Any two independent time scales will exhibit time departure due to two main causes, i.e., systematic differences in the frequency standards and inherent random noise processes. For time synchronization, theoretical considerations indicate that a third-order feedback system will automatically remove the systematic difficulties of typical frequency standards used in time scale work. The whole system is simulated with a computer to determine the systems feasibility and operating parameters. In this treatment we assume that time comparisons for synchronization would be intermittent and that the frequency standard may be represented by a systematic linear frequency drift with a random flicker noise spectrum ($1/|f|$) of the frequency fluctuations.

On the basis of the computer results an electromechanical system was designed and built. When the input to the system is the frequency from a high quality quartz crystal oscillator, the output frequency has no measurable frequency drift. If synchronization is performed every 12 hours, the rms time error predicted by the system for the time of the next synchronization is 70 nanoseconds, which is near theoretical optimum.

KEY WORDS: Time scales; Time synchronization; Computer simulation; Flicker noise; Optimum prediction; Frequency drift; Third order feedback system.

AN ULTRA-PRECISE TIME SYNCHRONIZATION SYSTEM DESIGNED BY COMPUTER SIMULATION

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DESCRIPTION OF THE TIME CONTROL SYSTEM

Consider a *master clock* to which synchronization of an independently operating secondary clock is desired. By *independently operating* is meant a clock which is physically remote from the master and for which time comparisons with the master are only intermittently available. These conditions apply to an important class of timing problems, the case for instance, where a master clock exists at the U. S. Naval Observatory or the National Bureau of Standards and where numerous secondary clocks throughout the country or world require synchronization with the master. Usually comparisons may be made only intermittently either daily by radio or less often by the use of portable clocks. The master clock consists of an ultrastable frequency source, such as a cesium beam standard, driving a set of frequency dividers to produce a time scale. The secondary clock is similar but may contain a less stable oscillator, typically a high quality quartz crystal oscillator. Generally, crystal oscillators exhibit two types of frequency instabilities: (1) a systematic linear change in frequency with time, known as drift, and (2) a random fluctuation exhibiting a power spectrum proportional to $1/|f|$ (flicker noise), where f is the fluctuation frequency. These instabilities occur simultaneously.

Both types of instability will cause the time scale kept by the secondary clock to depart from that of the master clock.[1] The systematic instability may in principle be predicted and is thus removable while the random fluctuations are not. It is these random fluctuations which prevent independently running clocks from remaining synchronized without intercomparison and indeed also prevent the precise determination, and thus removal, of the systematic instability. The present discussion describes the use of computer simulation techniques to determine the design parameters and the behavior of a multiloop servo system which is to correct for the systematic instability of a secondary clock also having random frequency fluctuations. Some of the work presented here has already been reported at the 1966 Frequency Control Symposium.[2]

