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Fundamentals of MEASUREMENT

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Editor's Commentary

Measurement has been the means by which man has sought, from his earliest days, to define, comprehend and utilize the physical phenomena of the world around him. He has needed, and has employed, measurement to gain sustenance and shelter, and to assure himself of survival. But beyond this he has found in measurement a means to enrich his mind and invoke his spirit. In the crude demarcations of prehistoric man in his cave, the monuments and temples of antique Greece and Rome, the precise geometries of the Pyramids, the ordered configurations of medieval cathedrals and palaces, indeed throughout man's history, there has been this evidence of a persistent endeavor to measure and standardize the world of dimension, weight, stress, strain and the many variables of environment.

In the complex probings of our interstellar missions to the Moon and to Venus, we are still pursuing the search begun by ancient astronomers to reduce the secrets of the universe to the precision of instrumented data. A crude surveying chain in a Paleolithic village is today's optical maser used as a measuring device; and it is today's wavelength of krypton 86 used as the standard of length. The span of measurement in man's history can be illustrated again and again. It permeates all aspects of this history — the home, industry and commerce, science and technology, war and peace.

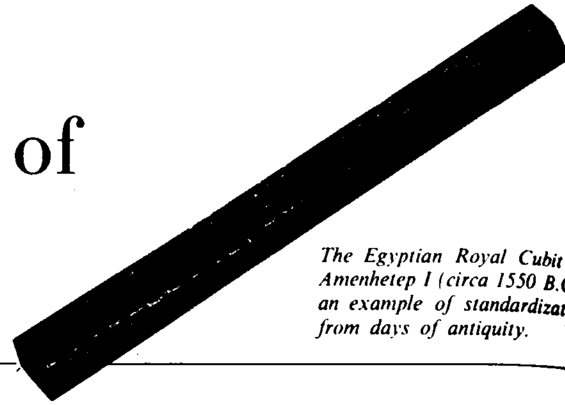
In the context of contemporary science and engineering, measurement has obviously assumed a greater role than it ever had in its history. To the design engineer, in particular, the standards and techniques of measurement are practical working tools. The engineer, after all, deals with physical substances, physical phenomena and the articulation of these into viable structures that are familiarly identified as discrete components, devices, equipments and systems. An understanding of fundamentals of measurement thus becomes essential in the proper and effective application of these tools. Lord Kelvin's frequently quoted statement bears repetition:

> I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of science, whatever the matter may be.

In the present article, "Fundamentals of Measurement," the author systematically examines both the historical concepts of measurement and the evolution of various measurement systems and techniques. He deals with primary quantities such as length, mass, time and temperature as well as with the many that are derived, such as the electrical quantities. Criteria for precision and accuracy, and the distinction between the two, are brought out. The significance of classical physical constants is explored, and also of those developed through modern research, for example, new methods for measuring the magnetic field by utilization of magnetic resonance principles. Finally, the keystone position of centralized standardization laboratories is emphasized.

ALEX. E. JAVITZ, *Senior Editor*

Fundamentals of Measurement



The Egyptian Royal Cubit of Amenhetep I (circa 1550 B.C.) an example of standardization from days of antiquity.

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"The trouble with the idea of measurement is its seeming clarity"—Henry Margenau

Nature of Measurement

Let us examine the conceptual basis of measurement, as one might examine the conceptual basis of language, starting first with words, then simple grammar, syntax, rhetoric, and so on to literature. This could not be accomplished in the space available here except that many of these concepts are already well known and others will become obvious once they have been pointed out.

Consider the nature of the measuring process. On the following line you will see an array of dots. Count them.

• • • • •

Having been given no further instruction than this everyone will agree that there are five dots. Now measure the distance between the extreme dots. Clearly, the instruction for the latter process is not adequate if there is to be even approximate agreement in the numerical result obtained by various individuals. To obtain such agreement it is necessary to specify the *unit* of length in which the result of the measurement is to be expressed and to supply a *standard* for the meas known length in terms of the unit. Let us agree that the shall be expressed in millimeters. Since most good scales, metric and customary, manufactured in the U States are accurate to better than 1 part in 1000, most will agree within a few tenths of a millimeter (1 in. mm) that the distance between the extreme dots is 50. but there will be disagreement. We have not yet agre just what distance is to be measured. Is it from cen center, or from inner edge to inner edge? Let us agree latter. Now there appears to be no problem in ach agreement if the measurements are carefully perform

If one looks at these dots with the naked eye or with power magnifying glass, each appears as a black circula But if one seeks to measure this distance with the utmo he would find a microscope very helpful in relating the tions of the dots to the lines on his measuring scale. another problem has been introduced. Under the micro there is no longer a sharp demarcation between blac white at the inner edges of the dots, but a shading from through gray into black, and the edges of the dots se fuzzily defined as the water line on a beach that is washed by waves. (How much more difficult it is to " ure" the inner Bohr radius of the hydrogen atom or the of the magnetic field of a spinning proton!)

The circumstances under which the distance between two dots is to be measured have not been defined. Th tance between them will be greater if measured at the Na Bureau of Standards laboratories in humid Washi D. C., than if measured in the National Bureau of Sta laboratories in arid Boulder, Colorado. The question

About the Author

Alvin G. McNish, the author of this article, hardly needs an introduction to those working in the field of measurement. At present he is the Chief of Metrology Division, National Bureau of Standards, a post he has held since 1960. Previously he was Acting Chief of the Optics and Metrology Division, and has held various posts at the Bureau in his long career there. Mr. McNish has performed distinguished research in many areas of science, among them geomagnetism and ionospheric physics, and also in the more general fields of metrology (measurement) and standardization. During World War II he was active in defense projects and holds, among other honors, the Presidential Certificate of Merit. He also holds the U. S. Department of Commerce Gold Medal for Exceptional Service. Mr McNish is a Fellow of the American Physical Society and the American Geophysical Union, a member of the Optical Society of America, a member and past vice-president of the Washington Academy of Sciences, and a member and past president of the Philosophical Society of Washington. He holds both the B.A. and M.S. degrees from George Washington University where he is now Adjunct Professor of Metrology.

ually arises, "Does the quantity we are trying to measure have a definite value within the limits of measurement we are trying to achieve?"

The foregoing considerations are not academic matters. They are encountered whenever accurate measurements are required in science or in engineering. They set a limit on what we can accomplish and point out a fundamental characteristic of the measurement process.

All measurements are inexact. Though we may usually improve our measurement techniques until the measurements are adequate for our needs there is always an uncertainty associated with the result of any measurement. It is this which differentiates measurement from counting, which is exact, and calculating, which is as exact as one chooses to make it.

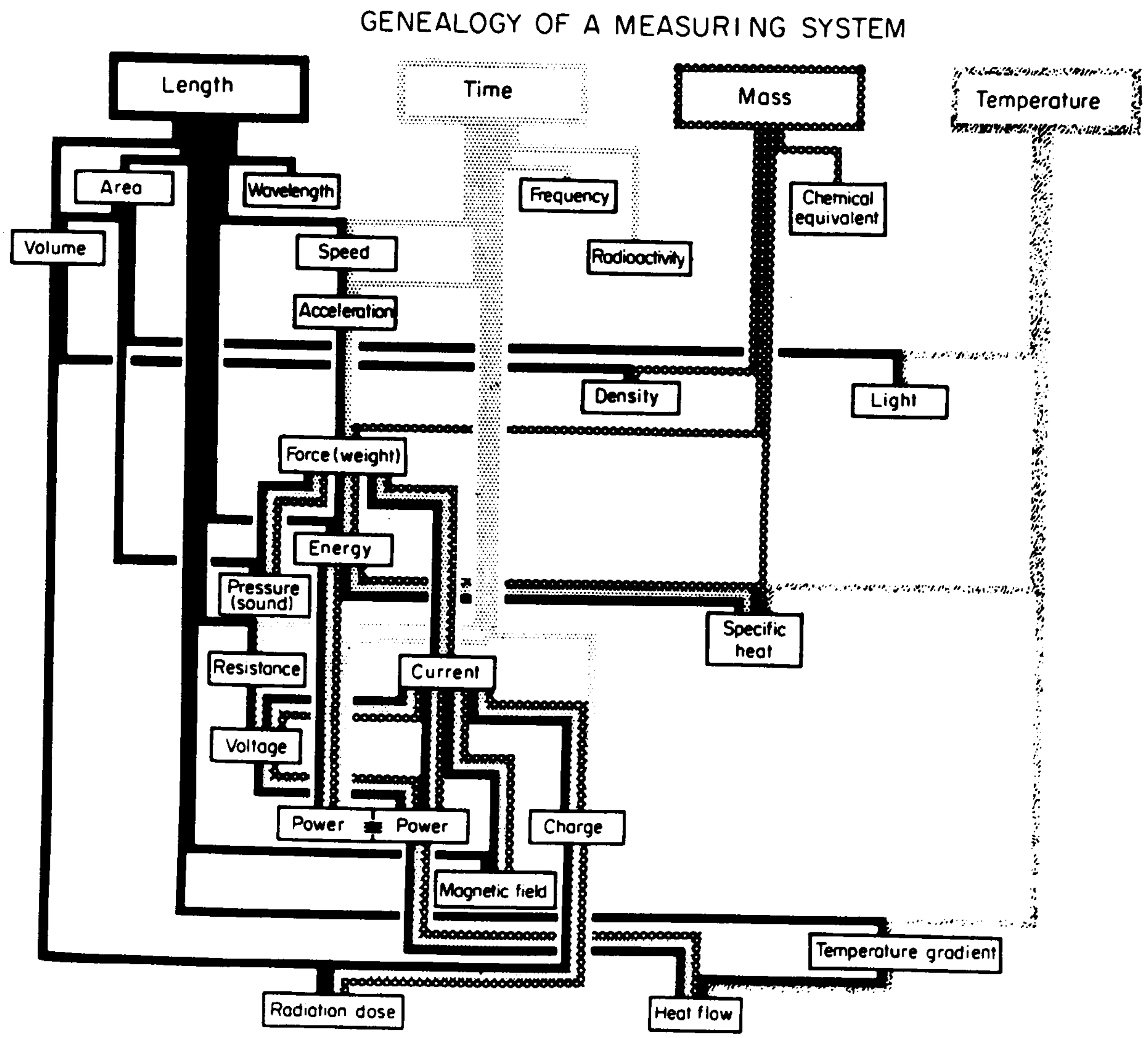
Now measure the mass of this magazine. Clearly, the instruction for this process is not adequate either, if approximate agreement is to be obtained, even though the unit and the standard for length measurement were previously specified. The magnitude of a mass can not be expressed in terms of a *unit* of length, and a *standard* of length is not a standard of mass.

