A NOVEL LOW NOISE REGENERATIVE DIVIDE-BY-FOUR CIRCUIT

A. Sen Gupta[1], J. F. Garcia Nava[2] and F. L. Walls[2]
[1] National Physical Laboratory, New Delhi, INDIA*
[2] Total Frequency, Boulder, CO, USA *

Abstract – **We discuss a novel design of a self starting regenerative divider that permits division by 3, 4, 5, 6... instead of the usual 2. This is accomplished by having the loop oscillate simultaneously at two different frequencies, e. g., at f/4 and 3f/4. A prototype of the divide-by-four circuit has been constructed for an input frequency of 400 MHz. This divider exhibits very low phase noise L(1 kHz) = –161dBc/Hz and L(100 kHz) = –169 dBc/Hz, , which is at least 8 dB lower than its constituent parts. Simple modifications of the feedback loop of the prototype enabled it to divide by 3, 5 and 8. Operation at divide by 10 appears feasible.***

I. INTRODUCTION

Frequency division forms an important aspect of frequency synthesis schemes and has been applied extensively. In particular, the regenerative dividers first proposed by Miller [1] has attracted a lot of attention mainly because they produce a clean sinusoidal output waveform and can be used at very high frequencies. Several design aspects of regenerative divider circuits have been studied in recent years – namely, their theory of operation [1-3] and their low phase noise performance [4-7]. Regenerative dividers have also been used in low-noise frequency synthesis applications [7-8]. Although in his original proposal Miller [1] had conceived the idea of a general division by N ($N \geq 2$), regenerative dividers subsequently designed and studied have been exclusively divide-by-two configurations. To obtain greater division ratios regenerative dividers have been cascaded, thus increasing the amount of hardware [9]. Also, it has been possible to achieve only binary division ratios unless multipliers have been added within the loop or the mixer is driven into hard saturation, which generates harmonics. Such dividers are generally not self starting.

In the present study we have attempted to overcome the above limitations by examining a more general regenerative divider configuration that permits division by 2, 3, 4, 5, 6, etc. A distinct difference in our approach compared to that of Miller is that the loop supports simultaneous oscillation at two different frequencies. This removes the necessity of using nonlinear elements (e.g., frequency multipliers) within the feedback loop to obtain integer frequency division higher than 2. The linear nature of this new loop makes it possible for our dividers to be self starting.

A very simple theory has been worked out for the specific case of division by four, giving the conditions under which such a divider would operate and the phase noise that would be expected due to noise in the constituent parts. However, the treatment is general enough to be extended to other division ratios. A prototype divide-by-four has been fabricated with an input frequency of 400 MHz, primarily motivated by the plan to replace the corresponding digital divide by four in our Cs synthesizer [8]. Phase noise measurements carried out on the prototype show reasonably good agreement with expectations. Finally, we have experimentally shown that by simple restructuring of the feedback loop of the divider it is indeed possible to get divisions by 3, 5 and 8. The extension to division by 10 seems possible.

II. OPERATION OF REGENERATIVE DIVIDE-BY-FOUR

In Fig. 1 we have shown the schematic of our proposed regenerative divide-by-four circuit. An examination reveals that it is essentially similar to a conventional divide-by-two, except for the transfer function of the feedback loop. Unlike the loop filter of the conventional divide-by-two, which consists only of a low pass (or a band pass) filter at f/2, we have a notch filter at f/2; a low pass filter at about 7f/8 and a high pass filter at about f/8. This makes the transfer function of the feedback loop have two broad gain peaks around f/4 and 3f/4. Let us assume that a signal e_1 at a frequency f/4 exists at one of input ports of the mixer. This mixes with the input signal e_0 at frequency f

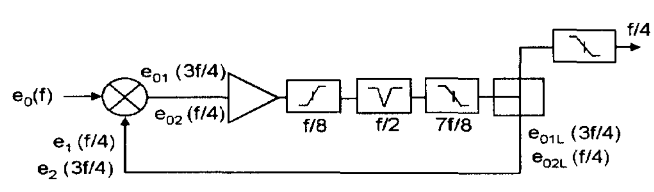

Fig. 1 Schematics of a regenerative divide-by-four

to produce the output e_2 at 3f/4. The signal e_2 can propagate around the loop, as the transfer function has a peak at this frequency and can appear at the input of the mixer. It can

