A SUPERCONDUCTING PARAMETRIC OSCILLATOR FOR USE IN INFRARED FREQUENCY SYNTHESIS

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INTRODUCTION

Superconducting cavities and oscillators have truly come into their own over the last few years in the field of time and frequency. Laboratories in Germany, Poland, France, England and Japan, in addition to the United States, are either currently involved in or starting projects which utilize superconducting oscillators. The purpose of this paper is to describe a superconducting parametric oscillator which will be used as a source for frequency synthesis experiments - from the microwave to the far infrared or higher. The goal of these experiments is to phaselock lasers at the highest possible frequencies to radio frequency sources. This will make it possible to transfer the full accuracy and stability of the cesium frequency standard to the infrared and to perform experiments which test theories of gravity and the constancy of certain fundamental constants.

Superconducting resonators have several properties that make them particularly relevant to the problem of frequency multiplication. The devices which have been used in recent years operate in the X-band near 10 GHz. This relatively high starting frequency helps to reduce the noise generated in the multiplication process. It may be possible to realize further advantages by using resonators at frequencies as high as 100 GHz. Compared to quartz crystals, superconducting resonators have small non-linearity. As a result it is possible to filter all the power needed to drive a multiplying junction through the resonator which also reduces the noise generated by the multiplier. The superconducting cavities which are currently being used are mechanically rugged structures constructed from solid niobium and consequently have small environmental sensitivities [1]. Finally, very narrow resonance widths can be achieved with niobium resonators. At 10 GHz, the maximum Q which has been reached is approximately 10^{11} while Q's greater than 10^{9} are achieved routinely [2]. The high Q relaxes the requirements on how finely the resonance must be split to achieve the necessary frequency stability.

A variety of oscillators can be built using a superconducting cavity. Some of the techniques which have been tried are: coupling a negative resistance device to the cavity [3]; coupling a free running oscillator to the cavity (stabilization) [4]; placing the cavity in a feedback loop around a unilateral amplifier [5]; filtering the output of a free running oscillator through the cavity; and using the cavity in a frequency discriminator circuit in order to feed-back to a voltage controlled oscillator (SCSO technique) [6]. Although each method has certain advantages and disadvantages, when the criterion is optimum spectral purity the best device appears to be a negative resistance oscillator which uses a parametric amplifier to generate the negative resistance. Such a device is compact, can be entirely located in the very favorable cryogenic environment which is needed for the superconducting cavity, and can directly generate the milliwatt power levels that are needed for frequency multiplication.

The Superconducting Parametric Oscillator

A parametric oscillator is simply a parametric amplifier which is pumped sufficiently hard that the generated negative conductance exceeds the net positive conductance of all loads and internal losses. The device which is described here is based on the three frequency non-degenerate parametric amplifier. A varactor diode is mounted in an assembly which permits it to be matched to a source of pump power at frequency ω_3. As a result it appears to be a time varying capacitance. The diode is at the same time coupled to two resonators - the idler resonator at frequency ω_1 and the signal resonator at frequency ω_2. The equivalent circuit is shown in Fig. 1.

Power can flow between the three frequencies provided they are commensurate:

$$\omega_3 = \omega_1 + \omega_2 .$$

The maximum power output which may be obtained at any frequency is determined by the requirement of energy conservation. For a parametric device consisting of a time varying reactance,the power conservation formulas are known as the Manley-Rowe relations [7]:

$$\frac{P(\omega_2)}{\omega_2} = \frac{P(\omega_1)}{\omega_1} = - \frac{P(\omega_3)}{\omega_3} .$$

