UNITED STATES DEPARTMENT OF COMMERCE • John T. Connor, Secretary
NATIONAL BUREAU OF STANDARDS • A. V. Astin, Director

NBS TECHNICAL NOTE 341

ISSUED JUNE 22, 1966

THE LONG-TERM PERFORMANCE OF TWO RUBIDIUM VAPOR FREQUENCY STANDARDS

B. E. BLAIR AND A. H. MORGAN

Radio Standards Laboratory
Institute for Basic Standards
National Bureau of Standards
Boulder, Colorado

NBS Technical Notes are designed to supplement the Bureau's regular publications program. They provide a means for making available scientific data that are of transient or limited interest. Technical Notes may be listed or referred to in the open literature.

For sale by the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C., 20402
Price 25 cents.

TABLE OF CONTENTS

ABSTRACT

Since mid-1961 the National Bureau of Standards has used two rubidium vapor frequency standards as transfer or Working Frequency Standards (WFS). The WFS provides a continuously available frequency which is periodically calibrated in terms of the United States Frequency Standard (USFS). This paper evaluates these calibration data over a 44.5-month period of time. It gives the long-term performance of the rubidium standards in terms of the USFS, compares them with a commercial cesium-beam frequency standard, and presents their reliability characteristics as shown by the mean-time-between-failures (MTBF). The calibration data are graphed with applicable tolerance limits for the between-adjustment periods. The average standard error of estimate about least squares lines fitted to the frequency data is about 2 parts in 10^{11}. The rubidium standards also show a rather consistent daily aging rate of parts in 10^{13}. This would indicate that the frequency of either unit would change a few parts in 10^{10} if operated continuously for one year without any frequency adjustments.

Key Words: aging, calibration (USFS), frequency standards, gas cells, performance (long-term), reliability, rubidium, stability

THE LONG-TERM PERFORMANCE OF TWO RUBIDIUM VAPOR FREQUENCY STANDARDS

B. E. Blair and A. H. Morgan

1. INTRODUCTION

In the field of atomic frequency standards the arrival of commercial rubidium (Rb) gas cells has aroused considerable interest in what one can expect of their long-term performance and stability. However, as pointed out by Farmer [1963], there is a lack of such data simply because of their relatively recent appearance. Since the first two commercially-available Rb frequency standards have operated nearly continuously over a 44.5-month period at NBS, we can provide the record of their operation for appraisal and study. This paper reports primarily on such long-term performance based on periodic calibrations with the United States Frequency Standard (USFS). Data on a third Rb frequency standard, used for a period of 3 months in 1964, are included also.

The Rb gas cell standards have been extensively described in the literature [Bender, Beaty, and Chi, 1958; Whitehorn, 1959; Andres, Farmer, and Inouye, 1959; Carpenter, Beaty, Bender, Saito, and Stone, 1960; Packard and Swartz, 1962; Arditi and Carver, 1963; Reder, 1963]. These papers trace their development from the earliest prototype models to the present-day commercially-constructed units.

This paper briefly describes the two particular Rb atomic frequency standards used at NBS (called Rb-1 and Rb-2), and presents statistics on their long-term stability in terms of the USFS. It also includes comparison data between the Rb standards and a commercial cesium (Cs) standard for a 6-month period of time, and briefly discusses relevant characteristics of the Rb standards, such as short-term stability and reliability of operation as experienced at NBS.

2. BRIEF DESCRIPTION OF THE RUBIDIUM FREQUENCY STANDARDS

Early in 1961 NBS obtained the first two commercially-available Rb atomic frequency standards which, except for short periods for repair and modification, have operated continuously from then until the present time. Each standard occupies nearly 3 cubic feet in a cabinet package weighing about 100 pounds. (A later version of Rb standard is now available in which the volume, weight, and power requirements are all reduced by a factor of 1/5 or more, and the specified performance is comparable to the older model.) Environmentally, the vital elements of the instrument are well protected. Proportionally-controlled ovens maintain temperature-sensitive components within set tolerance limits of temperature, and a triple mu-metal shield surrounds elements susceptible to external magnetic fields. A brief description of the internal functioning of these standards follows; detailed operational features are available in the literature [Andres et al., 1959; Packard and Swartz, 1962; Arditi and Carver, 1963].

Basically, the Rb frequency standard consists of a 5-MHz crystal oscillator which is stabilized by an atomic resonance of rubidium through suitable frequency synthesizers, multipliers, and automatic frequency control. Optical pumping and light-transmission monitoring form the basis for detecting atomic resonance of Rb^{87} between the ground state levels $F = 2$, $M_F = 0$ to $F = 1$, $M_F = 0$. Figure 1 shows a simplified block diagram of the Rb frequency standard. The heart of the rubidium standard is the optical package which contains an Rb^{87} lamp, an Rb^{85} filter, an Rb^{87} gas cell within a microwave cavity, and a silicon photo cell detector.

Functional operation of the Rb frequency standard is as follows: after the light from the Rb^{87} lamp is directed through the filter and gas cells, it is detected by the photo cell. The intensity of light transmitted through the gas cell to the detector depends on the difference between the microwave excitation frequency and the Rb^{87} transition frequency. Initially, through a repetitive process called optical pumping [Kastler, 1957], an excessive number of Rb atoms jump from the F = 1 level to the upper F = 2 ground level, accompanied by high transparency and a consequently higher transmission of light through the gas cell. Exciting the cavity to the critical transition frequency between the two ground state energy levels (approximately 6834 MHz), results in a net gain of Rb atoms back to the F = 1 ground level and a degree of opacity to the light beam focused on the silicon detector. To provide an error signal for correcting the frequency of the radiation driving the cavity, a low frequency (107 cps) phase modulation of the microwave energy is applied to the cell cavity. Thus, the light reaching the photo cell fluctuates at this frequency. A phase detector compares the photo cell output signal with the input modulation frequency, and the resultant phase error signal continuously locks the quartz crystal oscillator frequency to that of Rb transition through an electronic servo system. Thus, all of the synthesized frequency outputs of the crystal oscillator are continuously related to the Rb^{87} transition frequency. Synthesis of the Rb transition frequency from the basic 5-MHz signal of the crystal oscillator occurs through harmonic multiplication and subtraction using transistors and varactor diodes.

Because buffer gases are used in the Rb vapor cell to facilitate optical pumping and to reduce the Doppler width of the hyperfine absorption lines, the effective Rb transition frequency may be shifted either up or down, depending upon the type and pressure of buffer gas used. Consequently, one cannot use the modified transition output frequency as a primary standard, but must reference it to a known frequency. On the other hand, with proper selection of the buffer gas and its sealed-off pressure at the time of manufacture, one can obtain any offset frequency that is required for different time scales. Fine frequency tuning, by about $+5 \times 10^{-9}$, is possible also through adjustment of the magnetic field. Thus, the limitations which preclude the use of the Rb vapor instrument as a primary frequency standard can prove of value if precise matching and adjusting of output frequencies is needed.

3. THE USFS REFERENCE BASE

The defined reference for all standard frequency measurements is the unperturbed Cs/transition frequency ($F = 4$, $M_F = 0$) $\leftrightarrow$ ($F = 3$, $M_F = 0$) of 9,192,631,770 Hz at zero magnetic field [NBS, 1964]. In the United States this primary reference frequency has been derived from NBS-constructed Cs-beam units which form the United States Frequency Standard (USFS) [Mockler et al., 1960; Mockler, 1964]. From 1960 to early 1965, the USFS consisted of two such units designated as NBS-1 and NBS-2 with an accuracy of 1 part in 10^{11}. Presently, the USFS is the newly-constructed NBS-3 which is a longer Cs-beam unit and accurate to $\pm 1.1 \times 10^{-12}$ (1 σ value) [Beehler et al., 1966]. These accuracy figures derive from the RMS combination of all estimated uncertainties related to the USFS which may cause deviations from the true cesium resonance.

3.1. Function of Working Frequency Standards and Relation to USFS

Because the USFS does not operate continuously, the NBS established a Working Frequency Standard (WFS), which consists of the Rb standards and stable quartz-crystal oscillators, and provides standard frequencies without interruption. Periodic measurements relate the WFS to the USFS. From mid-1961 to January 1964, the Rb standards were a direct link in the calibration and frequency control of the NBS standard frequency broadcasts [Blair and Morgan, 1965]. Presently, one of the Rb standards in the WFS contributes to the NBS-A time scale [Barnes et al., 1965; Andrews, 1965] which, since January 1964, has given time-base control to such NBS broadcasts. In addition, the WFS serves as a continuous frequency reference for the intercomparison of atomic frequency standards via VLF broadcasts [Morgan et al., 1965] and provides an internal calibration source to the NBS Boulder Laboratories.