* Authors are Guest Researchers at the Time & Frequency Division of NIST, Boulder, CO, USA

0-7803-7082-1/02/$10.00 © 2002 IEEE

then mix with the input e_0 to produce the output at f/4, which in turn can propagate around the loop and, given the right phase conditions, can reinforce e_1. Thus in equilibrium we can have a situation where two signals at f/4 and 3f/4 propagate around the loop and sustain each other. The outputs can be extracted using a power splitter or directional coupler inserted into the loop.

It is possible to work out the general conditions under which the regenerative loop will indeed function in the way described above. Let us assume that the signals at the RF port of the mixer are the input e_0 at frequency f and those at the LO port are e_1, e_2 at frequencies f/4 and 3f/4, respectively. Let us write these signals as

$$e_0 = A\cos(2\pi ft) \quad (1)$$
$$e_1 = B\cos(2\pi(f/4)t + \theta_1) \quad (2)$$
$$e_2 = C\cos(2\pi(3f/4)t + \theta_2) \quad (3)$$

The amplitude terms B and C may be very small initially. At the output of the mixer we get the following outputs if we ignore the signals at the sum frequencies, as they would not propagate around the loop anyway.

$$e_{01} = K_1AB\cos(2\pi(3f/4)t - \theta_1) \quad (4)$$
$$e_{02} = K_2AC\cos(2\pi(f/4)t - \theta_2) \quad (5)$$

where $K_{1,2}$ are the mixer conversion losses. Let us assume that the loop transfer function has the general form $G_1 \angle\varphi_1$ and $G_2 \angle\varphi_2$ at frequencies f/4 and 3f/4, respectively. Then, at the output of the loop filter and amplifier, the resultant signals are given by

$$e_{01L} = G_2 K_1AB\cos(2\pi(3f/4)t - \theta_1 + \varphi_2) \quad (6)$$
$$e_{02L} = G_1 K_2AC\cos(2\pi(f/4)t - \theta_2 + \varphi_1) \quad (7)$$

Now when we close the loop, in order to have equilibrium we must impose the conditions

$$e_{01L} = e_2 \quad (8)$$
$$e_{02L} = e_1 \quad (9)$$

Using eqns. (2), (3), (6) and (7) and equating amplitude and phase terms, we then get the conditions for sustained regeneration as

$$K_1K_2G_1G_2A^2 \geq 1 \quad (10)$$
$$\varphi_1 = \varphi_2 + 2n\pi \quad (11)$$

The loop starts to regenerate if the total gain in eqn. (10) exceeds unity and finally settles down to some nonzero values of B and C when the loop gains saturate. We can further impose the condition that the two signals are precisely harmonically related to each other, or that we have synchronous operation. In this case e_2 is the 3[rd] harmonic of e_1 or that $\theta_2 = 3.\theta_1$, then using eqns. (2), (3), (6) and (7) again we get the further conditions

$$\theta_1 = \varphi_1 / 4 \quad (12)$$
$$\theta_2 = 3\varphi_2 / 4 \quad (13)$$

Clearly, to satisfy eqns (12) and (13) would require that n in eqn (11) be equal to zero. In other words this would require that the loop be non dispersive and have a short transit time. It is also clear that no special starting conditions are required for initiating oscillations in the loop. Simultaneous oscillations at the two conjugate frequencies reinforce and eventually support each other.

We must of course realize that it is possible to have an asynchronous mode of operation [10], in which case the two frequencies propagating in the loop, rather than being precise harmonics of each other as above, just satisfy the condition that their sum is equal to f . If eqns (10) and (11) are satisfied, then the loop can regenerate with these two frequencies. There can be conditions when both synchronous and asynchronous modes coexist and compete with each other. This could happen particularly when the transit time through the loop is long.