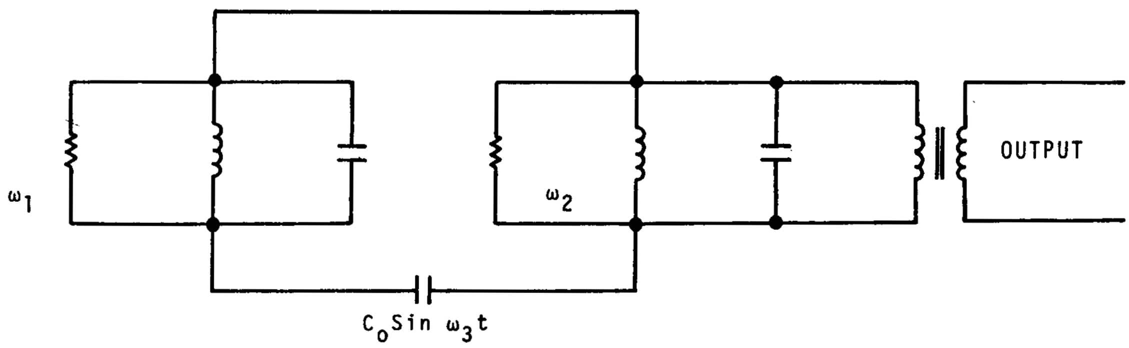

Fig. 1: Simplified equivalent circuit of parametric oscillator showing idler resonator (ω_1), signal resonator (ω_2), pumped varactor diode (time varying capacitance) and output coupling.

The oscillation properties of a parametric device must be calculated under the assumption that the oscillation signals may be comparable to the pump signal. It is then found that above a threshold level of pump power, the device oscillates [8]. However, the strength of the oscillation is not determined by a change in the diode characteristics at high drive level. Rather, saturation occurs due to the limited amount of pump power available. This method of saturation, which is different from other types of oscillators, may be useful in achieving low flicker noise levels.

Two parametric oscillators have been built and tested. The pump frequency for these devices is about 13.4 GHz with a signal frequency of 9.2 GHz and an idler frequency of 4.2 GHz. During operation at 290°K, the minimum pump threshold power which has been observed is about 1 mW and the maximum efficiency is more than 10%. Oscillation has been observed down to approximately 77°K and nonlinear behavior of the diode continues to at least 1.5°K. Oscillation with a superconducting cavity at 1.5°K is expected to be achieved in the near future.

The phase noise which results from thermal noise in a negative resistance oscillator has been calculated [9]. Noise in the parametric gain element or from internal losses results in what is known as perturbative phase noise and the spectral density of the phase fluctuations from this source has a random walk characteristic [10]. On the other hand, noise sources which are not filtered by the resonator cannot be distinguished from phase fluctuations and the spectral density has a white phase characteristic. For this reason the superconducting parametric oscillator has been designed with an independent output coupling probe so that the only white phase noise source is the user device. The total phase noise from thermal sources is the sum of thé additive and perturbative components:

$$S_\phi(f, \text{thermal}) = \left(\frac{\nu_o}{f}\right)^2 \frac{kT_e}{2P_a Q_E Q_L} + \frac{kT_e'}{2\,P_a}$$

$$= \frac{10^{-20}\text{Hz}}{f^2} + 2 \times 10^{-18}\text{Hz}^{-1},$$

where ν_o = 10 GHz, T_e = 1°K, T_e' = 300°K, P_a = 1 mW, $Q_L = 5 \times 10^9$ and $Q_E = 10^{10}$ have been assumed. The two sample variance which corresponds to this spectral density is

$$\sigma_y^2(\tau) = \left(\frac{8.3 \times 10^{-21}}{\tau^{1/2}}\right)^2 + \left(\frac{4.3 \times 10^{-20} f_h}{\tau}\right)^2,$$

where f_h is the bandwidth of the measurement system.

The performance limitations which are predicted by this noise analysis are very optimistic since there are additional effects which perturb the phase of the oscillator but do not fit this model. There are several phenomena which affect the frequency of the superconducting cavity directly. Mechanical vibrations in the frequency range of 1 Hz to 1 kHz couple to the cavity and the remainder of the apparatus. These vibrations produce FM sidebands which, unless considerable care is taken, can represent greater root mean square phase fluctuation than the integrated white phase noise [11]. The temperature of the superconducting cavity affects its frequency primarily due to variation in the field penetration into the superconductor. The sensitivity depends upon the operating temperature and is 6 parts in 10^{10} per degree Kelvin at 1.5°K [12]. The energy stored in the resonator causes a dc frequency shift of about 1 part in 10^4 per joule due to both the radiation pressure of the fields and the nonlinear surface reactance [1]. This offset allows AM noise in the oscillator to be converted to FM noise.

