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ON THE MEASUREMENT OF LIGHT VELOCITY AND DISTANCE

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In the discussion of the velocity of light and the measurements of distance by electromagnetic waves, Mr. Kenney stated that the International Association of Geodesy at Helsinki had passed a resolution adopting formulae for the refractive index, *viz.* the Barrel and Sears formula for light waves and the Essen and Froome formula for radio microwaves. He said that he would welcome the concurrence of URSI in the adoption of these working formulae, and proposed a resolution accordingly.

Mr. Thompson supported this resolution and proposed slight amendments in the final draft (p. 113).

REPORT ON THE VELOCITY OF LIGHT

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The purpose of this report is to summarize the status of our knowledge of the velocity of light at the time of the XIIth General Assembly of URSI (1957), to list significant results since 1957, to call attention to work in progress as far as known to the author, to mention promising proposals for future work and to collect a bibliography for the period January, 1957, to January, 1960. Emphasis is more on the reportorial task than on critical evaluation of individual work.

At the XIIth General Assembly it was possible to adopt a resolution recommending that the velocity of electromagnetic waves in vacuum be taken as 299,792.5 ± 0.4 km/sec. This value was based on work since 1954, particularly that of Froome (1954) with microwave interferometry and Edge (1956) with the geodimeter. The status in 1957 was that, since Essen's introduction of the subject at the General Assembly in 1950, the value of c as determined by microwave cavity, microwave interferometric, direct optical and band spectra methods had converged towards the value quoted above. The proper direction of further work seemed to be to develop further the accuracy of the most promising method and to examine the results of other methods for satisfactory agreement.

The outstanding work of the last three years is that of Froome (1958), who obtains

$$c = 299{,}792.50 \pm 0.10 \text{ km/sec}$$

where the range given is obtained from the standard deviation of a single observation statistically combined with estimated systematic errors. His method was a four-horn microwave interferometer operated at 72 kMc/s with corrections applied for diffraction and air refractivity. Most careful attention was paid to the accuracy of these corrections, to the identification and elimination of systematic errors such as temperature sensitivity and unwanted reflections, and to the calibration and measurement of frequency, length, temperature, etc.

Bibliography p. 97

The design of the experiment was well conceived, having been built upon experience of several years with previous instruments of the same type; the number of factors which can be listed as contributing to the uncertainty is small and each seems to have been dealt with in a satisfactory matter either independently or by internal consistency.

Two further contributions from the rotational spectrum method were published by Rank, Guenther, Shearer and Wiggins (1957) and Rank, Guenther, Saksena, Shearer and Wiggins (1957). The first paper was an extension of previous work on HCN, and the second dealt with CO. The method is rather indirect, involving the determination of the rotational constant *Bö* in wave number units by a process of fitting several rotational, vibrational and centrifugal stretching constants of the molecule to infra-red band wavelength measurements and the independent observation of the same constant in frequency units in the microwave region. The results depend on the adequacy of the theoretical expressions for the line frequencies and upon their applicability to both the infra-red and microwave regions. Estimation of errors by the authors gives results in agreement with all recent determinations and the uncertainty in the value so obtained has now been made less than 1 km/sec.

Janney (1957) has considered theoretically the effect of a general value of the surface impedance in the microwave cavity resonator method of measuring *c*. He obtains

$$\mathrm{Re}\ \omega = \omega_a\,[1 - (1/2Q)\ (X/R)]$$

where ω is the complex resonant frequency, ω_a is the angular frequency of the a^{th} normal mode, Q is the cavity Q, and X and R are the reactive and resistive components of the surface impedance respectively. For an infinitely thick material of constant conductivity, $X/R = 1$; but for lamina of different conductivities, as may unavoidably occur as a result of fabrication procedures or chemical reaction on exposure to the atmosphere, X/R may depart from unity by several percent. The author points out that this may occur even though the observed Q of the laminated cavity remains close to the Q expected from the base material alone. The conclusion is that the value of X/R for the cavity in use should be independently measured before this source of error can be positively eliminated.

Recent development at the National Bureau of Standards of an absolute standard of microwave conductivity may afford a means to make this measurement by observing both frequency shift and Q change caused by such a laminated material when used as the top plate of a standard cavity. Precision methods of measuring microwave voltage reflection coefficient, also recently developed there, may facilitate comparisons among differently prepared surfaces.

Williams and Williams (1957) have considered a number of errors which may limit the accuracy of the geodimeter. These are the proper averaging over the distribution of wavelengths in the geodimeter beam, the accuracy of the refractivity correction, and optical path length changes caused by curvature owing to atmospheric refraction. They believe that accuracy approaching 1 part in 10^7 can be attained by refinement.