One of the consequences of eqn (12) is that if there are any phase fluctuations in the loop, due to noise in the components used, then its effect is reduced by a factor of four at the output. Thus, the phase noise power, which is proportional to the square of the phase fluctuations, can be expressed for the ideal divide-by-four circuit as

$$\begin{aligned} S_\varphi(f)\,[\text{Divider}] &= (1/16)\, S_\varphi(f)\,[\text{feedback network}] \\ &= S_\varphi(f)\,[\text{feedback network}] - 12\text{dB} \quad (14) \end{aligned}$$

In practice it has been shown by Rubiola et al [5], that the mixer is a complex device whose phase gain can vary considerably by the phase shift in the loop, which can cause the noise reduction shown, e. g., in eqn (14), to be worse under unfavorable conditions. Thus, one has to fine-trim the loop's phase delay to get the phase noise reduction close to that given by eqn (14), even at the expense of slightly lower output power.

Finally, before concluding this section we must mention that the general conditions of synchronous operation that have been worked out should be generally valid for other pairs of harmonically related conjugate frequencies such as f/3 – 2f/3 or f/5 – 4f/5 etc. Thus given the right shape of the feedback-loop's transfer function provided the phase conditions are met, one can get regenerative division by 3, 5, 6, 7, 8, etc., without requiring highly nonlinear operation of the loop.

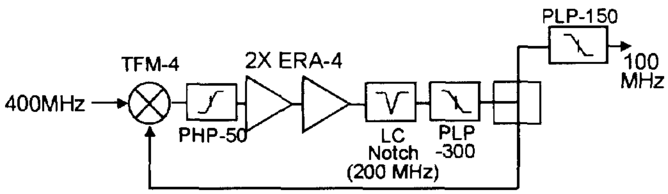

Fig. 2 Schematics of the prototype divide-by-four

III. DESIGN OF A PROTOTYPE DIVIDE-BY-FOUR

Figure 2 shows the schematic of the prototype regenerative divide-by-four circuit designed to operate at an input frequency of 400 MHz. We use a mixer, low and high pass filters, a power splitter, all in modular versions, and two low noise amplifier stages [11]. The notch filter at 200 MHz consisted of a chip inductor of 10 nH in series with a chip capacitor of about 40 pF to ground. This produces a sufficiently deep notch, which is not very sharp. The amplitude of transfer function of the feedback loop, shown in Fig. 3, clearly shows the two broad peaks around 100 MHz

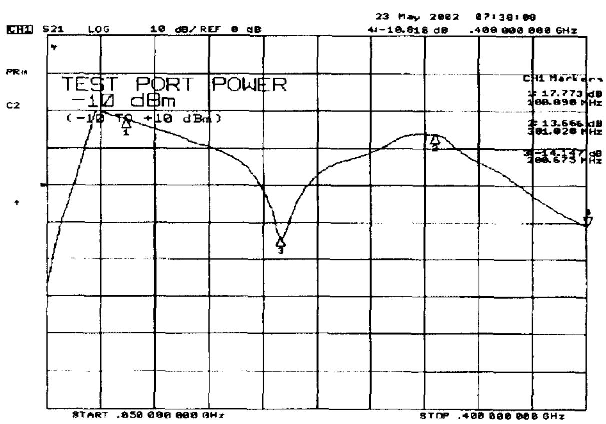

Fig. 3 Gain of the transfer function of the feedback loop

and 300 MHz. To close the loop from the output of the power splitter to the LO port of the mixer we used a piece of 50 ohm 0.085" coaxial cable. To get proper operation of the divider the cable length had to be very carefully adjusted. Figure 4 shows the output on a spectrum analyzer of the prototype divide-by-four circuit of Fig. 2. The input to the circuit was at 400 MHz and had a level between 5 and 10 dBm. The low pass filter at 150 MHz has been bypassed for this output to illustrate how even the 300 MHz output can be extracted if desired. This would then be an additional byproduct of the circuit - namely, division by ¾.