The frequency of the oscillator can also be perturbed via pulling which results from changes in external loading or in the oscillator components. In either case the pulling can be viewed as a change in the frequency of the idler resonator. The degree of pulling can be derived from the condition for oscillation and is

$$\frac{\Delta\omega_2}{\omega_2} \cong \frac{\Delta\omega_1}{\omega_1} \frac{Q_1}{Q_2}$$

A form of pulling which is unique to the parametric oscillator is due to variations in the pump frequency. The pulling equation combined with the requirement that the pump, idler, and signal frequencies be commensurate leads to the following relation:

$$\frac{\Delta\omega_2}{\omega_2} \cong \frac{Q_1}{Q_2} \frac{\Delta\omega_3}{\omega_3-\omega_2}.$$

Both cases lead to the same conclusion - the ratio of the Q's of the idler and superconducting resonators should be minimized. For the best noise performance the idler cavity should be loaded by the unavoidable losses of the varactor diode.

In order to best illustrate the potential advantages of the superconducting parametric oscillator for frequency multiplication to the infrared the results of multiplying oscillators with three different spectra to 100 THz are discussed below. The phenomenological theory of Walls and de Marchi is used to predict the phase noise after multiplication [13].

Figure 2 is a plot of the spectral density of phase fluctuations representing (A) the performance of a commercial 5 MHz quartz crystal oscillator with state-of-the-art white phase noise, (B) the performance of a 10 GHz Gunn-effect oscillator stabilized by a superconducting cavity via the SCSO technique, and (C) the predicted performance of a 10 GHz superconducting parametric oscillator with the assumption that the flicker level is the same as for the SCSO. The noise in each device is plotted referenced to its actual operating frequency.

