DIMENSIONS

UNITS

and STANDARDS

a talk presented
at a meeting of the staff of
the National Bureau
of Standards, December 14, 1956

By A. G. McNish

WHAT I wish to set forth here is somewhat in the nature of a sermon, and, as a sermon it should be based upon some text.

For my text I shall take the words of one of my favorite writers—Paul of Tarsus—as rather poorly translated in Acts 17: 22 of the King James Version. "Then Paul stood in the midst of Mars' hill and said, ye men of Athens, I perceive that in all things ye are too superstitious."

From this point on the words of Paul are no longer appropriate to my subject. For I am going to discuss dimensions, units, and standards. About these things I perceive that, like the men of Athens, most scientists are very superstitious.

The concepts I am going to discuss are derived from much talk with my colleagues and much meditation. Some are original, but, as I find on examining the literature, few, if any, are novel. Many are so old that they have been set in print long before I was born.

They have been in practice so long that they have come to be accepted with great faith, with little reason, and sometimes with much confusion. It is this irrational acceptance which I refer to as superstition. I find this trait in the speech and writings of some of the scientists I esteem most highly. Even though they confess vehemently their belief in the arbitrary nature of dimensions, units, and standards, they often seem to forget their confession and lapse into superstitious practices and expressions. For this reason an occasional exposition on the subject is warranted.

LET us proceed now to consider the concept of dimensions. What is a dimension? It is simply a tag we attach to a quantity in an equation expressing some physical law, no more. Some years ago I would have regarded this blunt statement as a heresy, but that was when I was even more superstitious than I am now.

I remember when I was first introduced to the concept of dimensions. I derived great esthetic satisfaction from the fact that the dimensions of various quantities,

like force and energy, could be expressed in terms of length, mass, and time. When, in electrostatics and magnetostatics, I encountered fractional exponents for the "fundamental" dimensions I was a little mystified. But when I learned that in the Gaussian system the dimension of capacitance was length and the dimension of inductance was length I became a complete mystic. After all, do we not measure the capacity of a sphere by its radius and the inductance of a wire by its length?

I think my feeling at that time was somewhat akin to what Leibnitz expressed when he spoke of the imaginary number as "A fine and wonderful recourse of the divine spirit, almost an amphibian between being and not being." Today I would rather regard the imaginary $\sqrt{-1}$ simply as a mathematical operator which, when twice applied to a quantity, reverses its sign.

How much better it would have been for me, and possibly for others as well, if the tags attached to the quantities in the simplest Newtonic equations had been only nonsense words or names like Louise, Mary, and Tom! Then Louise times Louise equals Alice, and Alice times Louise equals Victor. No one would then think that Louise was more closely related to Victor than to Tom. Yet, I am confident that there are many who feel that, because of dimensionality, volume is more closely associated with length than it is with mass, or than length is with time. If these people are argued with they will say that volume is closely associated with length, but time is of a different nature, and that one knows this *intuitively*. But I say to you, ask a small child to push a large box across a room. Without laying hand to it he is likely to say it's too *heavy*. It is clear that he associates the concept of mass with volume. The same child will speak of a *long* distance or a *long* time, showing that, in his mind, both distance and time have similar qualities.

The highly arbitrary and artificial nature of assigning dimensions to physical quantities may be recognized

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from the different results obtained by assuming certain quantities are "basic". Such results are shown in Table 1 where the dimensionality of various mechanical quantities is shown. The first column of dimensions is the familiar length, mass, time system, and the second, the less familiar length, force, time system. Both of these are three-dimensional systems, the other systems are two dimensional, obtained by assuming that at least one of the quantities in the table is dimensionless.

The two-dimensional systems are of no practical concern, except that presented in the fifth column, where the gravitational constant is assumed to be dimensionless. This is the system used in astronomy and results from the equation $F = M_1M_2/r^2$. It may shock some of us to see that in it the simple quantity, mass, has the dimensions L^3T^{-2}, that is, mass might be measured in cubic miles per hour per hour! Its dimensions indicate a change in rate of expansion!

Another interesting feature of the table is that in all of the systems represented the dimensions of torque and energy are the same. One might ask then, are torque and energy similar in nature? This is a nonsense question, the nonsensicality of which can best be exceeded by an affirmative answer. The dimensional identity of these two quantities is simply an artifact due to imperfect dimensional systems and is in no way related to the physical nature of the quantities.