Kinds of Quantities

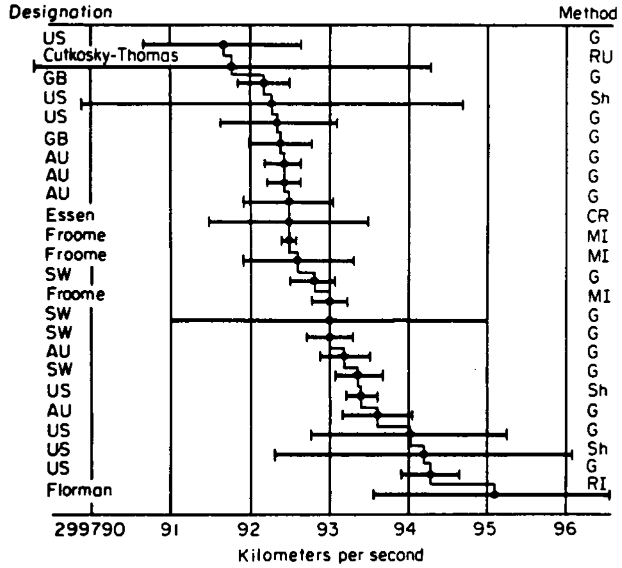
Length and mass are physical quantities of different *kind* and each requires its proper unit for measurement. If it makes sense to say $a > b$, $a < b$, or $a = b$, then a and b are quantities of the same kind, but if none of these statements is meaningful, then a and b are not quantities of the same kind.

The situation would be basically the same if the instruction were to measure the area of a page of this magazine, fot area is a quantity of different kind with respect to both length and mass. But it would not be surprising if many individuals, following the instruction to measure area, did so by extending the instruction for length measurement and expressed the result in square millimeters. This may seem the "logical" thing to do. The logic lies in inventing such a unit for area that one could measure the linear dimensions of a surface, calculate its area from geometric considerations, and obtain the result in terms of the chosen unit for area. It is certainly more convenient than expressing the length of the sides of a tract of land in feet and its area in acres.

To extend this procedure, one would invent as a unit of volume a cube each edge of which would be one length-unit.



Genealogy of a measuring system. Quantities at top of chart are those for which independent standards for defining their units have been established. Units and standards for all other quantities are derived from them. Solid and patterned lines show relationships. Thus area and volume derive parthenogenetically from length, length and time combine for speed, and time with speed for acceleration. Power may derive from energy and time, or current and voltage, but must be identical by either line. Quantities are representative, not exhaustive, and many important quantities are omitted. The particular system shown is the International System; others are possible.



Representative recent determinations of speed of light by various methods and estimated standard errors (horizontal lines). Values are arranged in order of value obtained. Initials to right indicate method: G, geodimeter (optical radar); MI, microwave interferometer; RU, ratio of units; Sh, shoran; CR, cavity resonance; RI, radio interferometer. Initials at left indicate country in which performed, names indicate experimenters. "Best value" is believed to be 299,792.5 kilometers per sec.

As with the unit of area, this unit of volume possesses a convenient relationship to the unit of length for measurement and calculation. But this does not mean that the length-unit squared and the length-unit cubed are the natural units for measuring area and volume. They seem natural because we are accustomed to them. Length, area, and volume are quantities of different kind. We cannot compare one with another, and therefore, having chosen a unit for one of these does not establish a natural unit for any other. For example, the electrical engineer expresses the cross section of electric conductors in circular mils, even extending this to rectangular bus-bars.

Measuring Systems

There is nothing natural about a measuring system. It is an invention of man, developed to serve his needs in commerce, industry, and science. Like all other human developments it is imperfect, but like all human developments it is a dynamic system, continually being changed to better meet our needs. It originated some time about the dawn of history when man's needs were few, precise measurements were not required, and simple specified relationships afforded no convenience. Early needs called for the measurement of only a few kinds of quantities, principal among which were length, area, volume, mass, time, and angle. As men began to traffic extensively with other men in goods and ideas, unification and extension of individual measuring systems became necessary. The greatest step in this direction was the introduction of the metric system nearly 200 years ago. Although it is almost universally adopted for commercial and technical purposes, many units of older systems are still in widespread use.

Scientific and technological advances have had the greatest impact on the development of measurement. Most significant of these was the recognition of a large number of quantities of different kinds as compared with the few kinds of quantities which were required for more primitive thinking. The history of scientific thinking is replete with illustrations of the development of these concepts, *e.g.*, the distinction between momentum and energy for a body in motion and the identification of heat as a form of energy. The final test of any of these concepts was accomplished through measurement, in some cases only by the most precise measurement, and measurement had to be developed to meet this need. Some of these developments may be well seen if we examine the evolution of the metric system historically.

The Metric System

The metric system was created by decisions made by the National Assembly of France in 1791 and 1795 to establish a uniform system of measurement throughout the republic. The original intent was to establish it primarily as a commercial system.

The framers of the system chose the earth as their standard for length and decreed the meter as 1/10,000,000 of the earth's quadrant. Since the cubic meter was inconveniently large, the liter (0.001 of a cubic meter) was designated as a convenient fluid capacity.

For a unit of mass which would likewise be reproducible they chose to define the gram as the mass of 1 cu cm of water. The concept of the simplest relationship between units for different kinds of quantities—length, meter; area, square meter; volume, cubic meter; etc.—was not embodied in the original form of the metric system.

The new system found favor with scientists of the day. C. F. Gauss in his famous treatise "Determination of the Earth's Magnetic Field in Absolute Measure" (1833) showed how an entirely different kind of quantity, magnetic flux density, could be measured in terms of combinations of units of length, mass, and time. It is interesting that in this treatise Gauss expressed measurements of length in millimeters, and mass in milligrams, rather than in meters and grams.

In the early years founders of the metric system encountered technical difficulties which illustrate some of the fundamental principles involved in the establishment of a satisfactory measuring system. More careful geodetic measurements showed that the meter bars in use did not conform exactly with the idealized definition. Since similar meter bars could be compared with each other more precisely than either could be related to the earth's quadrant it was decided to let the meter bar in the Archives of Paris define the meter. Similarly, in trying to realize the gram as defined, they found that two metal objects could be compared with each other more precisely than masses could be reproduced by measuring a volume of water. Accordingly, a metal kilogram was accepted to define the unit of mass.

The metrologists who made these decisions had these clear objectives: 1) to establish standards so that measurements could be carried out with the greatest possible accuracy, and 2) to replace (when necessary) idealized definitions for units of measurement with practical definitions. The liter, for example, was subsequently redefined as the volume of 1 kg of water at the temperature of its maximum density since a kilogram of water could be determined more precisely by weighing than a volume of 1000 cu cm could be established by linear measurements. The principle demonstrated in these objectives is basic in the development of an effective system of measurement.

Consistent Systems

As the insatiable curiosity of man urged him further and further to explore the structure of the physical universe about him, he apprehended more quantities of different kinds than were known to the ancients. To comprehend these quantities and the relationships between them, measurement of these quantities was necessary. Since it is not meaningful to compare quantities of different kinds it is not possible to express their respective magnitudes in a common unit. Therefore the number of different units required for a measuring system increases as the number of kinds of quantities to be measured. Considering some familiar equations of elementary physics such as $s = dL/dt$, $a = ds/dt$, $f = Ma$, etc., one can appreciate the unconscionable confusion involved if each of the quantities involved, s, L, t, a, f, M, etc., were to be measured in completely independent and unrelated units. Each equation would have to contain an appropriate constant of proportionality just as the familiar equation for area of a rectangle, $A = LW$, would have to be written $A = (1/14520)LW$ if area were expressed in acres, length in yards, and width in feet.

To keep the mathematical expression of physical relationships simple it is necessary to choose such units for quantities of different kinds that most of the equations can be written without constants of proportionality. Such a system of units is called a *consistent system.* To state the concept more exactly, one must say that the units are consistent with a certain set of equations expressing physical relationships. (Actually, the units acres, yards, and feet are consistent with the second equation for area in the preceding paragraph.)

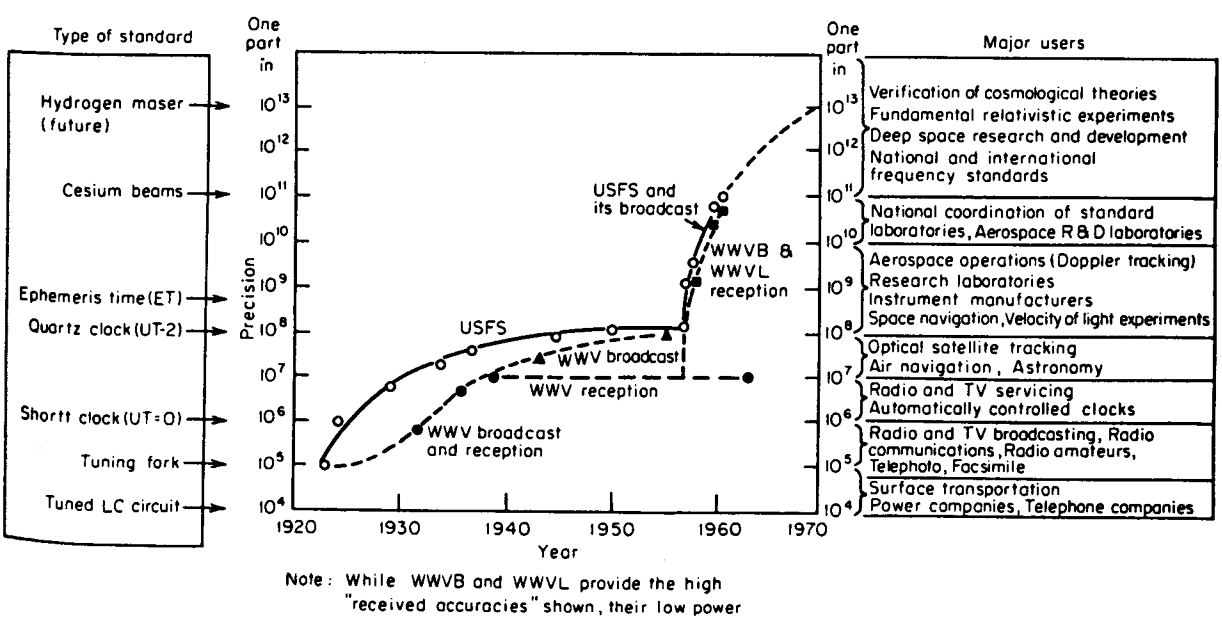
It is not desirable, nor is it possible without contravening some other concepts, to have all of the equations of physics freed of proportionality constants. For example, we like to think of potential energy and kinetic energy as quantities of the same kind. We express the potential energy of an electron in an electric field by the equation $e = qp$ where e is the energy, q is the electronic charge, and p is the potential at the position of the electron. Its kinetic energy after falling to zero potential is given by $e = ms^2/2$ where m is the electronic mass and s is its speed. If we use the MKSA system, e is in joules, q in coulombs, p in volts, m in kilograms, and s in meters per second, and the equations are correct. The elementary equations previously given are also correct because the system was designed to be consistent with them. If we eliminate the factor 1/2 in the last equation we must regard kinetic energy as a different kind of quantity from potential energy, and therefore not measurable in joules, or we must alter one or more of the other equations appropriately. If we do the latter, the MKSA system of units will not be consistent with the altered equations. Another way of looking at the matter is that, starting with the meter, kilogram, and second, these equations (or others consistent with them as is actually the case) define the units. The cgs electrostatic units are also consistent with these equations, and the cgs electromagnetic units are not.