Work is continuing at the National Bureau of Standards on a determination of *c* by a Michelson microwave interferometer operated in the Fresnel region at 6.28 mm. This work was described at the National Physical Laboratory Symposium on Interferometry by Culshaw, Richardson and Kerns (1959). The theory of diffraction in the interferometer has been put on a rigorous basis by Kerns and Dayhoff (1960) and is considered adequate to assess and correct for the diffraction error with confidence. A large 5 ft × 5 ft plane reflector and carriage and associated instrumentation are nearing completion and will be put in operation late this year (1960).

Instrumentation to measure simultaneously the refractivity of the laboratory air at 6.28 mm has been in operation for some time and is being used to determine absolute refractivities of He, N_2, O_2, A and other gases to 1 part in 10^7 or better. The values being obtained at 6.28 mm are in good agreement with values at 3 cm, 1.25 cm and 4 mm previously reported. The advantages of continuing this determination, in spite of the fact that the value of *c* now seems to fall within very satisfactorily narrow limits, are (1) independent determination by a microwave interferometer method of substantially different technique, and (2) benefit of a rigorous diffraction correction obtained from first principles of electromagnetic theory rather than from a semi-empirical correction judged by internal consistency after fitting adjustable values for effective aperture dimensions.

Another determination is in progress as reported in the Quarterly Reports of the M.I.T. Research Laboratory of Electronics by Zacharias and Stroke. They intend to use the cavity resonance method with careful refinements to overcome the limiting factors in previous cavity determinations. A fused quartz cavity of very accurate circularity has been constructed to avoid uncertainty from dimensional perturbations. The end plate will be swept through resonances while its position and orientation are measured and perhaps controlled by three optical interferometers on principles similar to those used in the interferometrically controlled M.I.T. grating ruling engine, in order to improve the length measurement and reduce the uncertainty from tilt of the end plate. Provided the deposition of the conducting layer can be carried out with sufficient skill to avoid difficulty in the "Q-correction" (better referred to as "surface impedance correction"), the method seems to offer the possibility of an improvement in accuracy and also to provide for periodic measurement of c in a search for possible secular variation. The investigators hope to approach accuracies of parts in 10^8.

Recently Zimmerer (1960) has measured 12 microwave lines of oxygen near 0.5 cm to a hundredfold greater accuracy than previously. The presently available expressions for the infra-red transitions on the one hand and the microwave transitions on the other hand in terms of the molecular parameters of oxygen are not sufficiently consistent to permit a competitive estimate of the velocity of light by this method. The reason is that oxygen has a more complicated structure than linear rotators such as HCN or CO, so that one is less sure of the appropriate parameters for the description of its spectrum. Nevertheless, the precisely observed transition frequencies are available for the time when the microwave theory may be suitably revised, and may give a result to a few parts per million.

Under advanced development at the National Bureau of Standards is the microwave Fabry-Perot interferometer, fed and observed by directive horn antennas. This device is proving to have extreme utility in the millimetre region as a resonator for wavemeter applications, dielectric constant and loss measurements, masers, microwave spectroscopy and length standards. It also appears very promising as a future method of determining the velocity of light.

The fundamental characteristic of this interferometer is its high Q, which is interpreted in the usual way as 2π times the ratio of energy stored in the reflector system to the energy dissipated per cycle. For plates assumed of infinite extent, this Q for normally incident waves is proportional to the order of interference n and the skin depth, δ. Thus since $n \propto \lambda^{-1}$ and $\delta \propto \lambda^{\frac{1}{2}}$, Q actually increases as λ decreases. Present techniques of design and fabrication indicate that Qs of 4,000,000 may be expected for wavelengths of 1 mm and interference orders of 2,000. Coupling power out to an external load, such as a receiving antenna, will lower the Q, but this may be kept as small as desired by design of the plates. If the plates are not infinite in transverse extent two other effects enter to reduce the Q. Power incident other than normally on the plates will be lost after multiple reflection and power incident normally will be partially converted by diffraction at the finite plate aperture to other angles, and will be lost by multiple reflection. The sharpness of resonance response as plate separation or wavelength is varied will depend both on the Q and on the directivity of the transmitting and receiving antennas. It seems to be possible to make the plates sufficiently large and the antennas sufficiently directive so that the sharpness of response is limited by the reflectivity of the plates alone. There may even be the possibility of "beam sharpening" by finite plates, in that the transmitted power may fall off even more rapidly with angles off the axis than does the antenna pattern, until the transmitted beam width is finally limited by the diffraction pattern appropriate to the aperture of the plates themselves. In practice, loaded Qs of 116,000 have been observed for 6 mm in the 136th order, and sharpness of resonance of 1/2000 wavelength has been obtained. This point emphasizes the precision of setting the

	$\lambda = 6.28$ mm	*$\lambda = 1.0$ mm*
Spacing, d	43.2 cm	100 cm
Order, n	136	2,000
$Q_o = \lambda/\Delta\lambda$ (computed)	368,000	4,250,000
Q_{loaded} (measured)	116,000	—
$Q_d = \frac{2}{n} Q_o = \lambda/\Delta d$	1,700	4,000 (est.)
Beam width for 15 cm × 15 cm horns ($\simeq \lambda/a$)	2.4°	0.38°

Bibliography p. 97

interferometer since one can readily set to 1/10 to 1/100 part of a fringe which is already localizable to 1/2000 part of a wavelength, which for an order of interference of 5000 gives an over-all precision of parts in 10^7 to 10^8. The advantages of the Fabry-Perot interferometer as the wavelength is reduced are summarized by some selected characteristics listed in the table above.