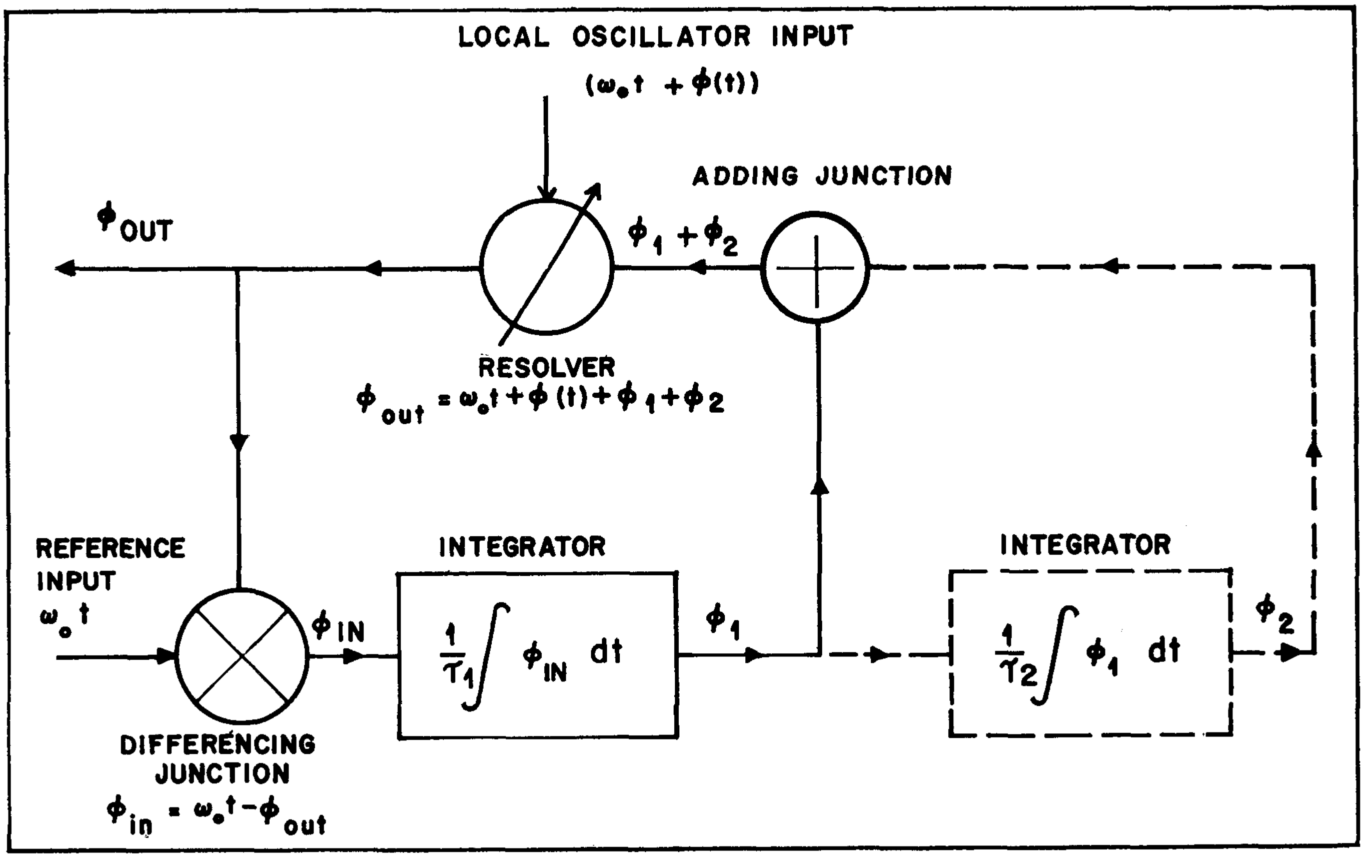

Fig. 1 — Block diagram of second order time control system.

The conventional idealized single loop servo diagram is illustrated in Fig. 1 (solid lines). The reference phase $\omega_0 t$ is assumed to result from a constant angular frequency, ω_0, from the master oscillator. The local oscillator phase from the secondary oscillator is equal to $\omega_0 t$ plus a small difference $\phi(t)$ which includes both systematic and random fluctuations about $\omega_0 t$. The function of the servo loop is to maintain the modified phase output from the local oscillator, ϕ_{out}, in agreement with the reference phase. This is done by integrating the difference between these two phases with respect to time to produce a change of phase in a direction that will reduce the phase error. By virtue of the fact that this is a negative feedback system, a step function in $\phi(t)$ will produce a servo correction which decreases as $\exp(-t/\tau_1)$, where τ_1 is the loop time constant. If instead of a step function phase error, a frequency difference, $\triangle\omega$, exists between reference and local oscillator, the uncorrected phase errors will increase linearly with time. The action of the single feedback loop will then be to supply a continuous phase correction while operating, of necessity, with a steady state phase error of magnitude $\triangle\omega\tau_1$. If now a second feedback loop is added as shown by the dotted line in Fig. 1, the phase error will be corrected as before with time constant τ_1. At the same time the sec-