The Electrical Units

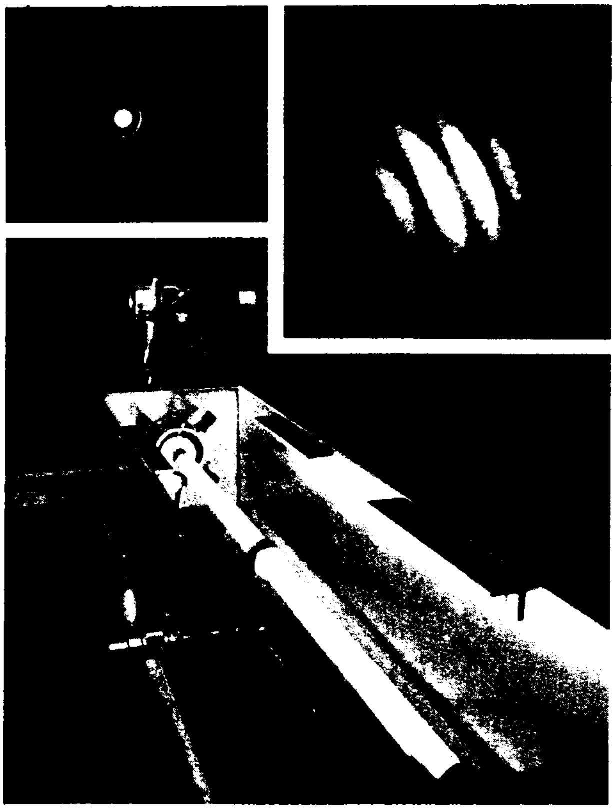
A measuring system is utilitarian; it is designed to satisfy man's need to know, whether the need stems from his curiosity or the practical necessity of his occupation. Measuring systems are designed to establish relationships between the quantities we apprehend. We do not devise units to measure quantities as yet unapprehended. We first become aware of something we need to measure, then we devise a way of measuring it. The history of measurement is fraught with the development of extempore measuring methods involving *ad hoc* units and standards. Usually these are subsequently revised, modified, or replaced by units which are consistent with the other units of the measuring system. Sometimes these *ad hoc* units and standards attain such widespread acceptance that they continue in use for a long time as nonconformal units, inconsistent with the measuring system. In one instance the *ad hoc* units attained such widespread use and the quantities measured in terms of them became so important in science and engineering that the measuring system itself was altered to include them. Such was the case with the practical electric units.

Probably no set of discoveries and technological exploitation has had greater impact on civilized society than that associated with electromagnetic phenomena. Discovery of the relationships between electric and magnetic phenomena during the early part of the last century brought to the attention of scientists a copious set of interrelated quantities of a new kind.

The first measurements in this new field of electromagnetism were of the simplest type, for example, how far a compass needle was deflected when a current was sent through a wire passing near it, and how the deflection was affected by doubling the length of the wire or doubling the number of electromotive cells in the circuit. Relative measurements they were, and rudimentary, but on such was Ohm's Law based.



Improvements in precision of the U. S. frequency standard. Prior to 1956 frequency standards required continual calibration through clocks which were rated by astronomical observations. Development of cesium beams led to frequency standards more constant than the rotating earth which need but once related to the defining standard for time.



Neon-helium cw optical maser set up in tape tunnel at National Bureau of Standards. The feasibility of using an optical maser for length measurements over great distances was demonstrated when fringes were obtained between two mirrors of a Michelson interferometer separated by a distance of 100 meters. The greatest distance over which interference fringes can be obtained with the krypton lamp is somewhat less than one meter. The path length obtained with the maser was limited only by the size of the tape tunnel and irregular variations in temperature and barometric pressure of the air in the interferometer. Interference fringes obtained with He-Ne optical maser in single-mode operation are shown in the insert. Left, *circular fringes obtained between reflecting surfaces of Fabry-Perot interferometer, separation 30 cm. If separation is increased rings expand and new ring forms at center. Shift of one ring corresponds to movement of 0.5 micron.* Right, *fringes obtained with Michelson type interferometer with reflecting surfaces separated by 100 meters, the greatest distance ever achieved interferometrically. If reflecting surfaces are brought toward each other fringes move across center of viewer, forming and reforming at edges. When zero separation of reflectors is reached, about 200,000,000 fringes will have moved across center of viewer.*

One of the faults of such measurements was that results obtained in the laboratory of one experimenter were not relatable to those in another. Uniformity of units and standards is a *sine qua non* in the interchange of scientific and technical information.

As pointed out before, Gauss showed how the earth's magnetic field could be measured in terms of the units of length, mass, and time, units which were well fixed in size by standards of a very constant nature. Also, length, mass, and time are quantities which can be measured with highly reproducible results.

It is difficult to appreciate fully the importance of Gauss' contribution to the science of measurement. The geometric relationships whereby area could be obtained from length measurements was known to the ancients. Newton formulated our thinking in dynamics. Measuring speed in terms of length units and time units seems obvious. That bodies in motion possessed some property that was a function of the mass of the body and of the speed of the body was common experience, although there were scientific arguments in the early days as to whether *ms* or *ms*^{2} better expressed this property. Gauss' dissertation extended these concepts to the quantities of electricity and magnetism which were not sensually perceptible. There were two quantities in the field of mechanics which could be identified as the same kinds of quantities in electricity and magnetism—force and energy. Gauss' method permitted electromagnetic force and energy to be expressed in the same units as mechanical force and energy.

W. Weber, a few years later, extended Gauss' method to the measurement of electric current and electric resistance in terms of mass, length, and time. Units for electric current could be expressed in terms of mass, length and time, and electric resistance in terms of length and time, even as speed is measured, and Clerk Maxwell was audacious enough to state that the speed of light was about 30 ohms!

The scientists of those days were developing a system of units for electricity and magnetism decimally related to the metric units of length and mass. The cgs (centimeter-gram-second) system found increasing acceptance because in it the density of the common substance, water, was unity.

While the natural philosophers were exploring the new mysterious relationships between electric and magnetic phenomena and pondering on their meaning, the engineers were applying these phenomena to the needs of society. They were not deceived by the philosophical niceties of an idealized measuring system. They knew that to engineer a telegraph system they had to make measurements but they found the units of the cgs emu system extremely inconvenient. They did not like to express the emf of a single-cell telegrapher's battery as 100,000,000 units or the resistance of a foot of ordinary copper wire as 1,000,000 units. They wanted units of a more practical size.

In a handbook of the mid-nineteenth century some interesting resistance standards are mentioned, such as "25 feet of copper wire weighing 345 grains" and "1 German mile of iron wire 1/10 inch in diameter;" these examples show the extent of the confusion which was developing. The situation was saved from complete chaos by a committee headed by that very practical scientist William Thomson, subsequently Lord Kelvin. The committee recognized both the advantage of having units for electromotive force, current, and resistance of convenient size and the desirability of having them simply related to the consistent units of the cgs emu system. The new practical units for these quantities (now known respectively as the volt, ampere, and ohm) were arbitrarily taken to be equivalent to 10^8, 10^{-1}, and 10^9 of the corresponding units of the idealized system.

Though the practical units were defined exactly, they could not be experimentally realized (with procedures then available) with the same reproducibility as two quantities of the same kind could be compared. This situation was analogous to the difficulty experienced in the early days of the metric system. The same solution was found. Separate standards for the volt, the ampere, and the ohm were established which embodied as accurately as the state of the art permitted the defined values of those units. Thus the ohm was defined by a specified column of mercury, the ampere by the deposition of silver at a specified rate, and a specified electrolytic cell was taken as the source of a fixed electromotive force consistent with the other two units. The units defined by these standards were called International Units.

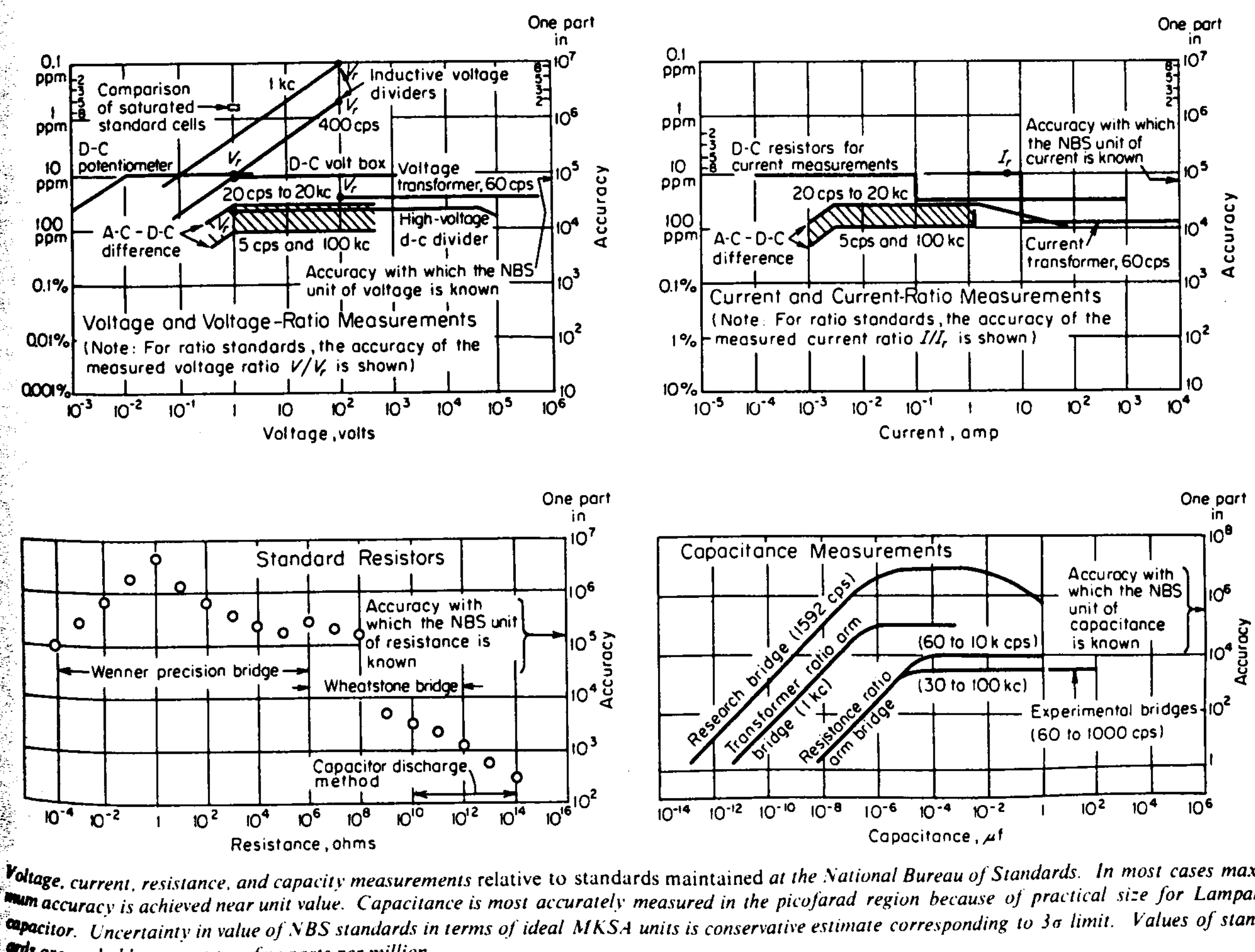
Eventually, the practical electric units were made part of a consistent system with predominantly unitary equatio[illegible] adopted by the General Conference on Weights and Measu[illegible] the only international body having legal authority in t[illegible] field. The Conference decided to take the meter, kilogr[illegible] and second as units of length, mass, and time and to inse[illegible] constant factor, 10^{-7}, in the force equation for current-ca[illegible]ing conductors to define the ampere. This was a distur[illegible] decision. In the new system the magnetic quantities B and H were no longer equal in free space. Equations in text books had to be rewritten. Most serious of all, a clarification of thinking was required.