The ambiguity arises because torque involves the concept of angle and, in all the systems, we have assumed that angles are dimensionless. This ambiguity may be resolved in a number of ways. I should choose to do it by introduction of a fourth dimension, namely, plane angle for which we shall use the symbol C (circle). The dimensions of torque and moment of inertia now become $L^2MT^{-2}C$ and L^2MC. Thus we see that to remove ambiguities in even the few mechanical quantities listed in the table four symbols are required. A little thought will reveal that if we extend the list we shall need at least one more dimension, the solid angle, to which we might assign the symbol S (sphere).

If we pursue this matter further we shall see that a lot of good has been accomplished by our forethought, and it is such things as this which generate a mystical aura about dimensions. The disputes which arise from the misunderstandings involved in the rationalization of electric units are reconciled if we introduce plane angle and solid angle as dimensions and the conversion between rationalized and unrationalized units is simplified. Planck's constant now has the dimensions $L^2MT^{-1}C$ and we don't have to worry about h and h-bar as long as we specify our system of metrics.

Thus we see that dimensions are only symbols of an elementary algebra, involving neither addition nor subtraction. To ask what are the true or natural dimensions of a quantity makes no more sense than to ask what is the true or natural word for goldfish.

Now I would not have you believe from what I have said that I think lightly of dimensions. Dimensional

Table 1

Dimensions in Various Systems

Quantity		LMT	LFT	LT $G=1$	LT $M=1$	LM $T=1$	TM $L=1$	LM $S=1$
Length	L	L	L	L	L	L★	1★	L★
Mass	M	M	$L^{-1}FT^2$	L^3T^{-2}	1	M	M✹	M✹
Time	T	T	T	T	T	1	T	L★
Area	A	L^2	L^2	L^2	L^2★	L^2	1★	L^2
Volume	V	L^3	L^3	L^3	L^3	L^3	1★	L^3
Density	D	$L^{-3}M$	$L^{-4}FT^2$	T^{-2}	L^{-3}	$L^{-3}M$	M✹	$L^{-3}M$✹
Speed	S	LT^{-1}	LT^{-1}	LT^{-1}	LT^{-1}	L★	T^{-1}	$L^{-3}M$✹
Acceleration	a	LT^{-2}	LT^{-2}	LT^{-2}	LT^{-2}✹	L★	T^{-1}	1
Force	F	LMT^{-2}	F	L^4T^{-4}	LT^{-2}✹	LM	T^{-2}	L^{-1}
Pressure	P	$L^{-1}MT^{-2}$	$L^{-2}F$	L^2T^{-4}	$L^{-1}T^{-2}$	$L^{-1}M$	MT^{-2}✾	$L^{-1}M$
Work	W	L^2MT^{-2}✾	LF✾	L^5T^{-4}✾	L^2T^{-2}✾	L^2M✾	MT^{-2}✾	$L^{-3}M$✹
Power	P	L^2MT^{-3}	LFT^{-1}	L^5T^{-5}	L^2T^{-3}	L^2M✾	MT^{-2}✾	M✾
Torque	t	L^2MT^{-2}✾	LF✾	L^5T^{-4}✾	L^2T^{-2}✾	L^2M✾	MT^{-3}	$L^{-1}M$
Moment of Inertia	I	L^2M	LFT^2	L^5T^{-2}	L^2★	L^2M✾	MT^{-3}	M✾
Gravitational Constant	G	$L^3M^{-1}T^{-2}$	$L^4F^{-1}T^{-4}$	1	L^3T^{-2}	L^2M✾	M✹	L^2M
Ambiguous: double		1	1	1	3	L^3M^{-1}	$M^{-1}T^{-2}$	LM^{-1}
triple		0	0	0	0	0	1	2
quadruple		0	0	0	0	1	1	1
						1	1	0

analysis is a useful tool for the physicist, but there is a time for using dimensions and a time for leaving them alone. To those of you who are interested in pursuing the matter further I commend the various textbooks on the subject.

I have avoided, so far, the assignment of dimensions to electric and magnetic quantities. In particular I have avoided the magnetizing field which many of us are wont to call H, and the associated quantity μH to which we give the nickname B. Our thinking on these matters will develop when we consider units of measurement.

I think the situation regarding units of measurement can be made clearer by a parable I have devised.

ONCE there was a great king who was an *absolute* monarch. But he was worried because his kingdom was surrounded by barbarians who threatened war against him. So he called his counsellors together and asked their advice.