3.2. Methods of NBS Frequency Calibrations in Terms of USFS

At regular intervals (usually daily except weekends and holidays) the Rb standards were measured in terms of the USFS. The NBS Boulder Laboratories have used two methods of frequency calibration with comparable results [Beehler et al., 1962]. In the initially established procedure, called the "manual" method, the 5-MHz output of the standard under calibration directly drives the multiplier chain of the USFS. For each measurement, the operator selects an upper and lower Intermediate Frequency (IF), equally offset from the cesium resonance center frequency, by judging the equality of the atomic beam detector current for each condition. For this method each reported value is the grand mean of from 20 to 50 independent measurements made over a period of 15 to 30 minutes. Typically, such measurements show a standard deviation less than 1 part in 10^{11}. (It should be pointed out that the manual method has not been used routinely since early 1963.)

In the second or "servo" method, similar multiplication and klystron control circuitry is used as in the manual method. In this case, however, the signal from a stable transfer oscillator becomes the input to the USFS multiplier chain and its frequency is then stabilized by the cesium resonance by means of an electronic servo system. Frequency comparisons are made between the transfer oscillator and the Rb frequency standard by measuring the average period of the difference frequency between them. In this case, the reported daily value is the grand mean of about ten 200-second averages taken over a measurement period of about 1/2 hour.

The standard deviation of such values is near 2 parts in 10^{12} for a typical measurement period. (Standard deviations of the means of such measurements in terms of the USFS are near 5 parts in 10^{13} [Beehler and Glaze, 1965].)

In a third measurement method, the Rb signal is continuously compared with a drift-corrected oscillator (DCO) in a phase-lock servo system [Looney, 1961]. Presently, the DCO is the direct controlling link of the NBS low frequency broadcasts from Ft. Collins, Colorado [Andrews, 1965; Barnes et al., 1965]. The frequency of the DCO is stable to several parts in 10^{12} of the zero offset frequency and is periodically compared to the USFS. By this method, used frequently in 1964, the daily value of the Rb frequency was determined by integrating the phase analogue records over 24-hour periods.

One additional method of assigning daily frequency values was used infrequently but should be mentioned. This technique consists of continuously comparing the Rb-1 and Rb-2 signals in a phase-lock servo system. When one standard is calibrated then, in terms of the USFS over a given time period, a frequency value of the other standard is determined for the same time period from the Rb-1 versus Rb-2 phase record. Such comparisons are made typically in parts in 10^{12}, and we used this technique for determining Rb-2 values for parts of periods 5 to 7.

4. PERFORMANCE DATA

This section presents the long-term performance of the Rb standards, gives their relative stability versus a commercial cesium standard for a 6-month period, and briefly discusses their short-term stability and power spectrum characteristics.

4.1. Long-Term Performance

Aging effects together with random day-to-day variability most often characterize the long-term stability of a frequency standard. Both factors may be seen in the data plots of the two Rb standards for the period August 1961 to April 1965, given in figures 2 and 3. Summary statistics of the least squares lines through these data are given in table 1. (These calculations were obtained with a least squares statistical program run on a high speed electronic computer. An appendix describes this program.) Each plotted point, as noted previously, is a frequency measurement in terms of the USFS or related thereto. However, it was not always possible to obtain a daily reading and the dotted points on the plots indicate such missing data. Since the use of any interpolated data would cause an unwarranted smoothing effect in the results, the analysis consists of actual measurements only. On the other hand, published values were omitted at times because of either faulty operation, service work, or scarcity of data for a given subperiod. Such values, although true and correct, do not contribute to the overall picture of what one can reasonably expect of a normally operating rubidium frequency standard.

From period to period a random aging (drifting) pattern for each standard is apparent in figures 2 and 3. This is more graphically shown in figure 4 where the average drift rates of Rb-1 and Rb-2 are plotted for periods between each adjustment during 1961-1965. In general, Rb-1 shows a persistent negative drift rate, whereas Rb-2 exhibits both positive and negative rates. During the first two or three years of operation, major component replacements were made, and it is not known what effect this may have had on the drift rates. Since we desired to keep the frequency of the WFS rather close to the nominal offset, quite a few frequency adjustments also were made periodically as is shown in the data plots. (The nominal offset is an approximation, agreed upon internationally, to the difference in the rate of occurrence of time ticks on the universal scale (UT2) and seconds pulses on the atomic, or ephemeris, scale [Hudson, 1965]. Such frequency offsets corresponded to -150×10^{-10} in 1961, 1964, and 1965; and -130×10^{-10} in 1962 and 1963.) The standards show an average daily drift rate of parts in 10^{13} during the period under study. The two curves in figure 4 show some cross correlation, but this is not too surprising since the standards are identical and operate in the same environment.

The variability about the least squares lines is indicated by the standard error of estimate, $S_{y/x}$, (a measure of the scatter in the observed points in the vertical or y direction about the least squares line) [Crow et al., 1960], shown in table 1. (The standard deviation of the frequency values, S_y, is usually larger, since it includes the effects of aging.) Table 2 gives some yearly statistics of the performance of the two standards for the 44.5-month period. Interestingly, from year to year there seems to be a decreasing trend in $S_{y/x}$. This results, perhaps, either through an improvement in measurement methods, or replacement of various components in the standards, or both. The average $S_{y/x}$ for these standards is 12 to 13 parts in 10^{12}. This figure includes the high initial variation in 1961 and 1962 caused by component failures and their subsequent replacement. It is noted that for several periods with substantial number of days, n, ($n \geq 20$), $S_{y/x}$ equals or is less than 6 parts in 10^{12}.

The Rb-2 standard experienced relatively small disturbance from March 1964 to April 16, 1965. During this period Rb-2 operated continuously and NBS made six discrete frequency adjustments, with no component alterations or replacements. Figure 5 gives a plot of these Rb-2 data with the frequency adjustments removed for convenience. A least squares line fits the points quite well; the average negative drift rate is about 8 parts in 10^{13} and the standard error of estimate, $S_{y/x}$, is

0.10×10^{-10}. There appear to be small changes in the drift rate, however, during each of the six different periods, as shown in figure 4, and this accounts for a somewhat high average $S_{y/x}$, for the overall period.

Data for a later-model standard, Rb-64, were also obtained for the period January 3 to March 30, 1964, and are shown in figure 3 and table 1. Generally, the performance statistics for Rb-64 agree well with those of Rb-1 and Rb-2; interestingly enough, however, Rb-64 showed very little frequency aging during this period.

4.2. Comparison of Two Rubidium and One Cesium (Commercial) Atomic Frequency Standards

For an approximate 6-month period (August 1962 to February 1963) a commercial Cs frequency standard was also used as a part of the WFS. Frequency calibrations of this standard were made and a plot of these values, together with those of Rb-1 and Rb-2, is shown in figure 6. (For ease of comparison, several discrete frequency adjustments were removed from the data of both Rb standards.) As may be seen, during this period, Rb-2 had an average positive, drift rate; however, Rb-1 and the Cs standard show average negative rates. The oven heater of the Cs beam tube was not functioning correctly for much of this period and what effect this had on its apparent drift rate is not known. During this comparison period, the Rb-2 data were the most variable with an $S_{y/x}$ of 0.2 parts in 10^{10}; but the Rb-1 and the Cs standard data each show an $S_{y/x}$ of slightly more than 0.1 part in 10^{10}.

4.3. Short-Term Stability

Although the major burden of this report is the long-term performance of the Rb standards (for periods of months to years), some mention will be made of their short-term stability. The best short-term stability (standard deviation) that has been reported was near 1 part in 10^{12} for averaging times of 24 seconds for each sample during an elapsed measurement period of about 13 hours [Packard and Swartz, 1962]. Packard and Swartz [1962] found that the measured stability is inversely proportional to the averaging time, due to both the low signal-to-noise ratio of the internal crystal oscillator output signal and the phase instability of the comparison instrumentation.

