cos(\varphi_H - \Sigma_{21})\right\} \tag{5}$$

Combining (4) and (5), the cross-spectral density of output voltage fluctuations due to the first noise source is obtained:

$$S_{u1,2}^{(1)} = \chi^2 \frac{k_B T_{inp1}}{4} \cos(\varphi_H - \Delta_{21}). \tag{6}$$

By analogy, the cross-spectral density of voltage fluctuations caused by the second noise source attached to port 2 in Fig. 1 is

$$S_{u1,2}^{(2)} = \chi^2 \frac{k_B T_{inp2}}{4} \cos(\varphi_H + \Delta_{21}). \tag{7}$$

Combining (6) and (7) results in the total cross-spectral density of voltage fluctuations at the output of the NMS:

$$S_{u1,2} = \chi^2 \frac{k_B}{4} \left\{ \begin{matrix} T_{inp1} \cos(\varphi_H - \Delta_{21}) \\ + T_{inp2} \cos(\varphi_H + \Delta_{21}) \end{matrix} \right\}, \tag{8}$$

which at $\varphi_H = \pi/2$ becomes

$$S_{u1,2} = \chi^2 \frac{k_B}{4} \left\{T_{inp1} - T_{inp2}\right\} \sin \Delta_{21}, \tag{9}$$

where the phase angle Δ_{21} is close to $\pi/2$, for identically tuned channels of the NMS.

The above result confirms that the two-channel NMS performs a differential temperature measurement. From previous discussion it also follows that the low voltage noise cross-spectral density must be observed with arbitrary noise sources no matter how high their intensities, provided that (i) noise sources are stationary and (ii) their intensities are matched.

4. DISCUSSION AND CONCLUSION

We have clearly shown that, in contrast to the traditional single-channel NMS, the two-channel NMS with cross-correlation yields the differential PM or AM noise between the two input ports. Since this noise difference is determined on a narrow bandwidth basis, these results apply whether the global character of the noise is white or varies with Fourier frequency.

The resolution of a two-channel NMS with good carrier suppression is typically $2/\sqrt{N_{avg}}$ lower than that of a single-channel system. For example, assuming that both channels are phase sensitive, the cross-correlation analysis yields

$$S_{\varphi}^{1x2}(f)=\left(S_{\varphi}^{dut}(f)-\frac{k_B T_o}{P_{imp}L_{DUT}}\right)$$
$$+\frac{2k_B T_o}{P_{imp}L_{DUT}}\left(\frac{2}{\sqrt{N_{ave}}}+\delta\right), \qquad (10)$$

where S_{φ}^{dut} is the spectral density of PM fluctuations in the DUT, k_B is Boltzmann's constant, T_o is the ambient temperature, P_{inp} and L_{dut} are the power at the input of the DUT and its insertion loss, δ is the parameter ($|\delta| << 1$) characterizing the asymmetry between the two-channels, and N_{avg} is the number of averages. This is to be compared with the results for a single-channel system of

$$S_{\varphi}^{1}(f)=S_{\varphi}^{dut}(f)+\frac{2k_B T_o}{P_{imp}L_{DUT}}. \qquad (11)$$

The statistical uncertainty of the measurements is

$$\sigma=\pm\left(S_{\varphi}^{dut}(f)+\frac{2\beta k_B T_o}{P_{imp}L_{DUT}}\right)\frac{1}{\sqrt{N_{ave}}}, \qquad (12)$$

where $\beta \approx 1$ for single-channel NMS and $\beta \approx 2$ for two-channel NMS.

These results demonstrate that both single- and two-channel measurement systems are capable of measuring the additive PM (AM) noise in the DUT with an effective temperature smaller than the ambient temperature. Moreover, the two-channel measurement system does not, in principle, offer an advantage over the single-channel one, as far as the spectral resolution of noise measurements is concerned, and it is more complicated. However, in practical terms, the non-stationary nature of the noise, the temporal separation of calibration and measurement, and the difficulty of reproducing the calibrations for two measurements make it extremely difficult to resolve noise which is more than 10 dB below the noise floor in a single-channel NMS.

5. ACKNOWLEDGEMENTS

The authors are grateful to L. Hollberg (NIST), Steve Jefferts (NIST), and A. Luiten (UWA) for useful discussions and valuable advice. This work is supported in part by the Australian Research Council.

6. REFERENCES

1. E.N. Ivanov and F. L. Walls, "Effect of anomalously low voltage noise in two-channel measurement systems", *submitted to IEEE UFFC in* November 1999.
2. E. N. Ivanov, M. E. Tobar and R. A. Woode, "Applications of interferometric signal processing to phase noise reduction in microwave oscillators', IEEE Transactions on MTT, **46**, 1537-1546, (1998).
3. E. N. Ivanov, M. E. Tobar and R. A. Woode, "Microwave interferometry: application to precision measurements and noise reduction techniques," IEEE Trans. UFFC, **44**, 1526-1537, (1997).
4. E. Rubiola, V. Giordano and J. Groslambert, "Double correlating interferometer scheme for measuring PM and AM noise," Electronic Letters, vol. **34**, 93-94, (1998).
5. E. Rubiola, V. Giordano and J. Groslambert, "Improved interferometric method to measure near-carrier AM and PM noise," IEEE Trans. Instrum. Meas. **48**, 642-646 (1999).
6. F. L. Walls, D. B. Percival and W. R. Irelan, "Biases and variances of several FFT spectral estimators as a function of noise type and number of samples," Proc. of IEEE Freq. Cont. Symp., 1989, 336-341.
7. J. S. Bendat and A. G. Piersol, Measurement and analysis of random data, Wiley & Sons, 1967.
8. F. L. Walls, S. R. Stein, J. E. Gray and D. Glaze, "Design considerations in the state-of-the-art signal processing and phase noise measurement systems," Proc. of the 30th Ann. Symp. on Freq. Cont., 1976, pp. 269-274.
9. P. A. Rizzi, Microwave Engineering Passive Circuits, Prentice Hall, pp. 347-411 (1988).
10. F. L. Walls, A. J. D. Clements, C. M. Felton et al, "Extending the range and accuracy of phase noise measurements", NIST Technical note 1337, pp 129-137(1990)