The sharp response, the indication that diffraction corrections may turn out to be small and thus easily applied, the small over-all size possible in operating at wavelengths of around 1 mm, and the ability to obtain a measurement by wavelength variation instead of mechanical movement, so that the experiment could be carried out *in vacuo*, all combine to make the microwave Fabry-Perot interferometer a promising possibility for a future determination of c.

Plate separation should be measurable in terms of the optical wavelength standard. It is likely that further work will give confident means of dealing with sources of systematic error that can presently be recognized. Among these are effect of slight misalignment (analogous to dimensional perturbations in a cavity resonator), effect of diffraction by both the antenna aperture and the plate aperture, and effect of phase shift or reflection (analogous to skin depth correction in a cavity resonator).

Still other methods have been proposed from time to time which are worthy of notice and evaluation. One of these is the modulation of visible light at microwave frequencies. Diffraction errors will be entirely negligible since they are associated with the visible wavelength and the propagation path can be reduced to laboratory dimensions because of the short modulation wavelength, thus combining the advantages of a refractivity correction better known in the visible than in the microwave region, and applied over a shorter path than in the geodimeter. A possible modulator for this application may be an ultrasonic crystal. Karolus and Fried (1959) have described such experiments at the lower frequency of 16 kMc/s.

Sanders (1959) proposes that the modulation be obtained from beats between coherent wave trains of visible light, as may possibly be obtained from optical masers. The beats would be detected in a photoelectric device, response of these devices having been shown to be proportional to the square of the incident amplitude. Recent encouraging progress in optical masers adds interest to this proposal.

Another proposal is the transit time oscillator in which a short pulse is recycled in a circuit consisting of pulsed light source, propagation path and electron optical image converter. The path (or path change) and the frequency of the oscillation are measured, and the velocity of light is computed as the product. A modification has been suggested by Vul'fson (1959) in which transit time is measured directly by the image converter. So far these methods have not been reduced to high precision because of difficulty of technique and systematic errors such as phase delays in the circuit portion of the apparatus.

Finally, in surveying the question of the velocity of light, the implications of general relativity, as summarized by Møller (1957) ought to be borne in mind, *viz.* that the observed velocity w depends on the scalar and vector gravitational potentials χ and $\vec{\gamma}$, such that

$$w = c'/(1 + \vec{\gamma} \cdot \hat{e})$$

where $c' = c(1 + 2\chi/c^2)^{\frac{1}{2}}$ and $\hat{e}$ is a unit vector in the direction of propagation. The magnitude of these effects, however, is so small as to offer little hope of their terrestrial verification until precisions of the order of parts in 10^{18} are attained.

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PROTON GYROMAGNETIC RATIO

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The proton gyromagnetic ratio has recently been measured by the NBS. A 12 gauss magnetic field supplied by a precision solenoid was used. The preliminary results were reported in 1958 by Driscoll and Bender. The precision solenoid was the same one employed in a recent determination of the NBS ampere in absolute units.

The technique used was free precession. The sample consisted of a 2 cm diameter sealed glass sphere containing distilled water. It was polarized in a 5000 gauss field and then shot through a 40-foot pneumatic tube into the centre of the solenoid. After arrival, about two seconds later, the magnetization was still large and lay along the direction of the solenoid field. A short pulse at the resonance frequency of about 52.5 kc/s was then applied perpendicular to the solenoid axis. This pulse left the magnetization perpendicular to the field direction and precessing about it. The precession frequency was obtained by measuring the period for a given number of cycles of the signal induced in a pickup coil surrounding the sample. The Q of the pickup coil was kept low to prevent radiation damping and possible associated frequency shifts. The average period over a three-second time interval could be measured to one part in ten million with this method.

The solenoid used in this experiment consisted of a single layer helical winding in a lapped groove on a fused silica form. The value of the magnetic field was calculated from the carefully measured geometry of the solenoid and the current through it. The only critical dimension was the pitch, which could be measured to about one part per million. Extensive efforts were made to prevent errors due to slightly magnetic materials used in or near the solenoid. In order to avoid gradients and man-made perturbations the original experiment was carried out at the Fredericksburg Magnetic Observatory of the U.S. Coast and Geodetic Survey.

References p. 100