Analogue records of the continuous phase difference between two frequency standards (as given by a phase-lock servo system [Morgan and Andrews, 1961]), provide some indication of their relative stability. Figure 7 shows a sample analogue record which gives the phase difference between Rb-1 and Rb-2. The slope of the trace indicates the frequency difference rate between the two standards, and the minor variations in the trace suggest good relative phase stability, as obtained with a servo averaging time of nearly one minute. The standard error of estimate about a least squares line fitted to points on this trace at 10-minute intervals is about 0.01 μsecond, corresponding to an RMS instability in frequency contributed by both standards of less than 2 parts in 10^{11}. Assuming that Rb-1 and Rb-2 contribute equally to this variation, one can compute an instability of about 1 part in 10^{11} for each standard. Because of the relatively long response time of the servo system (servo motor is gear-reduced by a factor of 6000:1) and a paper speed of 6 inches per hour, it is difficult to determine short-term stabilities for periods of 60 seconds or less from such records.

The output power spectrum of a frequency standard may also give some indication of its short-term stability. NBS has made power spectrum measurements of a rubidium standard by comparing a multiplied 5-MHz output to an ammonia maser reference signal at 24 GHz [Barnes, 1964; Barnes and Mockler, 1960]. These measurements gave a clean center spectrum with a bandwidth at the half-power points of about 4 Hz [Packard and Swartz, 1962]. (The comparison instrumentation, however, may have affected these measurements to some extent, and a more realistic bandwidth is probably less than 2 Hz.) Also, there were low level sidebands on the carrier (40 dB below the 24-GHz signal) at ±60, ±107, and ±120 Hz. These sidebands probably originated in the Rb standards. Note especially that the phase modulation frequency, which provides an error signal in the Rb standard, is 107 Hz. In addition, the noise level of the ammonia maser spectrum analyzer is at least -47 dB below the 24-GHz carrier [Glaze, 1966].

5. RELIABILITY OF Rb-1 AND Rb-2

Reliability of electronic equipment is often shown to follow an exponential frequency distribution [Lloyd and Lipow, 1962], such as $R = \exp - T/\bar{t}$, where

R = Reliability or probability of survival

T = Time of projected operation, hrs

$\bar{t}$ = MTBF (Mean time between failure - hrs).

Using the calculated MTBF, $\hat{\bar{t}}$, as an estimator of $\bar{t}$, one can show that, for a period of one year (8760 hrs), the Rb-2 maximum likelihood estimator, $\hat{R}$, is a factor of 5 greater than that for Rb-1. Such a comparison number will vary with the time, T, due to the exponential factor; nevertheless, it gives some measure of their relative service performance.

Table 3 gives the service performance data of the two frequency standards; bar graphs of their MTBF are shown in figure 8. Rb-2 shows over twice the MTBF of Rb-1 and about half the number of failures. However, the principal cause of failure, namely the tube-type lamp exciter, was the same for both standards. After the units were modified with a transistorized lamp exciter, considerably improved service resulted. For instance, a trouble-free period of 9040 hours followed the modification of Rb-2. A time weighted average for both standards covering the entire period of operation gives a MTBF of 5570 hours.

From the performance data over this 44.5-month study certain statistical inferences can be made about this model of standard in general. That is, one can derive confidence intervals which, with a stated confidence, should enclose the true MTBF. The following probability expression gives such confidence intervals:

$$P\left[\frac{2n\hat{t}}{\chi^2_{2n;\,\alpha/2}} < \bar{t} < \frac{2n\hat{t}}{\chi^2_{2n;\,1-\alpha/2}}\right] = 90\%,$$

where

α = 0.10

P = Probability

n = Number of failures

$\bar{t}$ = MTBF

$\hat{t}$ = Computed estimate of $\bar{t}$ (Total time between failures/ Number of failures)

$\chi^2_{2n;\,\alpha/2}$ = Percentage point of chi-square distribution with 2n degrees of freedom; the probability of χ^2_{2n} exceeding this is $\alpha/2$.

(NOTE: χ^2_{2n} is defined as $\frac{2n\hat{t}}{t}$ for a random sample of $\hat{t}$, [Lloyd and Lipow, 1962; Carroll, 1962]. From this relationship confidence intervals for reliability, R, also can be found.)

The 90% confidence interval, then, for MTBF of this model Rb standard, based on Rb-1 and Rb-2 service data, is $3610 < \bar{t} < 9930$.

The failures shown in table 3 are classed as catastrophic. That is, the standards completely stopped operation and/or it was impossible to phase-lock the controlling oscillator to the Rb resonance. In all fairness, we should state that at various times new components were installed in the standards, and there are transitory effects in the frequency data before and after such installation. It was also necessary from time to time to align the synthesizer and second harmonic amplitude. Rb-2 showed progressively increased time between alignments. From the middle of 1962 to early 1963, we note periods of 3000 to 4000 hours between alignments. From then to early 1965, such periods increased to 6000 and 9000 hours.

The continuous operation of the Rb frequency standards under laboratory conditions and over a long period of time at NBS has shown areas where improvements were necessary and required. (The transistor circuitry replacement of vacuum tubes in the lamp exciter is one such instance.) As further improvements are continuing, it is not difficult to foresee failure-free, unattended operation of Rb standards for periods greater than one year.

6. CONCLUSIONS

Over a period of almost four years these two particular rubidium frequency standards have given quite reliable service at NBS in performing the many functions of continuously-running, transfer and reference standards. After initial malfunctions were corrected, largely through the use of transistorized circuitry in the lamp exciter, improved operation resulted. For example, after such modification Rb-2 gave over 9000 hours of trouble-free service from April 12, 1964, to April 17, 1965.

In terms of the USFS, Rb-1 and Rb-2 each showed aging which averages to parts in 10^{13} per day for the periods under study. Assuming no malfunctions, this indicates that if each frequency standard were left unadjusted, at the end of one year's continuous operation its frequency would have changed only by parts in 10^{10}. If the performance of Rb-64 during a 3-month period is a good sample of the performance of improved models, their aging would be considerably less. The day-to-day average stability (standard error of estimate) of Rb-1 and Rb-2 advanced to about 0.1 part in 10^{10} from the initial value of 0.2 to 0.3 parts in 10^{10}. Further improvement is desirable in the general areas of aging and variability.

7. ACKNOWLEDGMENTS

We wish to thank Roger Beehler and his staff for the many calibrations of the Rb standards in terms of the USFS, and Vincent Heaton, Catherine Barclay, Joan Berube, Jean Petersen, and Ursula Palmer for their contribution to the data analysis work. We appreciate the help of Lyle Edlin in various aspects of the data collection. Thanks are extended also to Carole Craig for her careful typing of this paper.

8. REFERENCES

Andres, J. M., D. J. Farmer, and G. T. Inouye (October 1959), Design studies for a rubidium gas cell frequency standard, IRE Trans. on MIL ELECT., MIL-3, pp. 178-183.

Andrews, D. H. (December 1965), LF-VLF frequency and time services of the National Bureau of Standards, Trans. Instr. and Meas. IEEE, IM-14, No. 4, pp. 233-237.

Arditi, M., and T. R. Carver (January 1963), The principles of the double resonance method applied to gas cell frequency standards, Proc. IEEE, 51, pp. 190-202.

Barnes, J. A. (June 1964), Personal communication.

Barnes, J. A., D. H. Andrews, and D. W. Allan (December 1965), The NBS-A time scale - its generation and dissemination, Trans. Instr. and Meas. IEEE, IM-14, No. 4, pp. 228-232.

Barnes, J. A., and R. C. Mockler (September 1960), The power spectrum and its importance in precise frequency measurement, IRE Trans. on Instr., I-9, pp. 149-155.

Beehler, R. E., W. R. Atkinson, L. E. Heim, and C. S. Snider (December 1962), A comparison of direct and servo methods for utilizing cesium beam resonators as frequency standards, IRE Trans. on Instr., I-11, No. 3 and 4, pp. 231-238.

Beehler, R. E., and D. J. Glaze (December 1965), Private communication.

Beehler, R. E., D. Halford, R. Harrach, D. Allan, D. Glaze, C. Snider, J. Barnes, R. Vessot, H. Peters, J. Vanier, L. Cutler, and L. Bodily (February 1966), An intercomparison of atomic standards, Proceedings Letters, Proc. IEEE, 54, No. 2, pp. 301-302.

Blair, B. E., and A. H. Morgan (July 1965), Control of WWV and WWVH standard frequency broadcasts by VLF and LF signals, Radio Sci. J. Res. NBS 69D, No. 7, pp. 915-928.

Bender, P. L., E. C. Beaty, and A. R. Chi (November 1958), Optical detection of narrow Rb^{87} hyperfine absorption lines, Phys. Rev. L. 1, No. 9, pp. 311-313.