And they said to him, "Build a great scientific laboratory, tall like the Tower of Babel. Put into this tower the greatest scientists in the kingdom, and in due time they will find a solution for your problem; for is it not written in the First Book of Moses 'Nothing that they propose to do will be impossible for them'?"

So the king did as they advised. On the first floor of this tower he placed a group of men and ordered them to work in the field of mechanics, and in this field alone. And he placed a man in charge of them whose name was Isaac.

On the second floor he placed a group and charged them to work with electrostatics. And the name of the chief man was Benjamin.

On the third floor the men were charged to work with magnetics and current electricity under a man named Michael. The men on the fourth floor were to work only with heat. And so to each floor was assigned a separate field of science.

Since the purpose of this project was to provide for the security of the kingdom, the king ordered the project cloaked in the deepest secrecy. The men on one floor were forbidden to talk with those on another lest their secrets be disclosed. Although the men did not like this they obeyed, for the king was an *absolute* monarch and in those days the king's word was law.

Now Isaac was a man of great perspicacity. He perceived that to understand much it is necessary to measure, and in order to measure it is necessary to have units of measurement. So he decided on a unit of length which he called the centimeter, a unit of mass which he called the gram, and a unit of time which he called the second. And he said, "We will call these *absolute* units in honor of the king. For after all, are they not absolute in the same sense that he is, in that *they do not depend on any other units?*" And these units and all based upon them are called absolute units to this day. He also devised units for the other quantities he measured and expressed them in terms of his absolute units. The unit of force he called the dyne and the unit of work he called the erg, after the Greek words.

He also discovered that two masses exert a force on each other which is proportional to the product of their masses and inversely proportional to the square of the distance between them. This force was much less than his unit force even though the two masses were unit masses, and the distance separating them was a unit distance. So he named the ratio of this force to the unit force, G, and called it the gravitational constant. But he did not worry that this gravitational force was so much less than the unit force nor did he attribute its magnitude to a property of free space.

The men working on the other floors of the tower also set about establishing units for measurement, but their units, in most cases, were less satisfactory and they did not call them absolute units except on the fourth floor where they decided on a zero for their temperature scale which they called absolute zero, "For," they said, "this zero of temperature *does not depend on anything else.*" The temperature scale based upon it they called absolute also.

Now in Michael's group, which was working on magnetics and current electricity, were two men whose names were Samuel and Joseph. They said to Michael: "Since we are working for defense of the kingdom it has occurred to us that electric current can be employed for rapid signalling which is of great military value and we should like to do this." Then Michael said to them, "Be about it," and so they set to work.

As time went by the dangers to the kingdom decreased, security restrictions were relaxed, and the men on different floors began to mingle with each other. Each saw the work the others had been doing and he marveled at it. Each saw similarities in some of the phenomena that had been observed and relationships between them. Many of the people liked the absolute units that Isaac had devised and wanted to use them, "For," they said, "if we express our units in terms of Isaac's then we will have an absolute system too." This was because the power of the king was so great and the royal epithet *absolute* had made so great an impression on them. Furthermore, they saw that if they did this their units of power and energy would be the same as Isaac's.

But the men on the fourth floor were not concerned with this because they already had an "absolute" temperature scale. The fact that their unit of energy, the calorie, differed from Isaac's erg did not worry them for they regarded this as an idiosyncrasy of nature.

Then Isaac's men and Benjamin's men came together and they observed that two electric charges exert a force on each other. They decided to define the unit of charge as that charge which repelled an identical charge at a unit distance with a unit force. The remaining units of the system followed from this, and Benjamin's men had an "absolute" system of units.

Then Isaac's men and Michael's men came together and they observed that two parallel wires exert a force on each other when the same electric current flows in them. They decided to define the unit of current as that

current which, flowing in two parallel wires, repels with a force of 2 units for each unit of length when a unit distance apart. Doubling the force, they thought, was required because the wires were assumed to be very long. The remaining units of the system followed from this, and Michael's men had an "absolute" system of units.

At this time both Benjamin's men and Michael's men were very happy. But when they discussed these things which had been decided upon, they found that Michael's unit of charge was 30 billion times as great as Benjamin's and Benjamin's unit of potential was 30 billion times as great as Michael's. There were also many other discrepancies in their systems.