Digression on Dimensions

Area is calculated as the product of two lengths, corrected by a numerical shape factor; therefore, we say area has the dimensions L^2. Similarly, using length (L), mass (M), and time (T) as our arbitrary starting quantities we determine from the equations of physics the dimensions of various other quantities such as speed (LT^{-1}), acceleration (LT^{-2}), force (MLT^{-2}), energy (ML^2T^{-2}), etc. These are quantities of different kinds and have different dimensions. Dimensional analysis, to have its maximum usefulness in analysis of physical and engineering problems, requires that all quantities of the same kind have the same dimensions, and corollary to this, quantities having the same dimensions are of the same kind. In the cgs emu system, magnetic pole has the dimensions $L^{3/2}M^{1/2}T^{-1}$; in the cgs esu system electric charge has the same dimensions. Chasing the familiar equations down, inductance is found to have the dimension L in the emu system and capacitance the dimension L in the esu system. Does it make sense to say that the inductance of a solenoid is greater or less than the capacitance of a capacitor, or that either is greater or less than the distance from New York to Chicago? In the emu system resistance has the same dimensions as speed. Did Maxwell have his tongue in his cheek when he gave the speed of light as about 30 ohms? Does it make sense to say that there are circuit elements in a TV set that have resistances greater than the speed of light?

The requirements that quantities of the same kind have the same dimensions and units, and that quantities of different kinds have different dimensions and units are not imposed by any law of nature but by convenience, just as we want different names for different things to avoid ambiguity in language. Incorporation of the practical electric units into our measuring system focused attention on the fact that the venerated trinity of length, mass, and time are inadequate to provide an unambiguous dimensional identification of the quantities of mechanics and of electricity consistent with the equations of physics. (In fact, they are not adequate even in mechanics.) Although the three dimensions L, M, and T, with various exponents, afford a three-fold infinite set of compound dimensions, only a few of this set are useful. To identify dimensionally the various quantities of physics, more than three elemental quantities are necessary. It appears that seven properly selected elemental quantities are sufficient for most of our needs. Such a set is supplied by mass, length, time, plane angle, solid angle, electric current, and temperature, but this is not the only possible set, nor can we prove that seven constitutes a sufficient set. One can get along with fewer dimensions if he is willing to tolerate and guard against the pitfalls of some dimensional ambiguities.

Dimensional analysis, like a measuring system, is an invention; there is nothing natural about it. Its purpose is to assist in relating our concepts of the physical world about us, both for our individual understanding and for communicating our



Voltage, current, resistance, and capacity measurements relative to standards maintained *at the National Bureau of Standards. In most cases maximum accuracy is achieved near unit value. Capacitance is most accurately measured in the picofarad region because of practical size for Lampard capacitor. Uncertainty in value of NBS standards in terms of ideal MKSA units is conservative estimate corresponding to 3σ limit. Values of standards are probably correct to a few parts per million.*

interpretations to one another. From our point of view, its most useful function is to keep our measurements and our measuring system on a consistent basis.

Techniques of Measurement

Measurement is the comparison of a quantity of unknown magnitude with a standard for that quantity. The result is the ratio of the magnitude of the unknown to that of the standard and is expressed in terms of the unit for that quantity. The defining standard for the quantity need not be directly involved in the measurement; the defining standard may be replicated and multiples or submultiples of it employed to effect the comparison.

In this sense, no one has ever measured the speed of light, for there is no standard with which it can be compared. Many experimenters have measured a distance and the time required for light to traverse that distance from which the speed of light was calculated; but the speed of light itself was not measured. This does not mean that the only quantities which can be measured are quantities like length, mass, and time for which we have arbitrarily chosen defining standards. Standards can be constructed for numerous other quantities, such as standard resistors and standard cells, whose values in terms of the units for those quantities can be determined in accordance with the defining physical equations. In many cases comparator devices, such as thermocouples, are employed by which the unknown quantity may be compared with the standard even though the two are remote from each other.

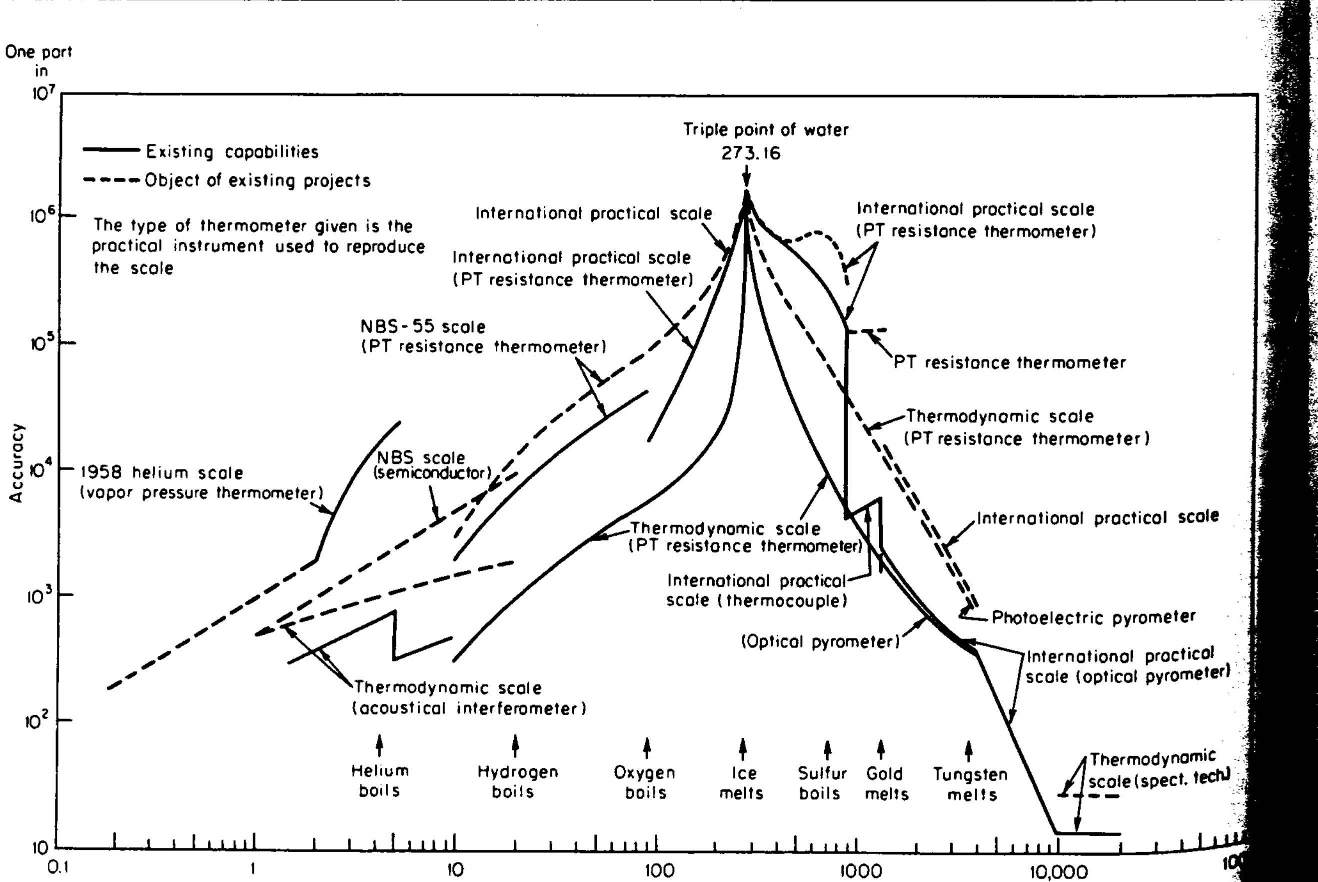
Length

Measurement of length is the easiest to comprehend and the simplest to perform of all the measuring processes, because the demarcations of the length are visually discernible in many cases. For this reason it serves to illustrate many of the problems of measurement which are of a general nature. Let us go back to our measurement of the distance between the dots shown at the beginning of this article. We placed a graduated replication of the length standard alongside the length to be measured, presumably with its zero graduation centered on the inner edge of one of the dots, and observed where the inner edge of the other dot came on the scale. We were not content to say the actual distance of 50.4 mm was greater than 50 mm or less than 51 mm; we estimated as closely as we could the residual difference and added this to the graduated interval which was closest to the length to be measured.