Carpenter, R. J., E. C. Beaty, P. L. Bender, S. Saito, and R. O. Stone (September 1960), A prototype rubidium vapor frequency standard, IRE Trans. on Instr., I-9, pp. 132-135.

Carroll, J. M. (November 1962), Reliability: 1962, Electronics, 35, No. 48, pp. 53-76.

Crow, E. L., F. A. Davis, and M. W. Maxfield (1960), Statistics manual (Dover Publications, Inc., New York, N. Y.), pp. 147-165.

Farmer, D. J. (May 1963), Performance and application of gas cell frequency standards, Proc. 17th Ann. Freq. Con. Sym. (U. S. Army Res. and Dev. Lab., Ft. Monmouth, New Jersey), pp. 449-461.

Glaze, D. (April 1966), Personal communication.

Hudson, G. E. (August 1965), Of time and the atom, Physics Today, pp. 34-38.

Kastler, A. (June 1957), Optical methods of atomic orientation and of magnetic resonance, J. Opt. Soc. Am., 47, pp. 460-465.

Lloyd, D. K., M. Lipow (1962), Reliability: management, methods and mathematics (Prentice-Hall, Englewood Cliffs, New Jersey), pp. 190-195.

Looney, C. H., Jr. (February 1961), A very low frequency synchronizing system, Proc. IRE, 49, No. 2, pp. 448-452.

Mockler, R. C. (May 1964), Atomic frequency and time standards, Radio Sci. J. Res. NBS 68D, No. 5, pp. 523-527.

Mockler, R. C., R. E. Beehler, and C. S. Snider (September 1960), Atomic beam frequency standards, IRE Trans. on Instr., I-9, pp. 120-132.

Morgan, A. H., and D. H. Andrews (April 1961), Méthodes et techniques de contrôle des ondes kilométriques et myriamétriques aux Boulder Laboratories, Comité Consultatif pour la Définition de la Seconde auprès du Comité International des Poids et Mesures, 2e session, Annexe 6, (Gauthier-Villars & Cie, Paris), pp. 68-72.

Morgan, A. H., E. L. Crow, and B. E. Blair (July 1965), International comparison of atomic frequency standards via VLF radio signals, Radio Sci. J. Res. NBS 69D, No. 7, pp. 905-914.

Natrella, M. G. (August 1963), Experimental statistics, NBS Handbook 91, Chapter 22 (Notes on statistical computations), pp. 22-1 to 22-4.

NBS (December 1964), World sets atomic definition of time, NBS Tech. News Bull., 48, pp. 209-210.

Ostle, B. (1963), Statistics in research, (Iowa State University Press, Ames, Iowa), 2nd Edition, pp. 159-206.

Packard, M. E., and B. E. Swartz (December 1962), The optically pumped rubidium vapor frequency standard, IRE Trans. on Instr., I-11, pp. 215-223.

Reder, F. H. (May 1963), Achievements and problem areas of atomic frequency control, Proc. 17th Ann. Freq. Con. Sym. (U. S. Army Res. and Dev. Lab., Ft. Monmouth, New Jersey), pp. 329-371.

Whitehorn, R. M. (May 1959), Gas cell frequency standards using buffer gases and buffer walls, Proc. 13th Ann. Freq. Con. Sym. (U. S. Army Res. and Dev. Lab., Ft. Monmouth, New Jersey), pp. 648-654.

9. APPENDIX

9.1. Description of Least Squares Statistical Program for Use With High Speed Electronic Computer

The least squares program was written to obtain simple linear regression coefficients of standard frequency data which show an aging or drift effect from day to day. This program analysis assumes an independent variable, y, corresponding to daily frequency values. The program requires a set of data such as

$$x_1 \quad y_1$$

$$x_2 \quad y_2$$

$$\cdot \quad \cdot$$

$$\cdot \quad \cdot$$

$$x_j, \; y_j, \text{ where } j = 1 \text{ to } n$$

and assumes a straight line as the best fit to these paired data. By the method of least squares, one can determine the sample regression curve of y on x by minimizing the sum of squares of the vertical (or y) deviations of each frequency value from the average curve [Crow et al., 1960]. The equation of this sample regression line is $y = a + bx$, where a is the intercept and b is the slope of the regression line.

The program prints out both the input data (given in parts of 10^{10} to 2 decimal places) and standard regression coefficients in summary form. These regression coefficients are computed by formulas given in most statistical texts [see Crow et al., 1960; Ostle, 1963]. The program accepts a maximum of up to 370 days of y frequency values. Missing y values are neglected; however, the x values are advanced the corresponding number of missing days; thus, each frequency value matches the day or x variable it is reported for. Although the input data are given to 2 decimal places, double precision techniques are employed in the program whereby 18 significant figures are carried in the calculations. The printout gives half this number of figures, and the final results are rounded to be consistent with the original data. This follows standard statistical practice to insure that errors of computation are small in relation to the statistical quantity being determined [Natrella, 1963].