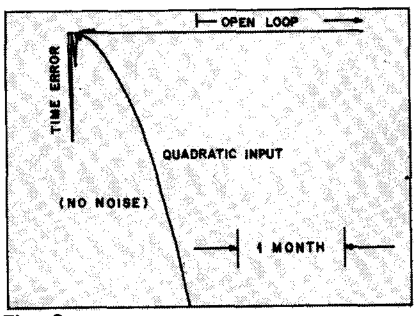

Fig. 2

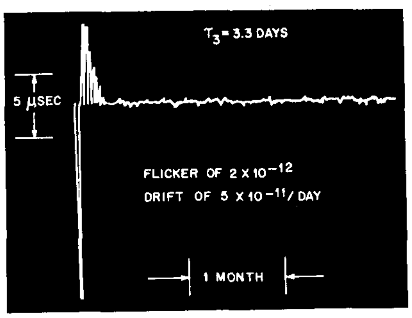

Fig. 3

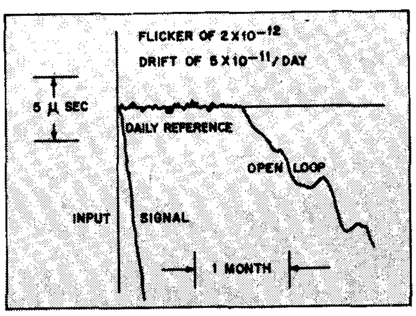

Fig. 4

Fig. 2 — Third order time control system error with linear frequency drift and no flicker noise.

Fig. 3 — Transient response of third order time control system with both linear frequency drift and flicker noise.

Fig. 4 — Open loop behavior of a third order time control system with flicker noise and after linear frequency drift removal.

ond loop integrator also contributes an error correction, reducing, with time constant τ_2, the correction necessary by the first loop until, at steady state, all the correction is supplied by the second loop, and no residual error is required as input to the first integrator. Since such a system now has the possibility of being unstable, the time constants τ_1 and τ_2 must be adjusted[3] to provide stability and desired transient response. After equilibrium has been reached, since no steady state phase error is required to actuate the servo loops, the reference may be removed and the output phase will continue to follow the reference phase even though a frequency difference exists between reference and local oscillator.

Carrying this procedure one step further, a third loop may be added, extending the performance capability to that of compensating for a linear frequency drift, that is, for oscillator aging. Then, as before, after equilibrium has been reached, the system will adjust itself so that the necessary oscillator drift correction is shared by the second and third loops with none required of the first loop. Consequently the output frequency will again remain constant after the reference frequency has been removed. After equilibrium has been reached no further comparison to the constant reference frequency would be necessary if it were not for previously mentioned random fluctuations which perturb the oscillator frequency.

If a third order feedback system is analyzed using Nyquist's stability criteria, the following time constant relationship is obtained:

$$\tau_3 > \tau_1.$$

This relationship is derived for a system where the reference is continuously available. Because the reference is typically available only intermittently, and because of the random fluctuations of the frequency of the clock to be synchronized, it is convenient to analyze the system via computer simulation techniques.

COMPUTER SIMULATION OF THE TIME CONTROL SYSTEM

A computer program was written to simulate the error response of a third order feedback time control system. The pertinent time constants and system parameters were left as variables so that optimization as well as stability might be achieved. The third time constant τ_3 is expected to be longer than the reset interval (about one day) for many of the system's applications.' The time required to reduce the system's transient response to a negligible value must be longer than τ_3, therefore, testing the system in real time for optimum operating conditions would be very impractical. If flicker noise is now introduced into the system, a determination of the characteristic response will be an overwhelming problem if the analysis need be done in real time. In real time several days would be required to evaluate one set of time constants even without random fluctuations. Several sets of time constants must be evaluated. With the addition of the random fluctuations several sets of such evaluations would be necessary for each set of time constants. Because of these problems, computer simulation becomes extremely expeditious, since it allows a time compression of at least 10^6. The systems feasibility, limiting characteristics, and optimum operating parameters may also be determined prior to construction, which in this case saved considerable time and money.

Fig. 2 is a computer plot of the output time error signal with the input being a simulated frequency drift (quadratic time departure as shown). The time is reset once each day and allowed to free-run between synchronizations. One can observe the transient response as the drift rate is automatically subtracted out; and then after several time constants the reference signal is no longer provided for synchronization (open loop), but the time error remains essentially zero.

Computer simulation of flicker noise is a recent and significant development.[4] This development along with the insertion of a linear frequency drift provides an excellent computer simulated model of a large and important class of frequency standards.

Simulated flicker noise was next generated in the computer and added to the frequency drift. Various levels of flicker noise and of frequency drift were tried and the system's simulated response was analyzed. Variation of the system's time constants showed pronounced effects on the transient response, and time constants could be chosen that would make the system either stable or unstable.

Realistic levels of flicker noise had little effect on the simulated system's response except to increase the instability if the system were unstable and to slightly increase the length of the transient response. Fig. 3 shows the transient response and the error signal for a fractional frequency drift of 5 parts in 10^{11} per day and a flicker noise level of 2×10^{-12}. The level of flicker noise is measured by computing the square root of the variance of the frequency fluctuations for an ensemble of paired, adjacent samples.[5]

Fig. 4 shows the effect of providing no correction signal (open loop) after the system has adjusted for the frequency drift of a signal having both linear frequency drift and flicker noise. The accumulated error is about 9 microseconds after 1 month in the open loop condition.