Then a man whose name was Carl Frederick proposed that when they worked upon the second floor the units of Benjamin should be used, and when they worked upon the third floor the units of Michael should be used. This made both groups happy, and in honor of their benefactor they named the system after him and called it the Gaussian system. They got along well with the new system, but they became confused when they went from one floor to another.

At this time Samuel and Joseph, who had been working on electronic communication, made themselves heard. They said: "A plague upon both your houses, for we are *practical* men. We have had to measure to engineer the telegraph, and we have our own units which we have named after our mighty men and which differ from both the units of Michael and of Benjamin. Michael's unit of *potential* is far too small and Benjamin's unit of *current* is far too small, and there are other differences."

So these units of Samuel and Joseph were called practical units. But many people did not like them because they were not based on the units of Isaac, and consequently they would not call them *absolute* units, saying instead they were *arbitrary*, which means almost the same thing.

Others who worked in the tower were unhappy with the Gaussian system because, they said, it is dimensionally inconsistent and leads to a dimensionless quantity for the velocity of light.

In this winter of their discontent a man whose name was John, or Giovanni as he was called in his native land, perceived that if the meter were established as the unit of length, and the kilogram, of mass, and the unit of time left unaltered then a system could be devised in which the absolute electric units would have the same magnitude as the practical units provided they defined *one* electric unit as an absolute unit. In this system the electric constant became $\frac{1}{9} \times 10^{-11}$, instead of one as the Benjaminites had it, and the magnetic constant became 10^{-7}, instead of one as the Michaelites had it. Then they called these quantities the permittivity and permeability of a vacuum. But the speed of light now had the dimensions of a velocity.

Then followed much senseless chatter and confused thinking in the tower. Some said that the old system with three fundamental units had been good enough for

their forefathers and so it was good enough for them. Some called the proponents of the new system subversive because they chose to create a new unit where three were ordained by nature. Others said that it was creeping socialism for it undermined the *absolute* authority of the king and placed engineers on the same level as scientists. This confused thinking and senseless chatter continue even to this day. Those of us who hear it call it babble in memory of the tower in which it started.

THE story I have related does not conform strictly to facts, nor does it contain all the facts. Many systems of units have been proposed. One system took the earth's quadrant, ten million meters, as the unit of length, and ten micromicro-grams as the unit of mass and derived the volt, ohm, farad, and coulomb. Most used the second for the unit of time.

The assignment of values other than unity for the permittivity of space and for the permeability of space in the Giorgi system was done so that the units of force and work in the electromagnetic system would be the same as those derived from the arbitrarily chosen units of mass, length, and time in the mechanical system. To describe these quantities, ϵ_o and μ_o, as properties of space makes *exactly* as much sense as calling the gravitational constant, big G, a property of space, and the gas constant, R, the specific heat of vacuum.

On this last point I should like to expand. If one were to take the gas constant R as equal to unity then, in conjunction with the *absolute* units of the mechanical system, we should have an *absolute* temperature scale defined by the equation $PV = RT$. The experimental procedure in determining the value of R in the above equation involves an extrapolation to zero pressure, and thus, speaking of R as the specific heat of a vacuum involves no more nonsense than speaking of ϵ_o and μ_o as the permittivity and permeability of free space. All of these things are devices to relate *arbitrarily* chosen units of mechanics, electricity, magnetism, and heat into one coherent system.

The question naturally arises: How many dimensions and how many units do we want in our science? The answer is clear, as many of you have perceived. In the case of dimensions, if we choose to have dimensions, we want *exactly as many as we have individual quantities to describe and to measure.* But to simplify our thinking and to order our thoughts we can regard most dimensions as composites of certain arbitrarily chosen elementary dimensions which are related in accordance with the various laws of physics and mathematics.

And how many *elemental* dimensions do we want? Since dimensions are only the elements of a simple and limited algebra we should have enough that there are no ambiguities in the quantities we wish to describe and not so many that there are redundancies in the dimensions of any one quantity.

The problem is like deciding how many colors are needed to make a map, subject to the condition that no two adjacent areas should have the same color. The

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topologists tell us they can prove that not more than five are necessary, but they have never found a case where more than four were required. But it is not so in physical science, for we are here dealing with an n-dimensional system and we do not yet know (I hope) the magnitude of n.