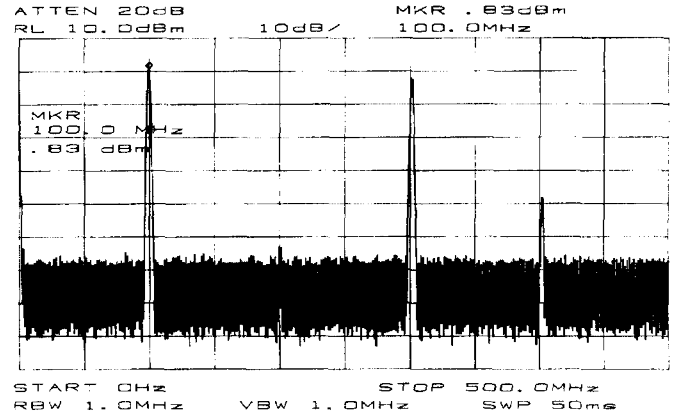

Fig. 4 Output of the regenerative divide-by-four

For a certain length of the feedback loop cable, the synchronous divide-by-four operation was found to take place over a variation of about 20 MHz in input frequency. Beyond this range the asynchronous modes start to compete with the synchronous operation and the output starts to look like that shown in Fig. 5. The way to increase the operational

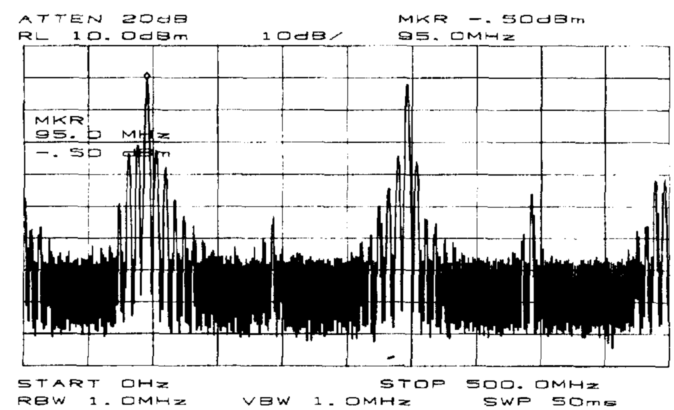

Fig. 5 Asynchronous operation of the divide-by-four circuit

bandwidth would be to make the loop delay as short as possible, and to use lower Q filters to reduce the dispersion effects. It has also been proposed [10] (in the context of the divide-by-two circuit) that one could increase the operational bandwidth by making the amplifier, rather than the mixer, the saturable element in the loop.

IV. MEASUREMENTS OF PHASE NOISE

Phase noise measurements were made on the prototype divide-by-four circuit using cross correlation setup [12]. The signal source was a low noise 100 MHz quartz oscillator that was multiplied up to 400 MHz using ultra-low-noise multipliers having a noise floor of –172 dBc/Hz. In the first part of the experiment, the noise of the entire feedback chain, comprising the amplifiers and filters was measured at 100 MHz. This is shown as Trace A in Fig. 6, where L(100 kHz) is within 2 dB of that expected from the input power

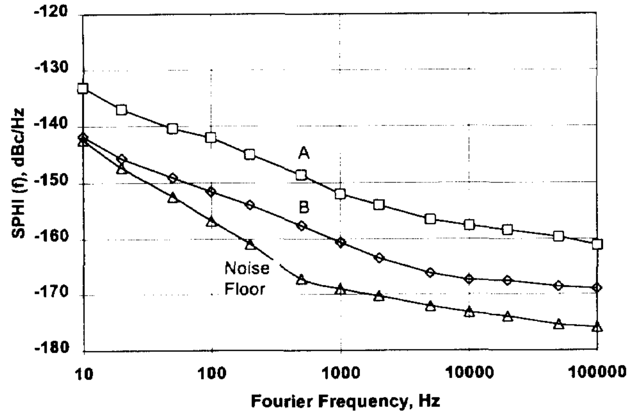

Fig. 6 Phase noise measurements of the divide-by-four circuit

and noise figure of the amplifier. Then the feedback loop was closed and phase noise of the divider was made with input frequency of 400 MHz. This is shown as Trace B in Fig. 6. The loop phase delay had to be carefully adjusted to get the lowest noise values. Between the best and the worst performances there could be a degradation of the noise by about 6 dB. This is in agreement with the work of Rubiola [5]. The best noise values for the divider were L(1 kHz) = -161 dBc/Hz and L(100 kHz) = –169 dBc/Hz. We observe that the values of Trace B are lower than those of Trace A by about 8 dB for all values of Fourier frequencies. This is a reasonable agreement with the expected reduction of 12 dB in eqn. (14). We suspect however that the phase noise measurement of the divider is being compromised by the noise of the multipliers used. A more precise measurement, that eliminates this limitation by actually doing the measurement on two dividers driven by the same source, should bring even closer agreement with eqn (14).