The following time constant information resulted from the computer simulation. If τ_1 is very short (about 1 second), then τ_2 need be longer than the time between synchronizations for the system to be stable. For critical damping during the transient response, τ_3 need be at least 3 times longer than τ_2. Longer times for τ_3 gave no noticeable degradation in the level of the residual error signal, but gave a proportional increase in the length of the transient response.

REALIZATION OF THE TIME CONTROL SYSTEM

On the basis of the computer results an electromechanical third order drift control system was constructed on a fairly compact layout occupying 7 vertical inches in a

19 inch relay rack. The main components include: two ball and disc integrators, three gear differentials, two 60-Hz synchronous motors, one dc servo motor and amplifier, one resolver, and one phase detector. Other components include: isolation filters and amplifiers, gear trains, mechanical counters, and a dc power supply.

The parameters such as loop time constants, frequency and drift ranges, allowable gear backlash, etc., were determined from the characteristics of the crystal oscillator to be controlled, the desired precision of the output phase, the specifications of available electromechanical devices, and the constraints determined by the computer analysis.

A 100 kHz signal derived from a high quality 2.5 MHz crystal oscillator was the signal to be controlled. This particular oscillator had a drift rate of -1×10^{-11} per day and exhibited a flicker noise level of 0.86×10^{-12}. The nonlinearities of the system contributed no more than ± 25 nanoseconds time error. The system's time can be synchronized to the reference to within 10 nanoseconds.

Since it is necessary to operate the control system continuously for a period of at least a few months, optimum loop time constants as determined by computer were modified somewhat by the limited range of the mechanical integrators and discrete values of stock items such as motor speeds and gear combinations.

The loop time constants used in this device are:

Loop 1 (phase loop), $\tau_1 \approx 2$ sec;
Loop 2 (frequency loop), $\tau_2 = 1.56$ days;
Loop 3 (drift loop), $\tau_3 = 8.68$ days.

The frequency range of the system is limited by the integrators to 18.75×10^{-10}. This allows for about six months of operation with this oscillator before both the integrator in the drift loop and the oscillator must be reset.

Synchronization between the input signal and the 100 kHz reference, which is derived from the master clock cesium beam standard oscillator, occurs automatically every twelve hours. A record of the time difference between the output of the automatic time control system and the cesium beam standard is taken on a strip-chart recorder.

The accumulated time error of the system was analyzed 12 hours after each synchronization (just prior to the next synchronization). The rms time error was 70 nanoseconds. If the systematic frequency drift is assumed to be effectively removed by the automatic time control system, the accumulated time errors will be due to the flicker noise fluctuations. An equation may be written which gives the optimum predicted rms error obtainable when flicker noise fluctuations predominate:[5,6]

$$\delta t_{rms} = \tau (\ln 2)^{-\frac{1}{2}} [\sigma(2,\tau)],$$

where $[\sigma(2,\tau)]$ is the previously defined flicker noise level. The calculated optimum predicted rms error for a flicker noise level of 0.86×10^{-12} and a prediction time, τ, of 12 hours is 45 nanoseconds. This is to be compared with the experimental result of 70 nanoseconds. The computer simulation of the system on the same basis gave an rms error of 65 nanoseconds in good agreement with the physically realized system.

REFERENCES

1. Barnes, J. A., "Atomic Timekeeping and the Statistics of Precision Signal Generators," *Proc. IEEE,* Vol. 54, February 1966, p. 207.

2. Fey, L., J. A. Barnes, and D. W. Allan, "An Analysis of a Low Information Rate Time Control Unit," *Proc. 1966 20th Annual Symp. on Frequency Control,* pp. 629-635.

3. Chestnut, H., and R. W. Mayer, *Servomechanism and Regulating System Design,* New York, Wiley, 1951.

4. Halford, D., to be published.

5. Allan, D. W., "Statistics of Atomic Frequency Standards," *Proc. IEEE,* Vol. 54, February 1966, p. 221.

6. Barnes, J. A., and D. W. Allan, "An Approach to the Prediction of Coordinated Universal Time," *Frequency,* November/December, 1967, pp. 15-20.