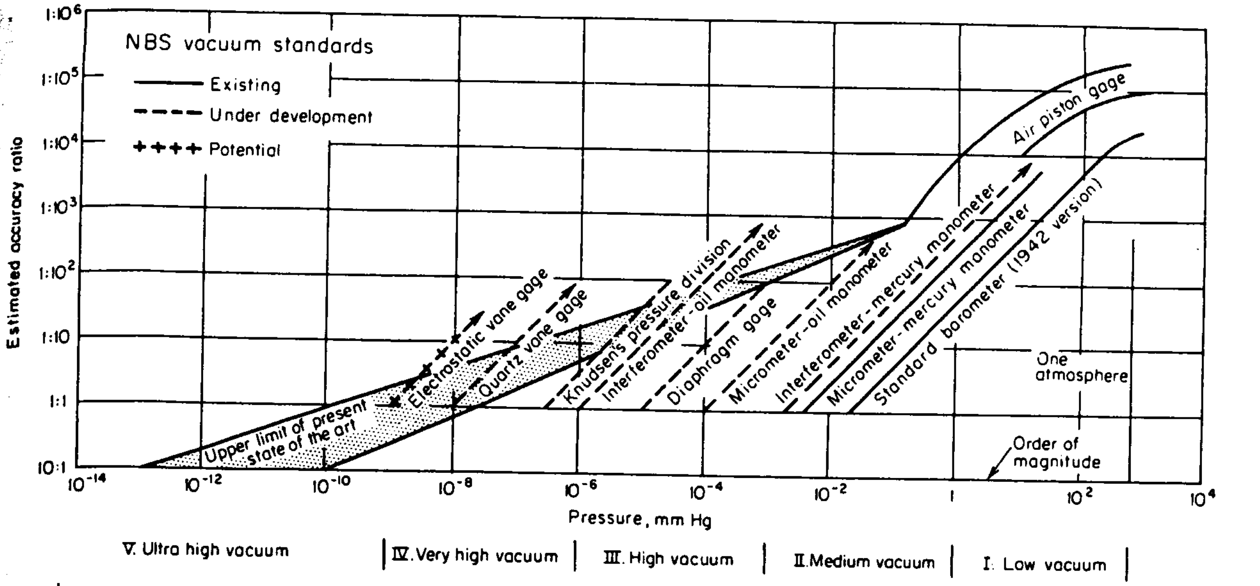
If we carefully repeat this measurement a number of times, removing and replacing the scale for each measurement as we did before, we obtain a series of values for the length, x_1, x_2, $\ldots x_n$, which are probably not all the same. If they are not, we take the average, $\bar{x}$, of all the values as the best value for the length. The scatter of the x's indicates the *precision* of the process, which is inversely proportional to σ, the standard deviation of the process. An estimation, s, of σ is obtained by

$$s = \sqrt{\sum_{1}^{n} (x_i - \bar{x})^2/(n - 1)}$$

This is valuable information because we may now estimate the precision of the average as $s_a = s/\sqrt{n}$. Subject to a few very general assumptions, such as that the value obtained in one measurement had no effect on the value obtained in any other, we may draw a useful conclusion. When n is sufficiently



Accuracy of temperature measurements by various methods over realizable range relative to thermodynamic scale. Triple point of water is ex[...] definition; accuracy limitation is due to experimental difficulties in reproducing it.



Vacuum and pressure measurements. Curves in upper half show conservative estimates of accuracy of vacuum and pressure measurements on an absolute basis. Gages in lower half must be calibrated in terms of them. Extreme vacuum beyond range of chart must be estimated by extrapolating from calibrated gages.

large s approaches σ and we may regard the standard deviation of the process as known. Limits for the errors of measurement may then be set for a normal or nearly normal error distribution. The chance of an error having an absolute value greater than σ is 1 in 3; 2σ, 1 in 20; 3σ, 1 in 400, etc.

Had all of our measurements given the same result, we would have found $s = 0$. This should not be interpreted to mean that $\sigma = 0$ and that, therefore, we have infinite precision, but rather that we probably used too large a "least count" and did not avail ourselves of the maximum precision obtainable by this method. We should have estimated our residual differences more finely. If several individuals, each attempting to achieve the maximum precision, performed this series of measurements under the same conditions, using the same linear scale, etc., it is unlikely that their individual averages would agree. It is not unlikely that the differences would be even greater than to be expected from the calculated standard error of each measurer. What, then, is the significance of the calculated precision? How can we decide which is the correct result?

A trained metrologist would have proceeded with the measurement in a slightly different way. He would not have lined up the zero graduation on his scale with one of the dots but placed it more or less at random so that where the edges of both of the dots fell on the scale had to be estimated. In successive measurements he would have placed the scale so that different length intervals on it were used. The standard error calculated from results obtained by this method is likely to be greater than for the other method, but we can reduce the standard deviation of the average as much as we wish just by taking more measurements.

In the first method there are possible sources of error which could have biased every one of the separate measurements in the same way. Setting a line to the edge of a circle is not the same process as estimating where the edge of a circle falls between two lines and consequently may cause an observer consistently to underestimate or overestimate the total distance. Also, it is difficult for an observer to avoid being influenced by previous readings. If the graduations toward one end of the measuring scale were in error, then all measurements made with that end of the scale were in error. Such errors as these are called *systematic errors*. They cannot be reduced or eliminated merely by taking more measurements in the same way. The calculated standard error does not give any evidence of their presence or absence and may lead to false confidence in the *accuracy* of the result.

The second method, shifting the scale, changes the nature of some of these systematic errors by randomizing them so that each will not affect all the observations in the same way. This may lead to a larger calculated standard deviation, but since the standard error of the average can be reduced by taking more measurements, we can reduce the contribution of these systematic errors to negligible proportions. We must regard the result of the second method as more accurate than that of the first.

Does the second method lead to an accurate value of the quantity to be measured, and what do we mean by accuracy? Let us say that by accuracy we mean the degree of agreement between what we measured and what the National Bureau of Standards would have obtained by measuring the distance between the dots on the matrix from which the page was printed, expressed in millimeters as presently defined by the wavelength of light. With this concept of accuracy it is foolish to talk about the superiority of the second method over the first, in spite of its technical refinements, because both methods are subject in the same way to considerably larger systematic errors, such as the overall error in the scale used and dimensional changes in the paper since printing.

We can never know completely the contributions of these sources to the error of our measurement but we can make good estimates of their magnitudes. If the scale was traceable to the National Bureau of Standards we can ascertain the possible error in it. We could have it calibrated, in which case the error would be known even better. Estimating the contribution from dimensional changes of the paper is more difficult but not hopeless. There is much information on how paper changes with environmental influences and from this we may estimate how much the paper could have changed and how much it is likely to have changed between printing and measurement.

The accuracy of measurement is limited by a number of more or less independent errors, $\epsilon_1, \epsilon_2, \ldots, \epsilon_n$ only one of which is due to the random errors of the measurer which we called the standard error, but it is one we can best assess. How should these errors be combined to give the overall error? There is no accepted doctrine on this. Before combining the error in any way they should be normalized. Since there is 1 chance in 3 that the random error exceeds the calculated error, the other errors should be estimated on the same basis if we are to combine them with it.

Adding all estimated errors arithmetically is likely to result in an overestimate of the total error because some of the errors may be of positive sign, others negative, thus tending to cancel each other. If there is a large number of independent sources of error, the addition of the errors in quadrature is sometimes justified, giving for the total error

$$\epsilon_T = (\epsilon_1^2 + \epsilon_2^2 \ldots \epsilon_n^2)^{1/2}$$

In many cases only a few independent sources of error can be identified and the chance that all of the errors are of the same sign is not trivial; thus this method will often substantially underestimate the total error. The mistakes which are made, even by experienced scientists, in assessing the error of an experiment are illustrated by the disparate values obtained in determining the speed of light and the uncertainties assigned to the results.

In assessing the accuracy of a measurement one must be guided by what hangs upon the assessment of accuracy. The situation is analogous to that which confronts the civil engineer who includes substantial safety factors in designing a bridge and the aeronautical engineer who cannot afford this luxury, for if he does his planes might never rise from the runway. This principle applies throughout measurement: The accuracy to be sought should be fixed by the needs that are to be met.

Accuracy in measurement is not always important—often *precision* is all that is necessary. In determination of the speed of light, accuracy is of primary concern because the speed of light is involved in many interrelated measurement complexes such as values of the electrical units, geodetic surveys, missile guidance systems, and physical theory. Touching thus on so many fields, the most accurate value of this constant of nature must be known in terms of meters per second (unless we are to accept separate and distinct values for this constant in each of its multifarious applications). The needs of many scientific and engineering situations are met by knowing values of quantities of the same kind with great precision relative to each other without knowing equally accurately how any one of these can be expressed in terms of the defining standards for that quantity. This point is well illustrated by measurement of mass.

Mass

Mass has long been the most precisely and most accurately measurable of all physical quantities, although during the past decade it has yielded this distinction to time. Two masses of about one kilogram may be compared with each other with a precision of a few parts in 10^9 under favorable conditions by means of an equal-arm balance, an instrument devised by the ancients. The measurement of mass is ordinarily performed by comparing the force exerted upon an unknown mass by the earth's gravitational field with that exerted on a standard mass. The international standard for mass is a certain cylinder of platinum-iridium, called the International Kilogram, kept in the vault at Sèvres, France. Its mass is compared with that of various national standards (also platinum-iridium) by the weighing process. But what is this standard mass? Is it the mass of the myriad of metallic atoms of which it is composed or is the mass of the adsorbed gases of the atmosphere included? Since the mass of this cylinder is compared only with that of geometrically similar cylinders of almost identical composition the question is academic. But what of other standards for mass having different composition, different surface areas, and different adsorbtion properties?

The more alike two things are, the more precisely some property of them can be compared—this is a principle which is axiomatic in measurement. If we compare the mass of a stainless steel cylinder with that of a platinum-iridium cylinder by weighing, some aspects of this principle are evident. Each cylinder is buoyed up by a force equal to the weight of the air it displaces. This is about three times as great for the stainless steel cylinder as it is for the platinum-iridium kilogram, a force equal to about 2 parts in 10^4 of the gravitational attraction on the mass we are trying to measure to a few parts in 10^9. To avoid difficulties arising from surface adsorption and air buoyancy one might resort to vacuum weighing, but this creates more difficulty than it cures, for mass standards placed in a vacuum out-gas and lose mass interminably. The practical solution (and measurement is *always* practical) is to express the mass of a standard as the mass it appears to have when compared, by weighing, with other mass standards of specified density under "normal" atmospheric pressure.

Much useful application in science and engineering proceeds from knowing very precisely the magnitudes of quantities of the same kind relative to each other. Take for example the masses of the atomic nuclides. We cannot measure the mass of a single nucleus in terms of the international kilogram with any satisfying accuracy, but we can compare the masses of the thousand-and-odd known nuclei with great precision by mass spectrometry. This is accomplished by assigning to one nucleus, carbon-12 (consisting of 6 protons and 6 neutrons) the value 12 atomic mass units, an *ex tempore* unit. In measuring mass by weighing, we make use of the earth's gravitational attraction for comparing masses; in mass spectroscopy we employ directly the inertial properties of mass for our measurement.

Roughly the mass of any atom or molecule is the sum of the masses of the neutrons, protons, and electrons of which it is composed, but it is not *exactly* that sum. Thus the molecule CH_4 has almost the same mass as the atom 0-16 and the masses of the two may be compared with great precision much greater than that attainable in comparing C-12 with O-16. If the masses of C-12 and O-16 are compared directly with each other in a mass spectrometer the electric and magnetic forces acting on each must be controlled in a ratio of 12 to 16 with a precision equal to that which we want to attain in the mass measurements. This is difficult to do.