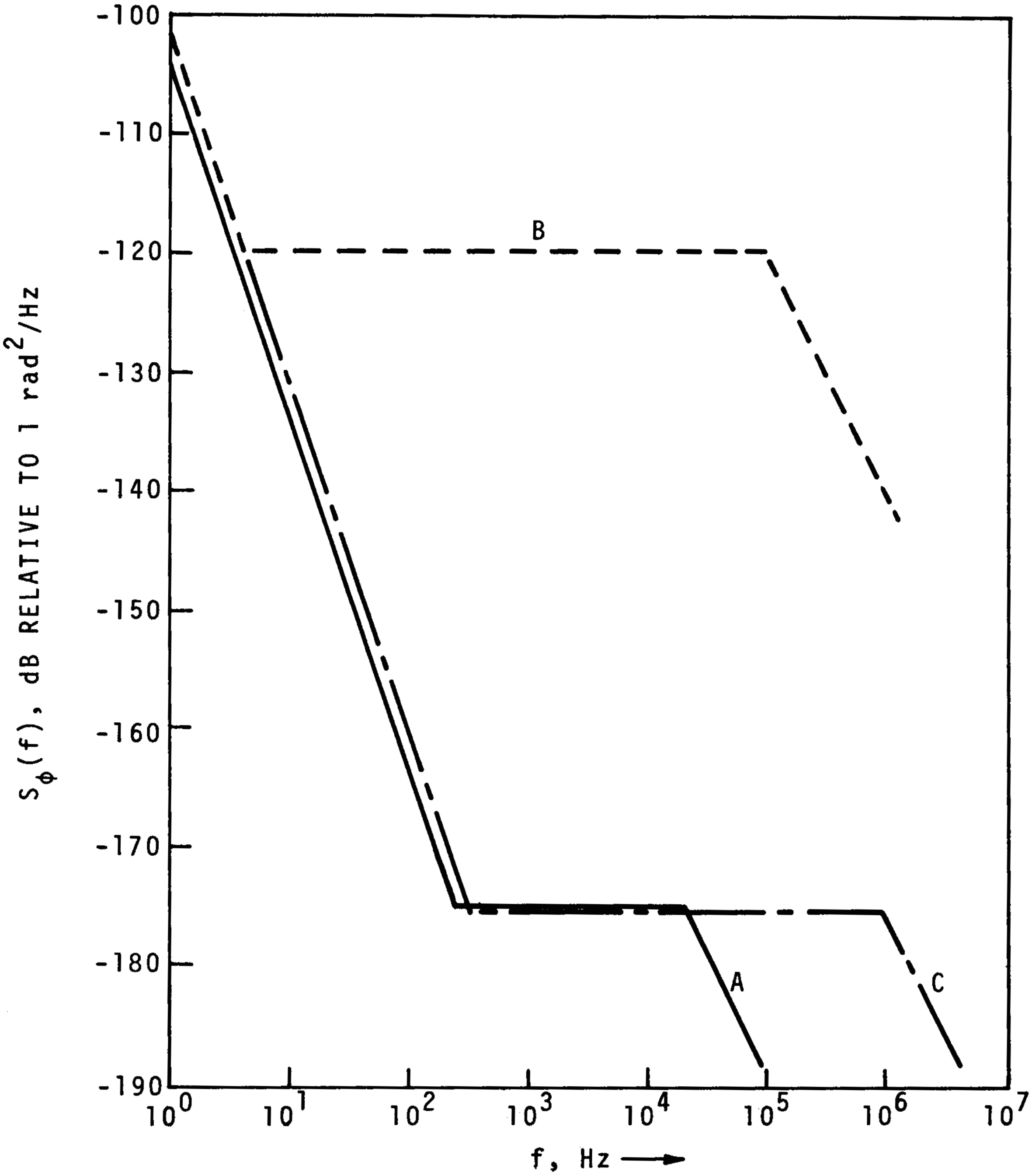

FIG. 2 Spectral density of phase fluctuations for (A) a 5 MHz quartz oscillator; (B) a 10 GHz superconducting cavity stabilized oscillator; and (C) a 10 GHz superconducting parametric oscillator (predicted performance).

Each spectrum is characterized by a carrier which behaves like flicker of frequency,

$$S_{\phi_o}(\text{carrier, f}) = \frac{K_c}{f^3},$$

and a white phase noise pedestal,

$$S_{\phi_o}(\text{pedestal, f}) = \frac{K_p}{1 + (f/fp)^2}$$

which is band limited with a 3 dB frequency f_p.

The phenomological theory predicts the results of multiplication as a function of three variables: The integrated mean square phase in the pedestal, Φ^P, is defined by the relation:

$$\Phi^P \equiv \int_{\text{pedestal}} S_\phi(\text{pedestal, f})\, df.$$

The bandwidth of the carrier, $\Delta\nu_c$, is defined by the relation:

$$\ln 2 = \int_{\frac{\Delta\nu_c}{2}}^{\infty} S_\phi(\text{carrier, f})\, df.$$

The bandwidth of the pedestal, $\Delta\nu_p$, is defined by the relation:

$$\ln 2 = \int_{\frac{\Delta\nu_p}{2}}^{\infty} S_\phi(\text{pedestal, f})\, df$$

The effect of frequency multiplication of order n is to increase the phase spectral density by a factor of n^2. The spectrum changes in the following way. The width of the carrier increases linearly with n:

$$\Delta\nu_c \simeq \left(\frac{2K_c}{\ln 2}\right)^{1/2} n.$$

The integrated mean square phase in the pedestal increases with the square of n:

$$\Phi^p \simeq \frac{\pi n^2 K_p f_p}{2} .$$

The power in the carrier relative to the total power decreases as $e^{-\Phi^p}$. The width of the pedestal is at first constant and then increases as the square of n when Φ^p exceeds ln 2:

$$\Delta\nu_p \simeq \frac{2 n^2 K_p f_p^2}{\ln 2} \qquad \text{for } \Phi^p > \ln 2.$$

For low orders of multiplication the primary effect is the increase in the bandwidth of the carrier. When n reaches the value n_1 for which $\Phi^p = \ln 2$ the power in the carrier is equal to the power in the pedestal. For n larger than this critical value the power in the carrier decreases exponentially. In this region it is necessary to decrease the observation bandwidth to less than f_p in order to observe the carrier with a power signal-to-noise ratio greater than unity. By the time the multiplication order reaches approximately $3n_1$, the power density of the carrier is smaller than the power density of the pedestal and it is no longer possible to observe the carrier.

Figure 3 illustrates the predicted results of multiplying the three signals of Fig. 2 to 100 THz. The power density of the carrier from the quartz crystal oscillator drops below that of the noise pedestal by the time it is multiplied to 50 THz. For this reason the noise pedestal is shown as a dashed line, indicating that there is no resolvable coherent signal. The same thing happens to the signal from the SCSO just at 100 THz. However, in the case of the superconducting parametric oscillator the theory predicts that the carrier could still be resolved at 100 THz. The signal to noise power ratio is predicted to be 2×10^3 at this frequency and it should be possible to phaselock a tunable laser to the multiplied signal permitting one to transfer the full stability and accuracy of frequency standards between the RF and infrared portions of the spectrum.