We saw in the table of dimensions I first presented that for the simplest mechanical quantities we need four dimensions to avoid ambiguities, five, if we include solid angle; that if we set one of these quantities equal to unity we can get along with four. But having one quantity equal to unity we cannot set another equal to unity without creating an ambiguity. Thus, unity itself becomes like a dimension, so again we may say we have five. Heat and electromagnetism add at least two more required dimensions. So I might venture to say that we should have seven elemental dimensions, at least, but I do not know, because I do not comprehend all of physics. One, of course, may get along with fewer dimensions if he will tolerate some ambiguities.

In the case of units the situation is different for we are not bound by the rules of an algebra. We can set as many quantities as we like equal to unity provided we do not violate any of the equations of experimental physics in doing so. Thus we can set big G equal to 1, or the velocity of light equal to 1, and the permittivity of a vacuum equal to 1. But in doing the latter we must also set the permeability of a vacuum equal to 1, for the equations of physics require that $c = 1/\sqrt{\epsilon_o \mu_o}$.

It can be shown that we can select a *single* quantity, assign a magnitude to it, and then, with this as our one and only *absolute* unit, derive a consistent system of units to measure all other quantities by arbitrarily assigning values to several physical constants. This is not a desirable thing to do because the units for many of the quantities which we frequently measure would have to be determined by difficult experiments which cannot be performed accurately.

It is therefore better to select certain appropriate quantities, the magnitudes of which we can reproduce and intercompare accurately, use these as our *absolute* units and base our other units upon them. This, then, leaves the various quantities like the gravitational constant, G, the magnetic constant, μ_o, the electric constant, ϵ_o, and temperature constant, °K, to be determined experimentally. Thus we see that the magnitudes of these quantities are not determined by nature, but by the units we have arbitrarily chosen for our measuring system.

We have wide choice in the units and the magnitudes which we may select and are limited only by the algebra of dimensions in obtaining an adequate and sufficient system. By an appropriate choice of units some things can be simplified.

Some of the difficulties we have long suffered are due to the magnitude of our common unit of time, the second. This is too small a unit for most human purposes and too great a unit for atomic processes. It is convenient for timing the 100-yard dash, but who ever heard of the 240-second mile? Some physicists have used the "shake" which is 1/100 of a microsecond as a unit of time for measuring nuclear processes and found it convenient.

For sport I have constructed a table of units based on the meter, gram, and a unit of time which I call the wink. It is approximately ⅓ of a shake. Now a number of interesting results are obtained by doing this. True, some of the units are of such size that they are not useful in everyday life but they are useful for measuring many of the quantities of modern physics. (See Table 2.)

Table 2

MGW System

(1 Wink = 1/300 Microsecond)

Quantity	Unit	Dimensions	Magnitude
Length	Meter	L	1 meter
Mass	Gram	M	1 gram
Time	Wink	T	$1/3 \times 10^{-8}$ sec
Force	Samson	LMT^{-2}	9×10^{18} dynes
Work	Einstein	$L^2 M T^{-2}$	9×10^{20} ergs (approx. 0.5 megaton)
Charge (es)	Charles	$L^{3/2} M^{1/2} T^{-1}$	100 coulombs
Charge (em)	Charles	$L^{3/2} M^{1/2} T^{-1}$	100 coulombs
Current	André	$L^{3/2} M^{1/2} T^{-2}$	3×10^{10} amperes
Potential	Alessandro	$L^{-1/2} M^{1/2} T^{-1}$	9×10^{11} volts
Resistance	Simon	$L^{-2} T$	30 ohms
μ		$L^{-2} T^2$	1

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I have pointed out the usefulness of the wink in timing nuclear processes. The unit of work, which I have called the Einstein, is very large, but it is approximately equivalent to the amount of energy released by the explosion of a half-megaton nuclear weapon. The unit of charge is equivalent to 100 coulombs, and, of resistance, to 30 ohms, not too different from our present units.

The pleasing feature of this system is that the unit of charge is the same for both the electrostatic and the electromagnetic systems. Also the permittivity of space, the permeability of space, and the speed of light are unity. For this purpose the system was designed. How much easier it is to remember the value unity than all the miscellaneous constants which our present system requires!

But another and even more striking result is obtained. You may remember that in every dimensional system the dimensions of energy are those of mass times velocity squared. This reminds us of Einstein's equation $E = mc^2$. If we take for m the gram and write 3×10^{10} centimeters per second for c, the value of E becomes 9×10^{20} ergs, which is exactly the magnitude of the unit of energy in our new system. We might say that in this system the unit of energy is the amount of energy which is obtained when one unit of mass is annihilated and call it the "gram-annihilate".