Every atom and every molecule possesses a quality which we identify as the physical quantity, mass, the characteristics of which are described in Newton's laws. When one of these particles is ionized it is subject to electric forces and, if in motion, to magnetic forces. Its reaction to these forces is governed by its mass. These principles allow us to compare masses of atoms and molecules with great precision. The path followed by an ionized particle in a mass spectrometer can be closely governed by the electric and magnetic fo[illegible] applied. Any deviation from this path must be due to a [illegible]ference in mass of the particle from the mass of the par[illegible] with which it is being compared. The paths of a C-12 a[illegible] and an O-16 atom differ much more under the same ele[illegible] and magnetic field conditions than do those of a CH_4 mol[illegible] and an O-16 atom, and consequently the former pair ca[illegible] be compared as precisely as the latter. On the other h[illegible] if we adjust the fields so that the two paths are nearly [illegible] same (as is actually the case in precise mass spectrosc[illegible] the range of adjustment required in the former case is [illegible] greater than in the latter and consequently can be ac[illegible]plished with less precision.

The mass spectroscopist thus applies the principle in [illegible]uring nuclidic mass of comparing the masses of par[illegible] which are almost equal so that he can achieve the utm[illegible] precision and accuracy in his measurements. Fortuna[illegible]

Large number of such mass doublets are available, most of which involve the carbon atom. For this reason a new scale of nuclidic masses and atomic masses was internationally adopted by the Union of Pure and Applied Chemistry in 1961; the scale was based on the mass of the carbon-12 atom even though this meant that all tables of atomic masses became obsolete. But the new scale was so delicately chosen that only those people who required the most exacting values were affected by the change.

On the new scale some nuclidic masses are known in atomic mass units, as defined by carbon-12, with a standard error of only a few parts in 10^8, nearly the same precision which is attainable in comparing two standard kilograms. However, the size of the atomic mass unit can be related to the kilogram with a standard error no smaller than a few parts in 10^5. Why strive for this great precision in measuring nuclidic masses when the atomic mass unit is so inaccurately known? The small differences between the individual masses of nuclides and the sums of the masses of the neutrons and the protons of which each nuclide is composed are called "mass defects." They indicate the amounts of energy available in various nuclear reactions in accordance with the famous Einstein equation, $E = mc^2$. Since these mass defects never exceed about one per cent of the mass of the pertinent nuclides, the relative masses must be known with great precision to calculate the energies available. These mass-energy relations are so well known that the masses of many nuclides can be calculated from the energy relationships more precisely than they can be measured by mass spectrometry.

One might consider the practicality of defining the kilogram in terms of the atomic mass unit instead of a cylinder of platinum-iridium. While this would have the attractive aspect of establishing the standard for mass as an immutable physical constant it would degrade greatly the accuracy achievable in measurement of all masses except atomic particles. Ordinary mass measurements could be made with no greater accuracy than is attached to our present knowledge of the value of the atomic mass unit in terms of the platinum-iridium kilogram. The situation would be like that which confronts us now in the measurement of time and frequency.

Time

Time has long been measured in terms of the rotation of the earth, the scientific unit of time, the second, having once been defined as 1/86400 of a mean solar day. For centuries, astronomers have observed the meridian passage of stars to mark the rotation of the earth and shorter intervals of time were interpolated by means of mechanical clocks, although the clocks themselves did not run uniformly enough to be used as independent standards. But the rotation of the earth has proved to be too erratic a time-keeper to meet modern scientific needs and, therefore, the mean solar second was continually changing. It is subject to periodic fluctuations within a year and unpredictable fluctuations from year to year. These departures from uniformity are so great that had a perfect clock been set at the beginning of this century to tick off seconds precisely equal to the mean solar second of 1900 the time indicated by that clock would now differ by 1/2 minute from that derived from the earth's rotation. What can one say of the accuracy of a measurement if the standard defining the unit is changing?

A partial remedy was achieved in 1956 when the International Committee on Weights and Measures redefined the second for scientific use as 1/31,556,925.9747 of the tropical year at 12^h ephemeris time, 0 January 1900. This multidigited number was obtained from Simon Newcomb's equation for the celestial motion of the sun. The equation is quadratic in time and gives, subject to correction for periodic effects, the longitude of the sun in the plane of the ecliptic with respect to the vernal equinox. The particular time in the definition reduces the quadratic term in the equation to zero. This is the ephemeris second, the unit of time in terms of which all planetary motions were most simply expressed.

Krypton-86 lamp, used to produce light-wave as standard of length, being placed in metal-cased dewar. The lamp consists of a U-shaped tube containing krypton gas. Electrical discharge is maintained between a heated cathode and a cold anode. The lamp is operated in a dewar containing liquid nitrogen which is lowered to its triple point by pumping through hose at bottom of container. Since the triple point of nitrogen is 63 K and the freezing point of krypton is 114 K the nitrogen bath establishes the pressure of krypton in the lamp as the vapor pressure of krypton gas at 63 K as well as the temperature of the gas, assuring uniform operating conditions. Light from the lamp for measurement purposes is brought out through glass port at bottom of container.

This definition has one serious fault. No one can measure an interval of time by direct comparison with the interval of time defining the second. By a lengthy series of astronomic measurements extending over several years, data can be obtained from which a current value of the mean solar second can be related to the ephemeris second by calculation. This relationship can be established with an estimated standard error of about 1 in 10^9, a large uncertainty compared with the exactness implied in the definition.

A spectacular revolution in the measurement of time was taking place while this redefinition of the second was being formulated. Atomic beams, masers, and absorption cells were developed which proved to be very stable standards for frequency and time. Not only could they be compared with each other with a precision of 1 part in 10^{10} or better during an observing time of an hour or so, but independently constructed cesium beam resonators agreed in frequency to a few parts in 10^{11}. Even greater achievements loom on the horizon. Thus, if the second were defined as the interval of time corresponding to x cycles of a selected atomic resonant frequency, the second could be as exactly defined as it is now. Any competent laboratory could then construct a standard for time and frequency which could realize this unit of time better than the ephemeris second can be calculated from astronomical observations.

Modern metrologists are not slow to take advantage of technological progress. Plans are now being laid for redefinition of the second in terms of the frequency associated with an atomic transition, just as the meter was redefined in 1960 in terms of the wavelength of a specified spectral line. But this redefinition will not be done capriciously. The atomic transition selected to be the standard for the second must be the most suitable within our current knowledge and the number set into the definition must be such that the "atomic second" will be indistinguishable from the second of ephemeris time within the limits of current astronomic measurements. Then, with an atomic time scale, we shall be able to time the planets in their orbits with an accuracy hitherto impossible.

When this redefinition of the second has been settled, we shall have moved closer to the idealistic goals of the founders of the metric system. Another of our elemental units from which units for other kinds of quantities are derived will be defined in terms of a physical constant which we regard to be immutable. When we abrogated the definition of the meter as the distance between two engraved lines on a certain bar kept in the standards vault at Sèvres, France, we made a great advance in the potential accuracy of length measurements. After considering many possible atomic transitions the meter bar was redefined as "the length equal to 1,650,763.73 wavelengths in vacuum corresponding to the transition between the energy levels 2p 10 and 5d 5 of the atom of krypton 86." Discourse on the many considerations which led to this choice would be very involved but two should be mentioned. The new definition had the effect of making the angstrom unit, long used by spectroscopists for expressing wavelength, identical with 10^{-10} meter; also it made the defining standard of length directly available to every competent laboratory so that each could calibrate its standards directly instead of having to go back through a chain of intermediaries to the International Meter Bar at Sèvres.

Temperature

Standards for temperature have long been defined by physical constants. The Fahrenheit and Celsius* scales of temperature each originally had fixed points defined by the freezing and boiling points of water. Since temperature, together with length, mass, and time, is one of the quantities for which completely independent units and standards have been selected for our scientific measuring system, extensive consideration of this quantity and the techniques employed in measuring it leads to better understanding of the whole measurement problem. Also, in temperature measurements we can see most clearly the compromise which has been necessary between idealized definitions and practically realizable ones.

Temperature has a characteristic which differentiates it from the other quantities which serve as the basis of our measuring system. This is described by saying that temperature is an *intensive* quantity; the other quantities are *extensive*. What we mean is simply this: if we take two identical bodies having the same temperature and combine them, the mass of the new body will be double that of each component; its volume will be doubled, its heat content will be doubled, but its temperature will be the same as that of the original bodies.

Since temperature is an intensive quantity, one might suppose that the best that we can do is merely to order temperatures, that is, to assert that one state of matter has a higher temperature than another. But it is useful to have some quantitative expression for the degree of difference in the temperatures of various bodies or systems. This can be obtained in several ways. The early inventors of temperature scales seem to have failed in one important step. While at least two fixed points were established to define the scale no prescribed way of interpolating between them was stipulated, although by implication they appeared to rely for this on a property of the thermometric substance in their instruments. Two thermometer makers could not have their instruments in agreement at any but the fixed points unless they used identical materials.

To measure temperature two things must be established: a reference temperature and a rule for measuring the difference between a particular temperature and the reference temperature. Of the four quantities under discussion, temperature alone requires this. Each of the other three may be measured without universal agreement on a reference point at which the quantity is assumed to have a particular numerical value. Thus, every gram in a kilogram is equal to every other gram, etc., but in the measurement of temperature it is of limited meaning to say that one degree in the neighborhood of 0 Kelvin is equal to one degree near 273 K.

Lord Kelvin proposed a method for relating temperatures based on the Carnot cycle. Imagine two heat reservoirs of infinite capacity and a perfect reversible heat engine transferring heat from one to another. If Q_1 is the quantity of heat taken from one reservoir, and Q_2 the quantity of heat put into the other, then the temperatures of the two reservoirs θ_1 and θ_2 are given by $\theta_1/\theta_2 = -Q_1/Q_2$. A temperature scale is then completely defined by establishing one fixed point, that is, by assigning a number other than zero to a reproducible state of some substance. This has been done by assigning the number 273.16 to the temperature of "natural" water at its triple point—that state at which solid, liquid, and vapor phases of water are in equilibrium. This temperature can be realized with a repeatability of a few ten-thousandths of a degree.

The temperature scale defined in this way, known as the Kelvin thermodynamic scale, is identical with that defined by the gas equation, $pv = R\theta$, which holds approximately for all gases over a limited range of temperature but is exact only for a "perfect" gas. It is through this gas equation that temperatures for critical states of matter are experimentally realized. Real gases must be used in gas thermometry and corrections made to reduce the results to what would be obtained for a perfect gas by means of experimentally obtained virial coefficients.

Gas thermometry is a difficult and tedious procedure, completely unsuited to the practical needs of temperature measurement. Therefore the International Practical Scale of Temperature was devised in which values were assigned to fixed points other than the triple point of water: the boiling points of liquid oxygen, water, and sulphur and the freezing points of silver and gold, covering a range from 90.18 K to 1336.15 K. The temperatures for these fixed points were assigned after collating the results from carefully performed determinations by gas thermometry of the temperatures corresponding to these states. Thus, the temperatures for these other fixed points of the practical scale are in as close agreement with the theoretical thermodynamic scale as the state of the art permitted at the time the assignments were made. At one point, the triple point of water, the two scales are in exact agreement by definition.