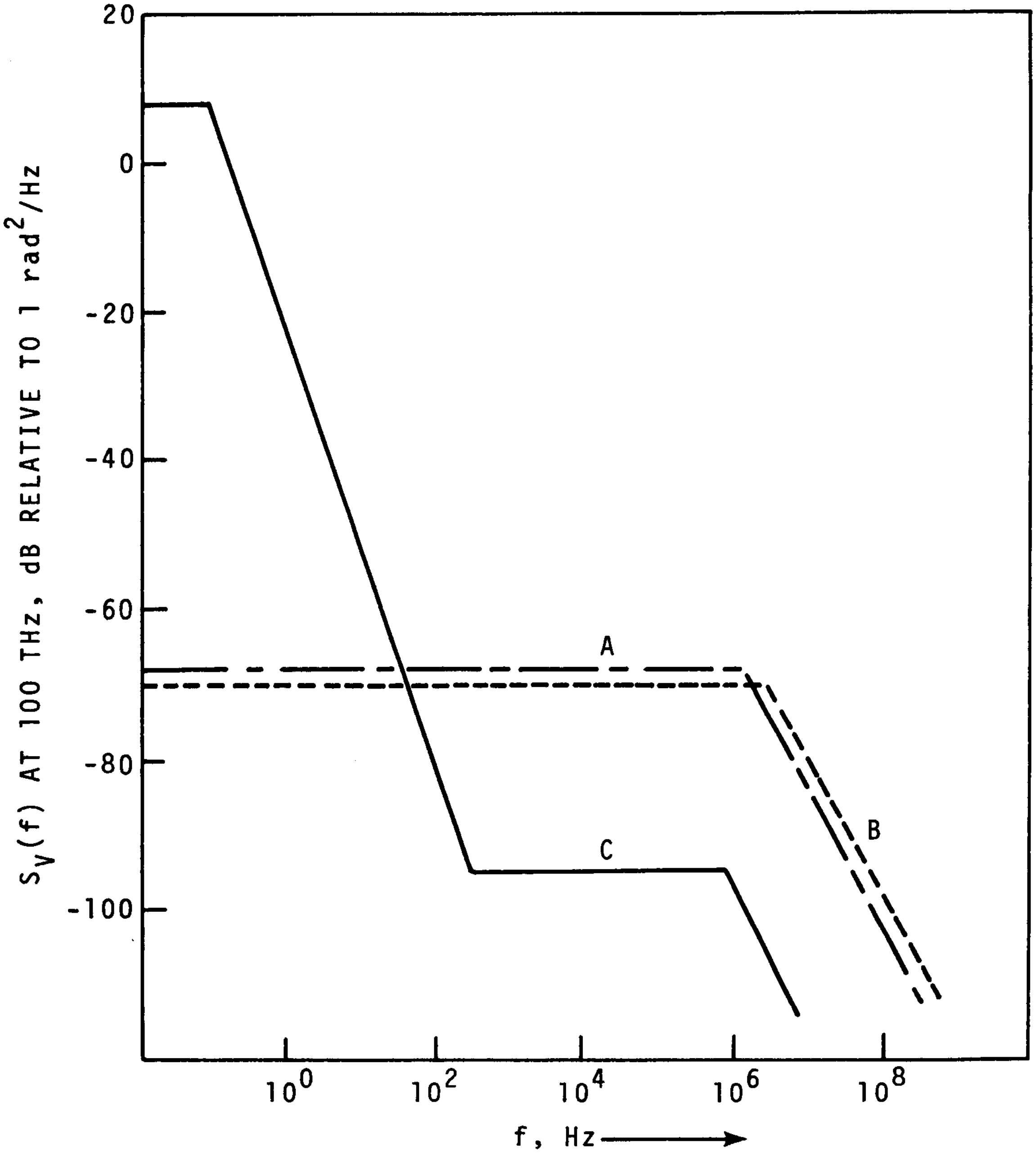

FIG. 3 Power spectral deeity of three oscillators of Fig. 2 after multiplication to 100 THz: (A) a 5 MHz quartz oscillator; (B) a 10 GHz superconducting parametric oscillator (predicted performance). Note the absence of any carrier in Curves A & B.

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