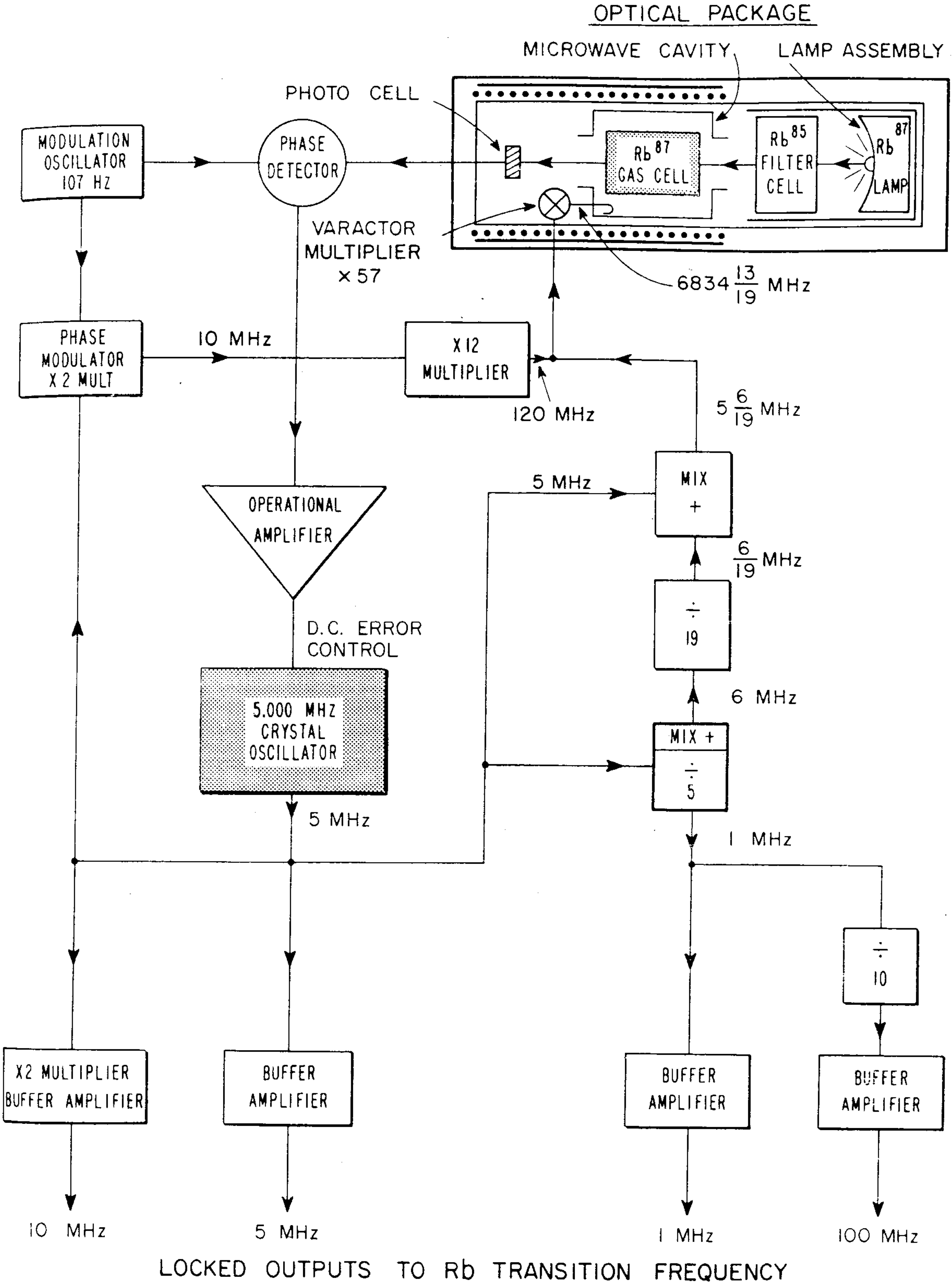


FIG. 1 BASIC SYSTEM OF Rb ATOMIC FREQUENCY STANDARD

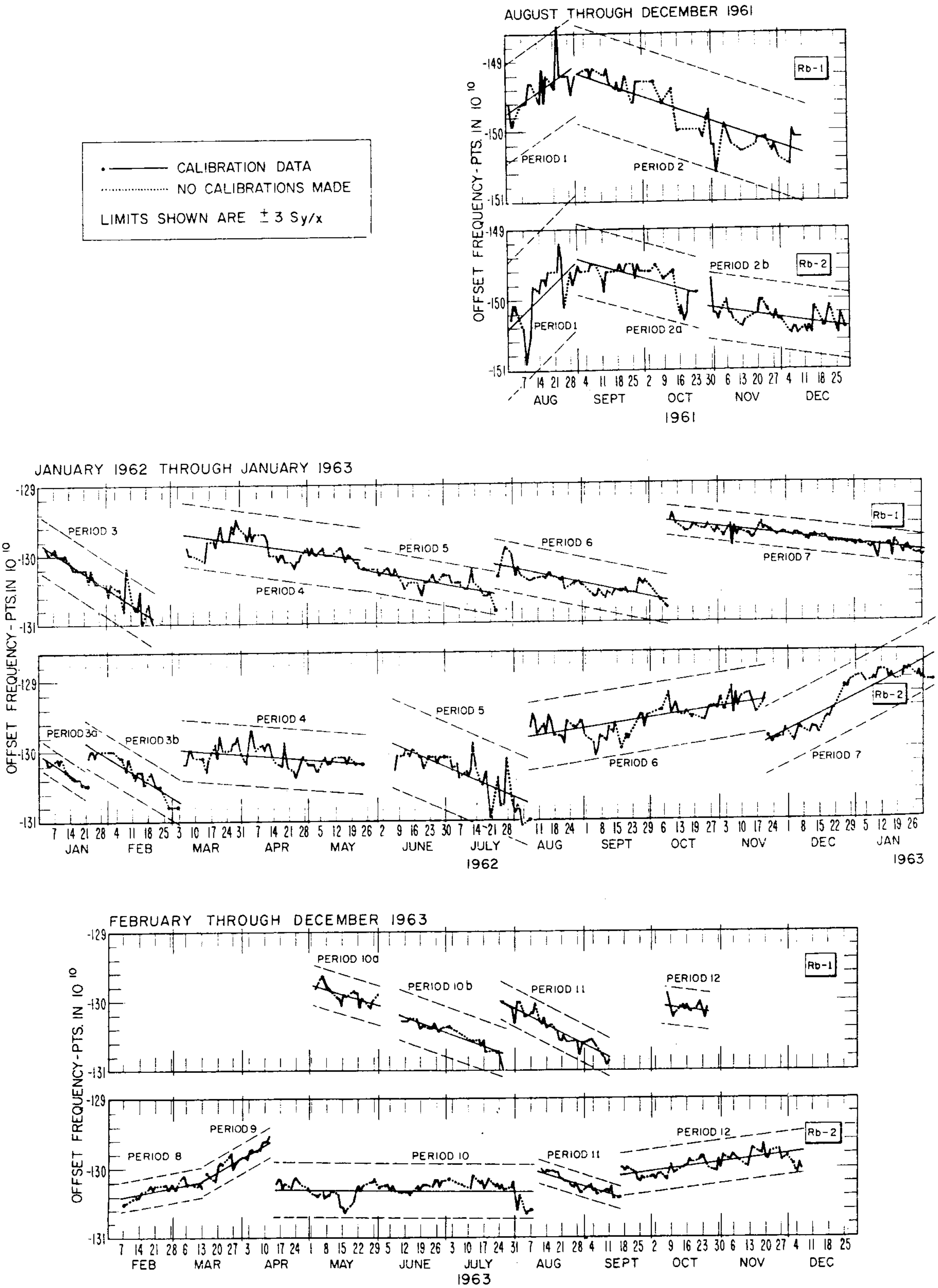


FIG. 2 CALIBRATION DATA OF Rb-1 AND Rb-2 FROM AUGUST 1961 TO DECEMBER 1963

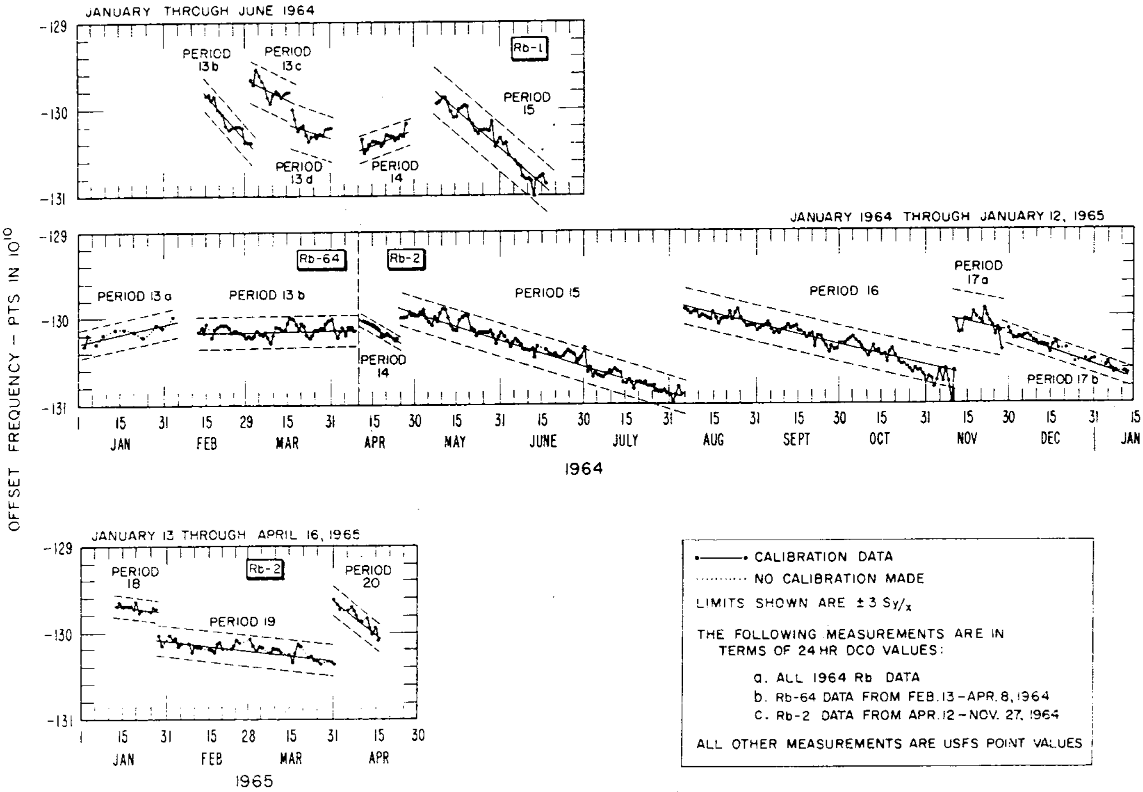


FIG. 3 CALIBRATION DATA OF Rb-1 AND Rb-2 DURING 1964 TO EARLY 1965

(Rb-64 DATA GIVEN IN PERIOD 13)