Interpolation between the fixed points is prescribed by the use of certain formulas for the resistance of a platinum resistance thermometer and of the emf of a platinum and platinum-rhodium thermocouple over specified ranges. Extrapolation of the scale above the gold point is prescribed by the ratio of energy radiated at a fixed wavelength at the higher temperature to the corresponding radiated energy at the gold point as given by Planck's Law.

Thus there are two versions of the Kelvin scale, the ideal thermodynamic and the practical. (Similarly, there are two versions of the Celsius scale, corresponding to those of the Kelvin scale, in which the zero value is shifted to 273.15 K. This makes the ice point, which is no longer a defining point, correspond to 0 C within the accuracy of measurement.) By selecting values in this way the practical and the ideal scales correspond as closely as possible to the old and less well defined Celsius scale and may be related to the Fahrenheit scale by known formulas. All competent laboratories may set up equipment for calibrating their thermometers in terms of the fixed point standards without reference to another laboratory but with firm confidence that the results will be

* By international agreement, Celsius officially replaces the Centigrade designation; the latter, however, is still generally used.

agreement (as already stated, such agreement is the *sine qua non* of a measurement system).

Other Physical Quantities

Although length, mass, time, and temperature are the only quantities taken as "basic," that is, quantities having independent, arbitrarily defined standards, these represent only a few of the quantities we have need to measure in science and engineering. Since the units for all of these other quantities are defined by simple equations relating them to the units for the "basic" quantities, anyone can determine the magnitude of one of the other quantities if he has access to standards for the basic quantities. But in most cases this is cumbersome, like the determination of a temperature directly on the thermodynamic scale.

To facilitate measurement, standards are constructed for the other quantities which embody as closely as is feasible the value of the unit as given by the defining equations. Some quantities, such as an area or a speed, can be calculated from easy measurements of the elemental quantities involved. But for many derived quantities standards are of great convenience and contribute to the accuracy and consistency of measurement. Take force, for example.

Force is defined by the equation $f = Ma$. To establish a standard for force one must first establish a standard for acceleration. The most convenient standard for acceleration is that of gravity, g. It is relatively uniform over the surface of the earth and spatial variations can be precisely measured. It is highly constant with time in geologically inactive regions (it is probably more constant than the mean solar second, except for short-term predictable periodic variations), and its noise level is very low.

Two methods have been extensively used to determine g. One involves the use of a pendulum from the period and effective length of which g may be calculated. The other involves determining the change with time of the speed for a free-falling body. The first method has long been used; the second has attained comparable accuracy only during the last few years with the development of electronic timing techniques. Though both methods are simple in principle, accurate determination of g by either requires careful and painstaking procedures.

Determinations of g performed by both methods at the principal metrology laboratories of the world agree within a few parts per million after allowing for local differences in g. These local differences can be established with high precision by comparison pendulums and gravimeters in accordance with the metrological principle that like quantities can be compared with each other more precisely than either can be determined in terms of the elemental quantities of which it is composed. Collation of the results of these variations leads to the belief that an adjusted average of them is accurate to within 1 part in 10^6 and that, by comparison measurements, a value of g may be assigned at any station with high confidence that it is not in error by any more than this amount.

Experimental determinations of the magnitudes of quantities, such as the acceleration of gravity, in terms of the quantities for which we have arbitrary standards are called absolute determinations, following the usage of Gauss. The result obtained is called an absolute value. In the strict sense used before, we cannot measure an acceleration until a standard has been established for it, either by absolute determination or by arbitrary adoption of a value for the standard, manifested under stipulated conditions, as was done for the International Electric Units.

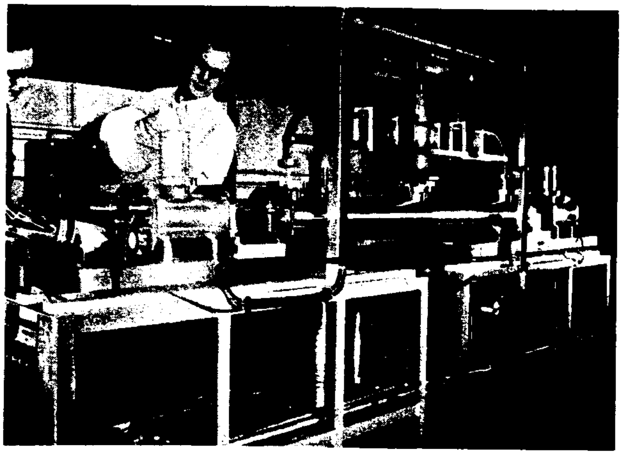
An absolute determination can never be as accurate as the potential accuracy of measurement for the basic quantities entering the experiment. Its uncertainty includes the accumulation of uncertainties attached to the measurement of the component quantities involved, which may be degraded from their pristine possibilities by the circumstances of the experiment. For example, in determining g by the pendulum method, it is the length of the pendulum while swinging, not its length at rest, that enters the calculation. Also, what forces other than gravity may govern the period of the pendulum? Such problems permeate all absolute determinations.

The utility of establishing derived standards by absolute determinations is clear. Through them measurements of quantities of the same kind can be performed on a uniform basis. In practice we are more frequently concerned with knowing precisely how two quantities of the same kind compare with each other than with an accurate knowledge of the value of either in terms of the elemental quantities of which they are composed.

Thus, if an experimenter has accurate standards of mass he can compare two forces by observing the mass which each force will balance against gravity, even though he does not know the local value of gravity. His measurements will be comparable with those of an experimenter in a different place if the gravity difference is known, even though its absolute value is not established at either place. If both places are tied into a world-wide network of gravity values, then all such force measurements will be relatable. If an absolute determination of gravity has been made at one or several of these places, the force measurements are expressible in terms of the units of length, mass, and time with an accuracy limited only by the accuracy of the absolute determination and the precision of inter-station comparisons.

Of what use is knowledge of an accurate value of g so long as our measurements and calculations are coordinated? For many technical purposes it is valueless. If we measure the strength of a cable, using an incorrect absolute value for g, then when we calculate the weight it will carry based on that same incorrect value, we will obtain a correct result. But this will not be true if we calculate the load it will sustain under centrifugal forces.

Consider the satellites and how they move. Their paths are not governed by the conventions of our measuring system but by the laws of physics. If we measure the thrust of a rocket motor in terms of a unit of force derived through use of an incorrect value for g, and calculate the height to which this thrust will carry the rocket while operating for a given



This cesium atomic clock at the Boulder Laboratories of the National Bureau of Standards measures frequency and time intervals to an accuracy equivalent to the loss of less than 1 sec in 1000 years. In the operation shown here, the atomic beam detector is being adjusted as liquid nitrogen is being poured into a cold trap. The nitrogen helps form a vacuum so that the cesium atoms can be beamed through the apparatus without being deflected by molecules of air.

time, again using the same incorrect value for g, our height calculations will be correct. For the rocket to go into a circular orbit of radius r it must have a tangential speed $s = r_0 \sqrt{g/r}$, in which r_0 is the radius of the earth, assumed to be spherical and non-rotating. In this case it is the correct value for g that is involved and any error in its assigned value requires a compensatory change in r to satisfy the equation. Since g is known to within an assured accuracy of a few parts per million it is unlikely that a better value for this constant would contribute much to improving our space program.

For convenience in measurement, standards are established for many of the quantities we need to measure, and standard instruments are constructed which are calibrated by comparison with established standards. Even for such a simply measurable quantity as force (if the value of g is known) we have devices such as proving rings and load cells, some capable of performing measurements of forces as great as that exerted by gravity on 3×10^6 kg. In addition to convenience another great advantage is attained through these devices; they tend to keep measurements for quantities of the same kind on a closely related and comparable basis.

Electrical/Magnetic Quantities

Measurement of electrical and magnetic quantities directly in terms of length, mass, and time is difficult, and unless performed with great care, inaccurate. Therefore, in this field establishment of derived standards is of great advantage.

By any of several well known procedures the resistance of a length of wire in ohms can be established to within a few parts in a million in terms of the units of length and time in accordance with the equations relating these quantities. Such a piece of wire may then be regarded as a standard for resistance. Similar resistors may be constructed and compared with it with great precision. If the quality of the wire has been wisely selected, and if the resistors are preserved with care, their resistances are not likely to change appreciably. By intercomparison with each other, each serves as a witness to the fidelity of each other. Serious deviates may be rejected from the group. From time to time the average resistance of the group may be checked by repeated absolute determinations. Such a group serves as a standard and experience leads to the view that the average resistance of the set is constant to within much closer limits than its accuracy is known.

Similarly, a current flowing in a wire can be determined in absolute units (through weighing) by the force it exerts on an identical current. Since we cannot preserve a standard of current as we can preserve a standard of resistance, we compare the voltage drop which that current produces when flowing through a standard resistor with the electromotive force of an electrochemical system, a standard cell. A group of such cells then serves as a standard of voltage just as a bank of resistors serves as a standard of resistance. In this case, too, the constancy of the set of cells appears to be better than the accuracy we ascribe to its value.

These standards of resistance and voltage furnish a basis for establishment of standards for other electrical and magnetic quantities in terms of which many of them may be measured with a consistency, with respect to the equations of electricity and magnetism, better than 1 part per 10^6. However, the consistency with the equations relating them to the standards for length, mass, and time is not nearly so good. The system is like a continent in the world of measurement, the relations between points of which are closely tied together although the distance to another continent is not nearly so well established.

It is useful to measure with consistency closely associated quantities, even though we cannot relate them with equal accuracy to those in another field, just as it is useful to tie together points on a continent so that we may join them roads and communication systems. We can pursue exte our research and engineering within a large but limited without regard to differences between fixed values in field and those in another. It is important only to be ful that such differences do exist. We have problems e in relating quantities within a limited field.

Standards for voltage and resistance have been set the national standards laboratories of the world and fully related to each other. Values have been assign these standards on the basis of absolute determin which are treated as exact. Other resistors and sta cells are calibrated by comparison with them and their given to a part in a million with respect to the na standards *as maintained* by the laboratories, even thou is recognized that these national standards themselves have a constant error. If we were to do otherwise, if we to include the estimated absolute error of the standa assigning the error to the calibrated device, its users be unable to perform measurements with it with surety the full precision of which it is capable.

Few users have concern with minor deviations of stan in their own field from their theoretical absolute value long as the standards pertaining to each field are coordi It is of little concern to our instrument maker if his i not exactly 1/12 of the foot a surveyor uses, but it concern to each that within each industry the units of surement conform so that the parts made by one instru maker will mate with those by another, that the lines o one surveyor meet those of another. How would the mu an orchestra sound if each musician played according sense of absolute pitch instead of the pitch selected b conductor?

Relaxing requirements for conformity, even between s ards within one measurement discipline, can be useful. example, by accepting the established standards for resi and d-c voltage as correct, other electrical quantities su current, charge, power, and energy can be realized in ter them to better than 1 part per 10^6 by d-c techniques, the corresponding a-c quantities cannot be realized with same limits. The difficulties become greater at h frequencies. The uncertainty in relating microwave pow the x-band to d-c power is estimated at 1 part per 10^3 when great pains are taken in the experiment.