V. DIVISION BY N

Although the discussion in Section II focussed on a division-by-four by considering a f/4–3f/4 conjugate pair, it is clear that the treatment is quite general. Therefore, given the appropriate filter transfer function, any other conjugate such as – f/3-2f/3 or f/5-4f/5, or even f/10-9f/10 could also work in the synchronous mode. Indeed, by making simple modifications of the transfer function of the feedback loop in our prototype divide-by-four circuit in the form of different values of the low and high pass filter cutoffs and adjusting the phase delay we could operate the circuit as divide by 3, 5, 8, etc . The spectrum analyzer outputs, in these cases are shown in Figs. 7, 8 and 9. The divide by 5 results shown in

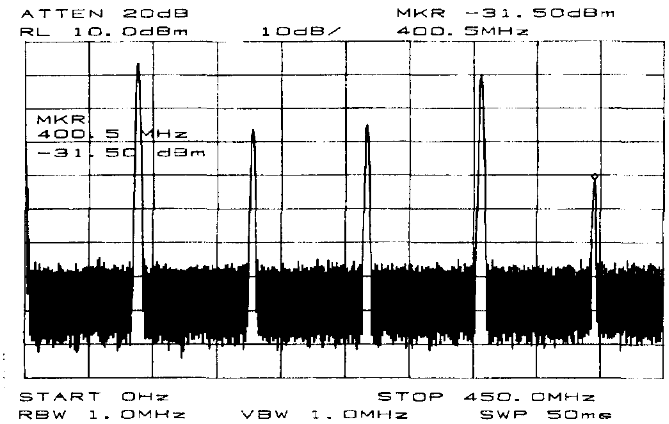

Fig. 7 Regenerative divide-by-three operation

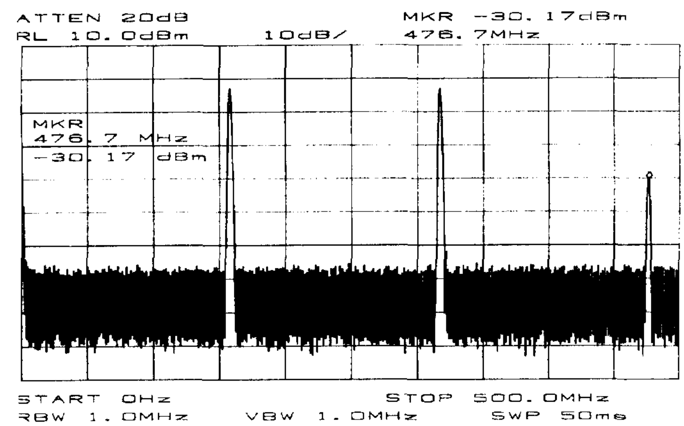

Fig. 8 Regenerative divide-by-five operation

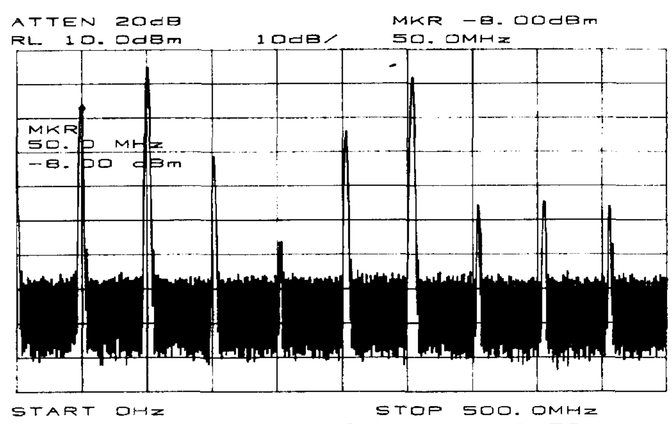

Fig. 9 Regenerative divide-by-nine operation

Fig. 8 have the most dominant outputs at f/5 and 4f/5 but other harmonics – 2f/5 and 3f/5 are also present. Likewise, for divide by 8 in Fig.9, we can see a whole comb of outputs nf/8, n = 1 to 7. It was however observed that the operational bandwidth of the loop decreased with increase in the division ratio.