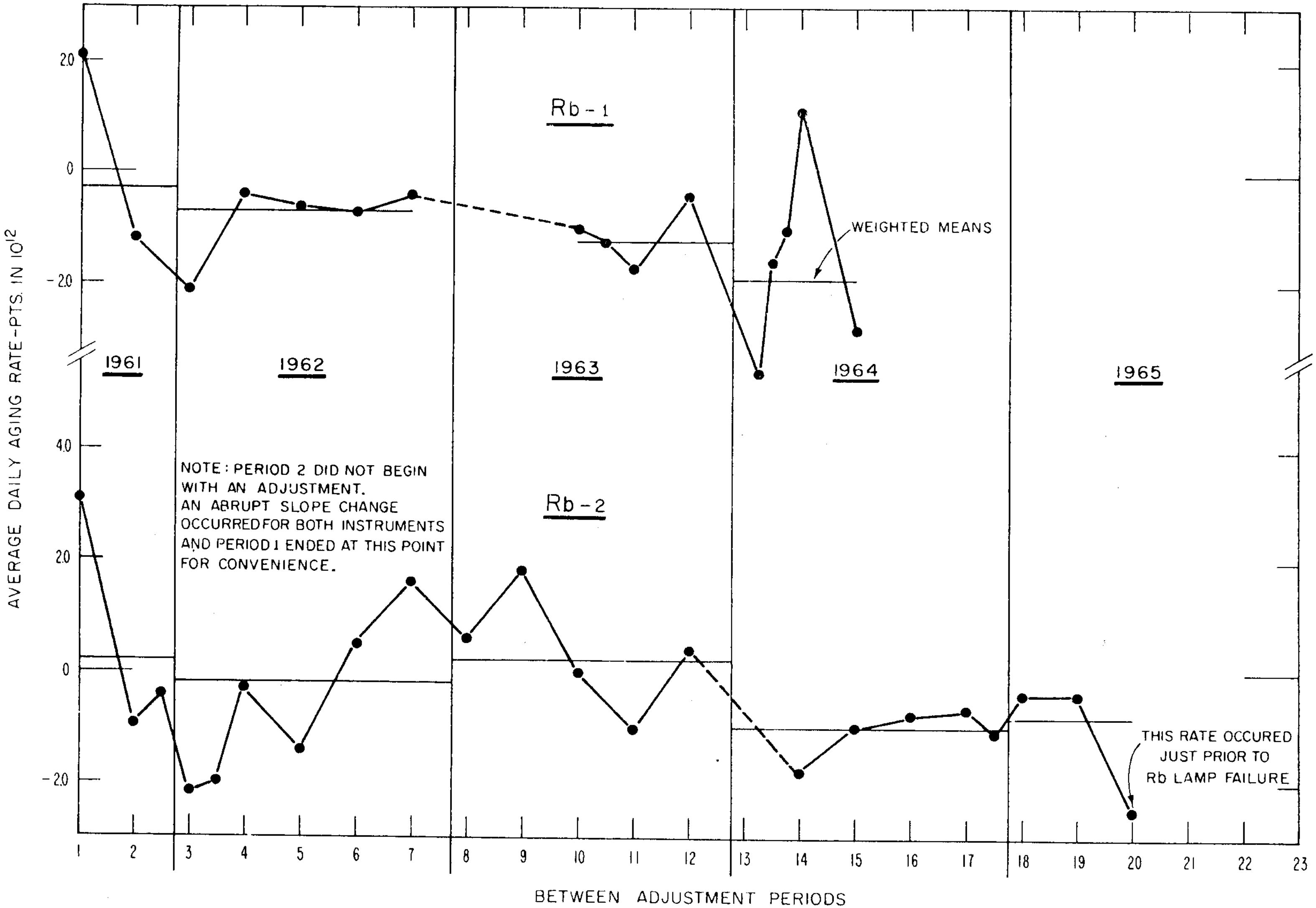


FIG. 4 AGING RATES OF Rb ATOMIC FREQUENCY STANDARDS