Often under similar circumstances arbitrary standards be constructed which are highly stable and closely repr ible without reference to the sacred trinity of length, and time. If these standards are maintained by a c laboratory so that all closely related standards ca referred to them throughout the industry, then all mea ments throughout the industry will be conformal. D matter if the unit which we call a watt in microwave is only approximately equal to a watt in d-c power? will be concerned about the difference unless he is go compare the two in producing some other form of pow as that to drive a motor or operate a water heater? which we call a watt, even in d-c measurements, approximately equal to the power which, applied for would give a 1-kg mass a speed of 4 meters per sec. serves our purposes very well, though it may be in e a few parts in a million.

There are few individuals who are concerned with t crepancies between one measurement discipline and a as well as with expressing the values of various p constants such as the speed of light, the charge an of the electron, and Planck's constant in terms of mental units of measurement. It is well to leave to t care of these things. Their task will be made easier who make measurements state explicitly the chain which the standards used in their experiments are t to the national standardizing laboratories.

Conclusion—Frontiers of Measurement

SCIENTIFIC KNOWLEDGE, TECHNOLOGY, AND MEASUREMENT move forward today hand-in-hand, helpless each without the others. Advances in the first two are demanding greater accuracy and precision in measurement, and metrologists, in turn, are quick to seize every scientific discovery that promises to advance their art. For illustration, take the action of the General Conference on Weights and Measures, which last met in 1960.

The Conference abrogated the meter bar as a definition of the unit of length, which had prevailed in scientific circles for nearly 200 years, and defined the meter in terms of a wavelength of light from the isotope krypton 86. Before the printed report had reached the conferees, another scientific breakthrough was achieved. Continuous maser action in the optical region of the spectrum had been demonstrated. Here was the possibility for a new standard for length!

THE IMPORTANCE OF THE OPTICAL MASER as a measuring device is clear when it is compared with the krypton standard. Length measurements with krypton light cannot be made in a single step by interferometric methods for distances as great as one meter because the light waves from a krypton lamp are not sufficiently coherent. The light from a krypton lamp is emitted by atoms radiating more or less independently so that individual waves separated by a few decimeters in a beam coming from the lamp have no fixed phase relationship. But the atoms in a maser are stimulated to emit radiation in phase with each other. Waves in the beam from a laser are coherent over great distances.

Quick to take advantage of this break-through, metrologists at the National Bureau of Standards demonstrated the feasibility of applying an optical maser for length measurements by obtaining interference fringes with a Michelson-type interferometer, using a maser source, in which the difference in length of the two arms was 100 meters. In Michelson's day the limit was a few centimeters; krypton extended it to a few decimeters, but the maser has not yet reached its limit. (Calculations show that present masers are capable of producing radiation that is coherent over a distance of several hundred kilometers in a vacuum.) The maser promises to make length measurements competitive with frequency measurements for the distinction of attaining the highest accuracy and precision. On the more practical side it opens the way to the control of machine tools by light waves.

ELECTROSTATICS IS THE OLDEST BRANCH of electromagnetic science, yet during the past decade a new electrostatic theorem was discovered. This is the Lampard theorem, which affords a means for establishing more easily and more accurately a standard for the unit of electric impedance. A capacitor, constructed to satisfy the conditions of this theorem, consists of four cylindrical conductors with parallel axes which intercept a perpendicular plane at the apices of a square, the conductors themselves being close to, but insulated from, each other. The Lampard theorem states that the average of the capacitance per unit length between diametrically opposite pairs of cylinders, one pair being grounded, is a constant, subject to corrections in the fourth power of the difference between the capacitances of the two pairs. Symmetrical arrangement makes this difference very small and its fourth power negligible; the theorem itself is more general.

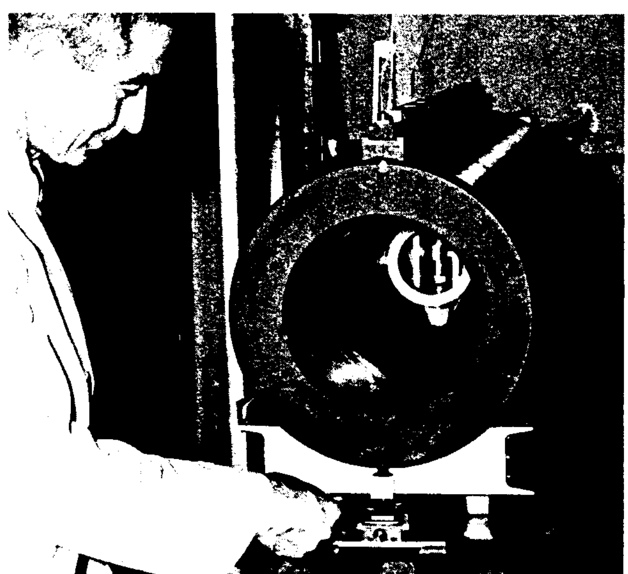
The great advantage of the Lampard capacitor for establishing a standard for capacitance is that its capacitance is proportional to the length of cylinders only, a quantity which can be measured with great accuracy. It requires no measurement of radii or calculation of areas as do ordinary spherical or cylindrical condensers. Previous to discovery of the Lampard theorem, the most trustworthy method for establishing a standard for electric impedance was by means of a calculable inductor.

Since the impedance of a capacitor is given by $Z_c = 1/C\omega$ and of an inductor by $Z_L = L\omega$, the two results are identical if expressed in MKSA units, Z_c and Z_L being given in ohms. But if the units we use are cgs esu for the first equation and cgs emu for the second, both C and L are expressed in centimeters and $Z_c \neq Z_L$. The quotient, Z_L/Z_c, then has the dimension of a speed squared, and this speed is identical with c, the speed of light. The value for c obtained by combining the value for an impedance determined by the Lampard capacitor method with that determined by the inductor method agrees to within 3 parts per 10^6 with the concensus value derived from a large number of determinators of c by more direct and accurate methods. This gives assurance that the value assigned to the standard for impedance is highly accurate, that is, it is strongly in accord with the equations by which the unit of impedance is defined.

DISCOVERY OF NUCLEAR MAGNETIC RESONANCE afforded another opportunity for advancement in metrology. A proton in a magnetic field, if disturbed in its orientation, will precess about the field like a gyroscope. The precession frequency, easily measured by electronic techniques, is proportional to the magnetic field. This is a new means for measuring a magnetic field. The precession frequency has been measured in magnetic fields established by accurate circuit configurations and electric standards. Using the value of the gyromagnetic ratio of the proton, the strength of any magnetic field may be measured from the relationship $B = \omega/\gamma$ where ω is the precession frequency in field B and γ the gyromagnetic ratio, that is, the precession frequency in unit magnetic field.

Applying this technique to the measurement of strong fields which cannot be calculated accurately, a value of e/m for the proton, that is, its charge-to-mass ratio, has been obtained. This quantity, multiplied by the mass of the proton in atomic mass units, yields the value for the important physical constant, the faraday, F, which is the charge associated with one gram-atomic mass of a monovalent element. The faraday may be determined also by the method of electrodeposition or electro-erosion.

The equations expressing F in terms of the unit for electric charge as determined by these two radically different methods are of such form that erroneous values for the electricity



Equipment for the determination of the gyromagnetic ratio of the proton. The value of this ratio can be used to measure the strength of any magnetic field.

A sixteenth century standardization proce for the German rute, *a variant of the English rod, is shown in this woodcut. S teen men, selected as "random sample" they emerged from church, were arranged a toe-to-heel formation and the overall len of their feet was used to set up a stand measure of length.*

standards would cause the values for F to deviate from each other. However, both methods lead to values which agree to about four parts in a million. This gives us assurance that the electricity standards for current and charge have been established in high conformity with their idealized definitions.

It is highly interesting to note that this quantity, the faraday, when multiplied by c^2, is the conversion factor for relating the atomic mass unit to electron-volts in accordance with Einstein's equation $E = mc^2$. This conversion factor, of great importance in nuclear physics, is estimated to have a standard error of 4 parts per 10^6 because of the circumstances just described. On the other hand, the standard errors of the conversion factors for relating electron-volts to joules and atomic mass units to kilograms are 12 parts per 10^6 in both cases.

AS MEASUREMENT IS INDEBTED to present-day scientific knowledge and technology for its advancement, likewise it is being taxed to pay that debt by the stringent demands of modern space and nuclear science. Greater accuracy and precision are now required in measuring quantities which were of everyday interest a few decades ago. The range of magnitude over which high accuracy is required has been greatly increased and environmental factors have been multiplied. Quantitative data are being called for where qualitative information sufficed before. In many cases the demands are so new that it seems as if metrologists were being called upon to measure new kinds of quantities. Examples are numerous.

We are called upon today to assess "radiation damage." Most physicists think of this in such context as the effects of high energy radiation on solar cells powering satellites or the effects on electronic components subjected to a nuclear blast. Who can assess, even to an order of magnitude, the genetic effects of high energy radiation? Though many talk about it none has accurately measured it. This is of the nature of a new kind of quantity, lacking in our metrology.

Demands are being made for optical properties of materials above and below the range of the visible spectrum where before it sufficed to designate them as merely opaque or transparent. Our space laboratories will view the sun and stars with ultraviolet and infrared telescopes to seek the information which our atmosphere has shielded from us. Design information for these devices is sparse. Methods of conventional optics must be revised to meet these measurement needs. Demands are also made for measurements at temperatures considerably above and below room temperatures.

WHEN OUR SATELLITE OR SPACE PROBES are beyo atmosphere they are subjected to conditions of vacuu tenuous then we can produce or measure in a lab When they are launched, the rocket gases are at a te ture which is difficult to measure, but on such measur their flight depends. Their courses through space are f by tracking systems which are dependent on distance m ments on the surface of the earth required to be a to one part in a million. Angles must be measured b and optical means with high accuracy. The refractiv variations of the atmosphere for radio and light wav be accurately measured to correct for bending of the Time must be coordinated at widely separated statio microsecond precision. These are some of the p which measurement must solve.

Perhaps, more than on anything else, the success o undertakings depends on the less glamorous asp measurement: the seemingly simple process of me diameters of bearings and shafts, roundness of balls, a shapes and spacing of teeth on a gear. The measur are made difficult by high accuracy requirements o devices as are used in inertial guidance systems. El components must be matched. Pressure gages an mometers must measure on the launching pad conf with the measurements made in laboratories and sites. With many agencies of our vast industrial c contributing to each project, coordination of standar be achieved by tracing the standards to a central such as the National Bureau of Standards. It is rem that our measuring system has worked so well and en to launch satellites and space probes which in turn m albeit roughly, the intensity of radiation in the Va belts, the magnetic field of the moon, and the temperature of Venus.

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