VI. CONCLUSION AND FUTURE WORK

We have shown that a modification of the feedback loop of a regenerative divide-by-two can work as a divide-by-four. A simple theory of the condition of stable operation of the divide-by-four circuit has been worked out. A prototype regenerative divider from 400 MHz to 100 MHz has been constructed and found to work well over a band of 20 MHz around the input frequency. Beyond this band, there seems to be competition between synchronous and asynchronous modes that inhibits stable operation. Preliminary measurements indicate that the divide-by-four circuit has a very low phase noise L(1 kHz) =-161 dBc/Hz and L(100 kHz) = –169 dBc/Hz. These values are about 8 dB less than the phase noise of the components constituting the feedback loop. This is consistent with theory that predicts 12 dB reduction of the noise. The disagreement is partly explained as being due to a limitation of our measurement process.

It was shown that by modifying the feedback loop transfer function it is possible to operate the divider as divide by 3, 5, 8, etc. In fact for the higher division ratios the output is actually a comb of outputs nf/N, $n = 1,2,.. N-1$. Such kinds of dividers could be extremely useful in frequency synthesis.

In the future, we would like to extend our work to higher input frequencies and more general loop architectures. We also plan to make phase noise measurements on the higher order dividers.

ACKNOWLEDGMENTS

We are thankful to Dave Howe for providing facilities at NIST to carry out most of the experimental work and for continued support and encouragement.

REFERENCES

[1] R. L. Miller, "Fractional-Frequency Generators Utilizing Regenerative Modulation" Proc. I. R. E., pp 446-457, 1939.

[2] V. F. Kroupa, "Frequency Synthesis: Theory, design and Applications", John Wiley & Sons, pp 62-63, 1973.

[3] R. G. Harrison, "Theory of Regenerative Frequency Dividers Using Double Balanced Mixers", 1989 IEEE MTT-S Symp. Digest Vol-1, pp 459-462, June 1989.

[4] M. M. Driscoll, "Phase Noise Performance of Analog Frequency Dividers", Proc 43rd Annual Freq. Control Symp., pp 342-348, 1989.

[5] E. Rubiola, M. Olivier and J. Groslambert, "Phase Noise of Regenerative Frequency Dividers", Proc. 5th European Freq. and Time Forum, pp 115-122, March 1991.

[6] E. S. Ferre-Pikal and F. L. Walls, "Microwave regenerative frequency dividers with low phase noise", IEEE Transactions on UFFC, Vol 46, No 1, pp 216 –219, Jan 1999.

[7] M. Mossammaparast, C. McNeilage, P. Stockwell and J. Searles, "Phase Noise of X-Band Regenerative frequency Dividers", Proc, 2000 IEEE Intl. Freq. Control Symp., pp 531-535, 2000.

[8] A. Sen Gupta, D. Popovic and F. L. Walls, "Cs Synthesis – a New Approach", IEEE Transactions on UFFC, Vol 47, No 2, pp 475 –479, March 2000.

[9] M. Mossammaparast, C. McNeilage, P. Stockwell and J. H. Searles, "Low Phase Noise Division From X-Band to 640 MHz", presented at 2002 IEEE Intl. Freq. Control Symp., in these proceedings.

[10] S. V. Ahamed, J. C. Irving and H. Seidel, "Study and Fabrication of a Frequency Divider Multiplier Scheme for High-Efficiency Microwave Power", IEEE Transactions on Communication, Vol 24, pp 243-249, Feb 1976.

[11] All parts used were modular units from Minicircuits as indicated in Fig.2. The amplifiers were constructed out of ERA-4's, also from Minicircuits. This however does not constitute an endorsement of Minicircuits parts as being the best or the only ones suitable for this application. They were used to make the prototype due to their ready availability in our laboratory.

[12] W. F. Walls, "Cross-Correlation Phase Noise Measurements" Proc. 1992 IEEE Freq. Control Symp., pp 257-261, 1992.