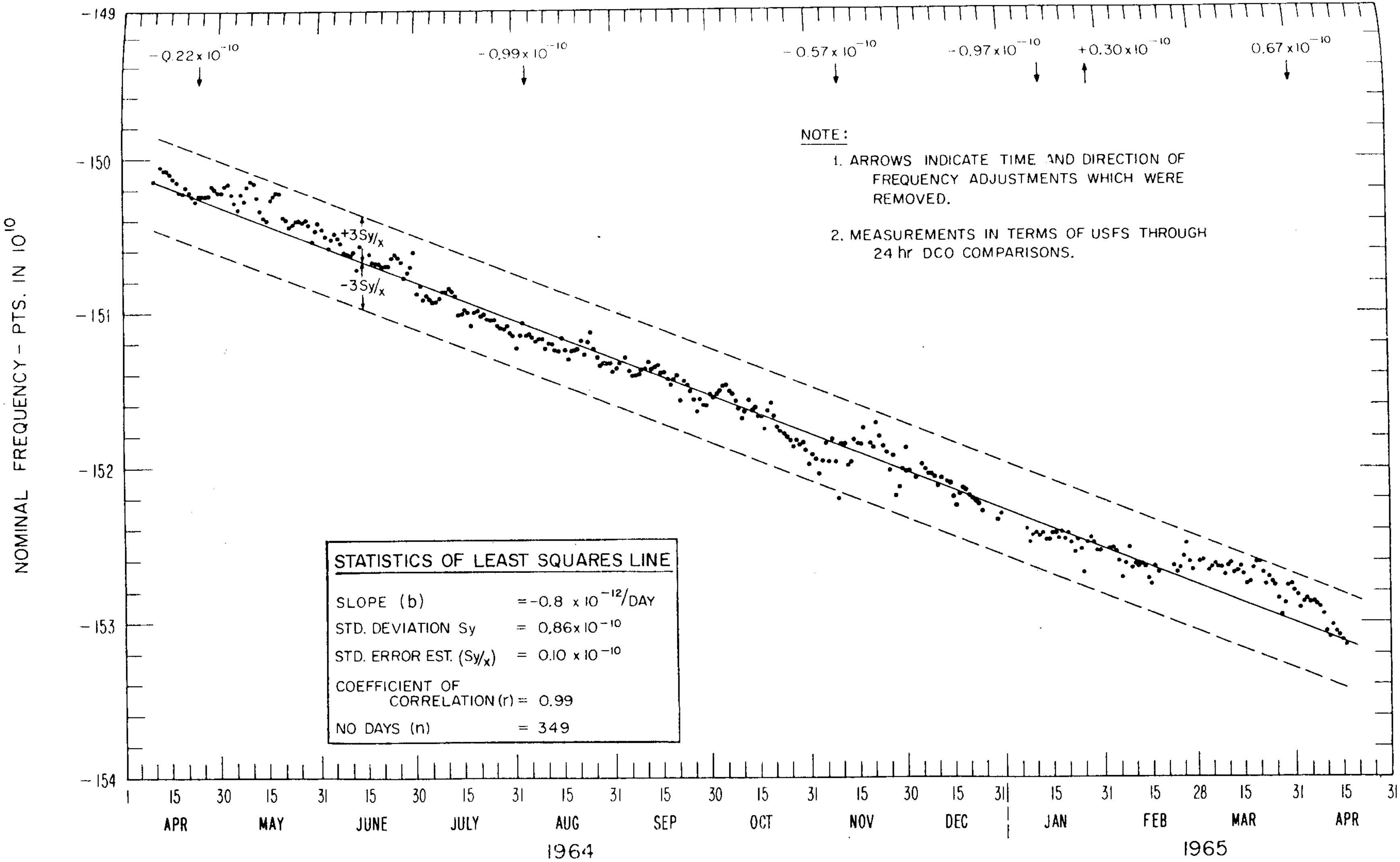
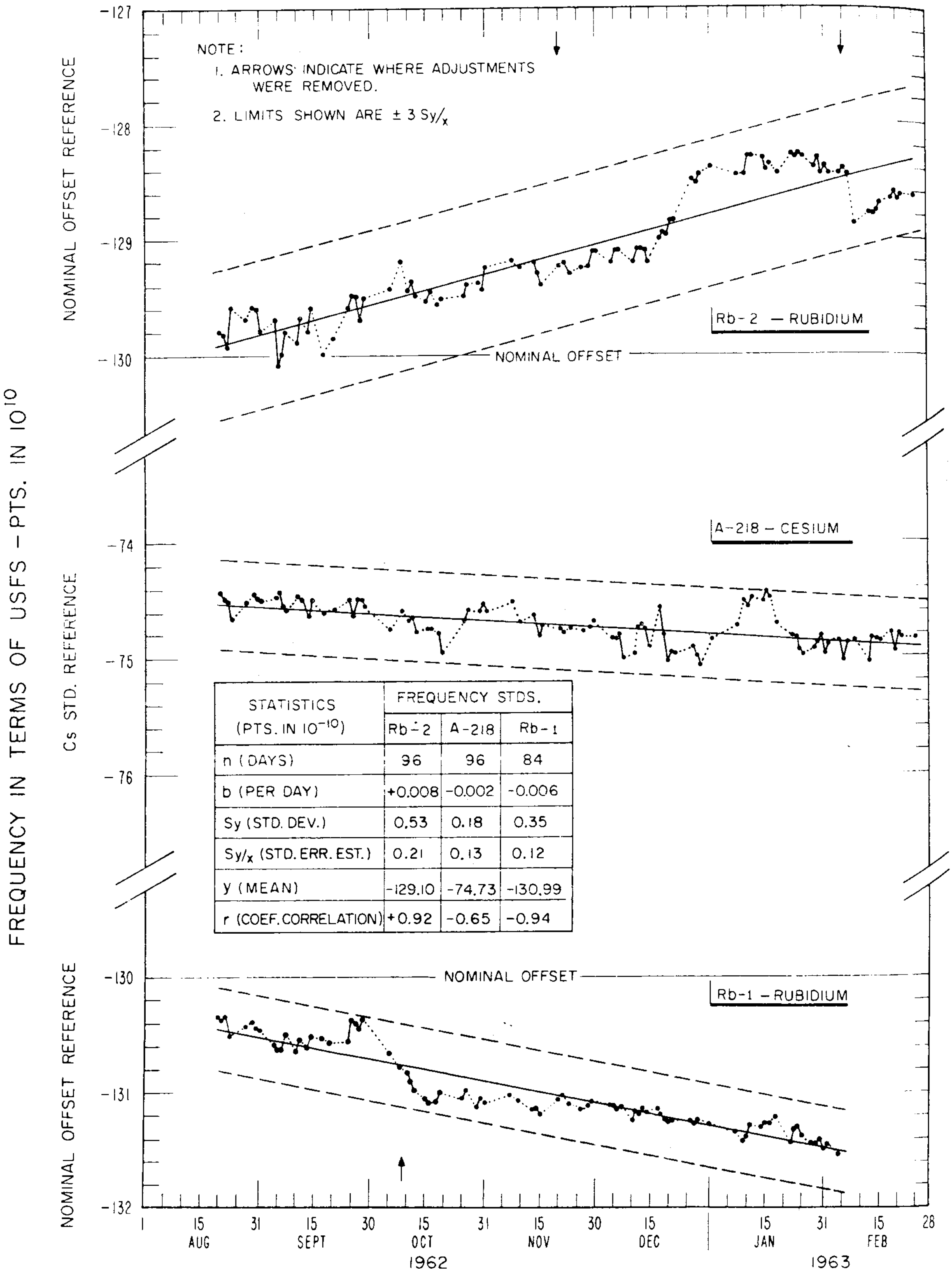
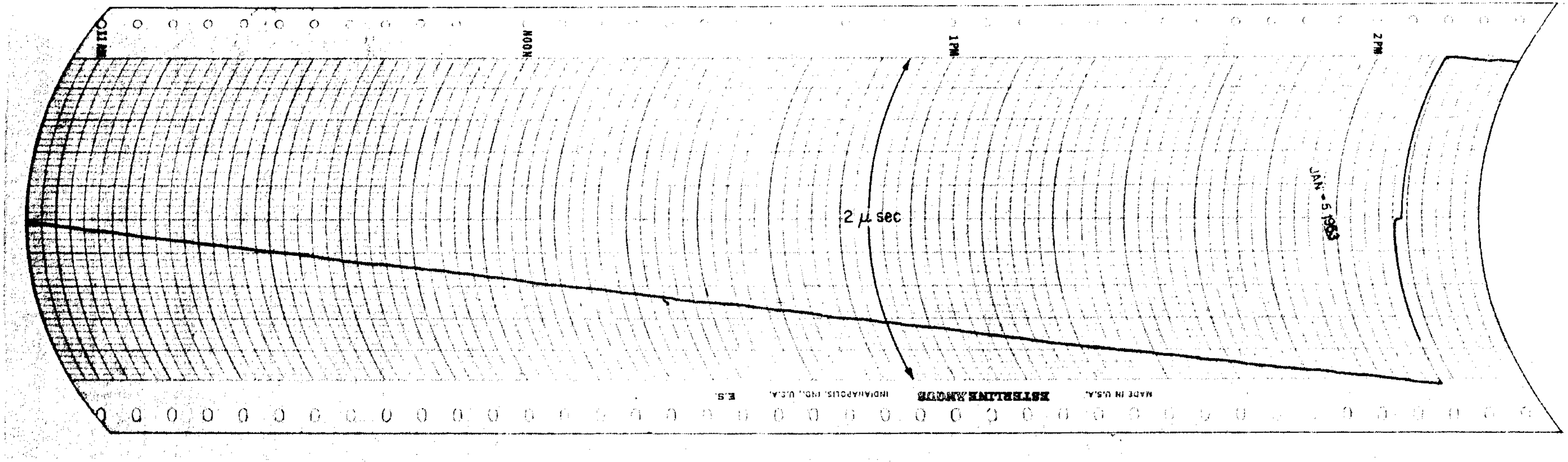


FIG. 5 CUMULATIVE FREQUENCY OF Rb-2 FROM APRIL 12, 1964 TO APRIL 16, 1965 (WITH 6 FREQUENCY ADJUSTMENTS REMOVED)



STATISTICS (PTS. IN 10^{-10})	FREQUENCY STDS.		
	Rb-2	A-218	Rb-1
n (DAYS)	96	96	84
b (PER DAY)	+0.008	-0.002	-0.006
Sy (STD. DEV.)	0.53	0.18	0.35
Sy/x (STD. ERR. EST.)	0.21	0.13	0.12
y (MEAN)	-129.10	-74.73	-130.99
r (COEF. CORRELATION)	+0.92	-0.65	-0.94

FIG. 6 FREQUENCY COMPARISONS OF Rb AND Cs ATOMIC FREQUEN[CY] STANDARDS FOR 6 MONTH PERIOD. (TERMS OF USFS)



DIFFERENCE RATE BETWEEN Rb-1 AND Rb-2 = 7.6 μsec/24 hr
(Rb-1 IS 0.88×10^{-10} LOWER THAN Rb-2)

VARIATION ABOUT LEAST SQUARES LINE $\cong$ 0.01 μsec.
(DATA POINTS TAKEN AT 10 min INTERVALS)

FIG. 7 COMPARISON OF 100 KHz OUTPUTS OF Rb-1 AND Rb-2

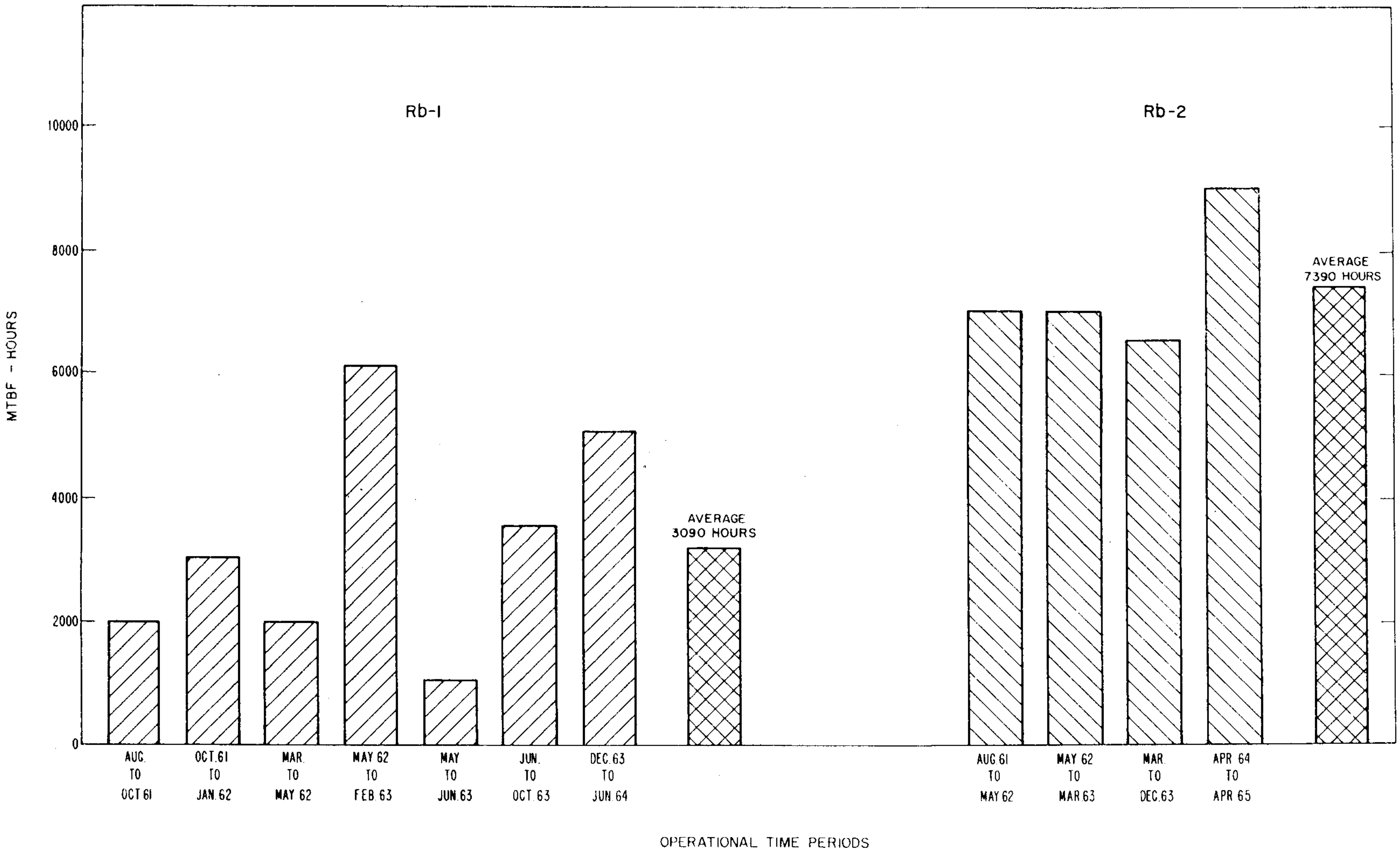


FIG. 8 SERVICE HISTORY OF Rb-1 AND Rb-2

TABLE 1

SUMMARY STATISTICS OF Rb STANDARDS FREQUENCY DATA

Year	PERIOD	Rb-1 STATISTICS n	b	S_b	S_y	$S_{y/x}$	$\bar{y}$	r	a	Rb-2 STATISTICS n	b	S_b	S_y	$S_{y/x}$	$\bar{y}$	r	a
1961	1	22	+0.021	0.006	0.31	0.24	-129.42	0.64	-129.74	22	+0.031	0.007	0.43	0.32	-129.93	0.69	-130.42
	2	56	-0.012	0.001	0.45	0.23	-129.81	-0.86	-129.17								
	2a									32	-0.010	0.002	0.23	0.17	-129.70	-0.66	-129.44
	2b									36	-0.005	0.001	0.18	0.16	-130.27	-0.49	-130.10
1962	3	32	-0.021	0.001	0.34	0.13	-130.41	-0.93	-129.87								
	3a									13	-0.022	0.003	0.15	0.07	-130.29	-0.90	-130.07
	3b									25	-0.020	0.002	0.27	0.11	-130.27	-0.91	-129.88
	4	53	-0.004	<0.001	0.17	0.15	-129.91	-0.54	-129.73	56	-0.003	0.001	0.15	0.14	-130.09	-0.41	-129.98
	5	38	-0.006	0.001	0.14	0.10	-130.39	-0.70	-130.21	38	-0.014	0.002	0.33	0.21	-130.37	-0.78	-129.91
	6	44	-0.007	<0.001	0.19	0.12	-130.41	-0.79	-130.17	62	+0.005	<0.001	0.22	0.16	-129.58	0.71	-129.83
	7	73	-0.004	<0.001	0.15	0.07	-129.79	-0.89	-129.56	48	+0.016	0.001	0.38	0.16	-129.28	0.91	-129.92
1963	8	*								22	+0.006	0.002	0.09	0.07	-130.29	0.66	-130.40
	9	*								20	+0.018	0.002	0.17	0.07	-129.85	0.91	-130.16
	10									72	-0.0002	<0.001	0.13	0.13	-130.31	-0.06	-130.30
	10a	20	-0.010	0.003	0.13	0.09	-129.91	-0.69	-129.75								
	10b	26	-0.012	0.001	0.21	0.07	-130.49	-0.94	-130.21								
	11	32	-0.017	0.001	0.26	0.09	-130.44	-0.94	-130.00	26	-0.010	<0.001	0.13	0.05	-130.24	-0.91	-130.05
	12	14	-0.004	0.004	0.09	0.09	-130.12	-0.25	-130.08	54	+0.004	<0.001	0.13	0.10	-129.96	0.65	-130.10
1964	13a	*								11**	+0.007	0.002	0.08	0.06	-130.18	0.81	-130.29
	13b	17	-0.036	0.003	0.19	0.06	-130.14	-0.95	-129.82	58**	+0.001	<0.001	0.06	0.06	-130.16	0.13	-130.18
	13c	14	-0.016	0.006	0.10	0.08	-129.79	-0.63	-129.68								
	13d	15	-0.010	0.005	0.09	0.09	-130.26	-0.46	-130.19								
	14	17	+0.011	0.003	0.07	0.05	-130.37	0.74	-130.47	13	-0.018	0.002	0.07	0.02	-130.17	-0.95	-130.04
	15	40	-0.028	0.001	0.34	0.09	-130.37	-0.97	-129.80	100	-0.010	<0.001	0.31	0.06	-130.42	-0.98	-129.89
	16	*								98	-0.008	<0.001	0.25	0.07	-130.30	-0.96	-129.88
	17a	*								17	-0.007	0.005	0.11	0.11	-130.10	-0.33	-130.03
	17b	*								30	-0.011	<0.001	0.16	0.04	-130.43	-0.97	-130.18
1965	18	*								11	-0.004	0.003	0.04	0.04	-129.73	-0.50	-129.69
	19	*								45	-0.004	<0.001	0.09	0.06	-130.20	-0.77	-130.08
	20	*								12	-0.025	0.003	0.13	0.05	-129.85	-0.92	-129.63

STATISTICS NOTATION (DATA GIVEN IN PTS. IN 10^{10} — 1961, 1964 AND 1965 DATA ADJUSTED TO NOMINAL OFFSET OF -130.00×10^{-10})

n = NUMBER DAILY REPORTED CALIBRATIONS

b = SLOPE OF LEAST SQUARES LINE THRU DATA (DRIFT PER DAY)

S_b = STANDARD DEVIATION OF SLOPE

S_y = STANDARD DEVIATION OF DAILY VALUES (INCLUDING SLOPE CONTRIBUTION)

$S_{y/x}$ = STANDARD ERROR OF ESTIMATE (VARIATION ABOUT LEAST SQUARES LINE)

$\bar{y}$ = AVERAGE DAILY VALUE

r = COEFFICIENT OF CORRELATION

a = INTERCEPT OF LEAST SQUARES LINE

* OUT OF SERVICE FOR MODIFICATION

** Rb-64 DATA – (Rb-2 IN REPAIR)

TABLE 2

Yearly Statistics of Rb-1 and Rb-2

YEAR	Rb-1					Rb-2				
	n	b	S_b	S_y	$S_{y/x}$	n	b	S_b	S_y	$S_{y/x}$
1961	78	-0.003	0.003	0.42	0.24	90	+0.002	0.004	0.28	0.21
1962	240	-0.007	<0.001	0.20	0.11	242	-0.002	0.001	0.27	0.16
1963	92	-0.012	0.002	0.20	0.09	194	+0.002	<0.001	0.13	0.10
1964	103	-0.019	0.004	0.23	0.08	258	-0.010	0.001	0.26	0.07
1965	-	-	-	-	-	68	-0.008	0.002	0.09	0.05
OVERALL	513	-0.010	0.002	0.25	0.13	852	-0.003	0.002	0.23	0.12

Note: 1. Statistics given in pts. in 10^{10} - Symbols as in Table 1.

2. Standard deviations are RMS of pooled variances, S^2_p where -

$$S^2_p = \frac{(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2 + \ldots (n_k - 1) S^2_k}{n_1 + n_2 + \ldots n_k - k}$$

TABLE 3

Service Performance of Rb-1 and Rb-2

FREQUENCY STANDARD	MTBF	NO. FAILURES	NO. ALIGNMENTS	OUTAGE TIME TOTAL PERIOD 2/	TOTAL OPERATING PERIOD 3/
Rb-1	3090 hrs.	7	16 (1380 hrs.) 1/	14/11%	25,239 hrs.
Rb-2	7390 hrs.	4	10 (3000 hrs.) 1/	9/0.2%	32,552 hrs.

1/ Value in parentheses indicates the average time between alignments.

2/ Two values are given. The upper value includes the time of factory modification; the lower value omits this time.

3/ Including outage time.

GPO 858-646