In spite of its obvious advantages there are a number of reasons why such a system should not be adopted for general use. The strongest of these is that we are already familiar with several systems the units of which are well established.

IN selecting units for a measuring system it is necessary for its experimental application to embody each unit in some material standard. The standard can be some graven image like the platinum-iridium meter bar, kept at Sèvres, France, or it can be a natural standard like the wavelength of a certain spectral line. If a unit is arbitrarily chosen it is called an absolute unit since it is independent of all other units, and I choose to call the standard which embodies it a prototype standard because the word "absolute" has so many different meanings.

At the present time there are four such prototype standards: the meter bar, the kilogram, the tropical year 1900.0, and water at the triple point. The electrical standards are not prototype standards in this sense for they are dependent on one or more of the above prototype standards. If we were to define the coulomb as the quantity of electric charge on 6.24192×10^{18} monovalent ions, then the coulomb would be an *absolute* unit embodied in a prototype standard of nature, the charge of the electron. If, however, it were defined in terms of the electrochemical equivalent of silver, or some other element, it would cease to be an absolute unit embodied in its own prototype standard, for it would depend on the standard of mass. However, although the coulomb defined in terms of the electrochemical equivalent of silver would not be an absolute

unit it would be a highly precise and accurate unit if we specify a suitable experimental procedure for realizing it. The current balance experiment would then become a determination of μ_o which might not be exactly 10^{-7}, the ampere being given by dQ/dt.

It is desirable that the ampere, volt, and ohm should be related by the equation $E = IR$ and that the quantities I^2R, E^2/R, and EI give the same magnitude for power as the mechanical units give. To satisfy this desire the value of the ohm as determined experimentally must be adjusted to the value of μ_o determined by the current balance, if the coulomb were adopted as an absolute unit.

It is instructive to examine the experimental uncertainties in the values of the principal electrical standards as obtained from the present definitions, and compare them with the uncertainties which would result if a unit of charge were taken as a prototype standard. These uncertainties can be treated in accordance with simple "error theory". For brevity we shall refer to these experimental uncertainties as errors.

At the present time the ampere is determined experimentally from a relationship expressed by the equation

$$\mu_o I^2 = K_1 F \pm e_1 \tag{1}$$

in which I is the current; F, the force (in MKS units); K_1, a geometric dimensionless factor in the experiment; μ_o, the magnetic constant, defined implicitly to have exactly the magnitude 10^{-7} MKS unit per ampere squared; and e_1 the over-all experimental error (fractional). Since I enters as its square, the error in the standard of current so determined is $e_I = e_1/2$. This is the error in the standard for the so-called "absolute" ampere.

While the ohm is defined in terms of the ampere and the MKS mechanical units by $I^2R = W$ where W is in mechanical watts and R is the resistance in ohms, a more accurately realizable experimental relation can be represented by an equation of the type

$$R = \omega \mathcal{L} + e_2 \tag{2}$$

where ω is a frequency; $\mathcal{L}$, an inductance given by $\mu_o K_2 L$ (K_2 being a dimensionless geometric factor, and L a representative length of the circuit); and e_2, the over-all experimental error for this experiment. Thus the error in the standard for the "absolute" ohm is $e_R = e_2$. The standard for the volt is derived from the ohm and the ampere by an experimental relation expressed by the equation

$$E = IR \pm e_3 \tag{3}$$

in which E is the potential difference; I and R are the current and resistance expressed in "absolute" units derived from the other experiments; and e_3 is the experimental error. If the three experiments are performed independently then e_1, e_2, and e_3 are independent. (It is very difficult to determine the extent to which the three experiments are independent. This may be done

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only by the most diligent inquisitorial examination of the experimental procedures. However, since the experiments on which the electrical standards are based are performed in different laboratories at different times, and since the values adopted for the electrical standards are obtained by combining the results of these laboratories, the condition of independence may be assumed.) Therefore to estimate the error of the standard for the "absolute" volt the three errors should be added as orthogonal quantities and the error becomes

$$e_V = \sqrt{e_1^2/4 + e_2^2 + e_3^2}.$$

We can now write the error for the standard of the "absolute" watt as given by electrical units as compared with that given by mechanical units. From the relation $I^2R = W$ it becomes

$$e_W = \sqrt{e_1^2 + e_2^2}.$$

(Since I enters as I^2 the error contributed by I must be doubled before squaring.) From the relation $E^2/R = W$ the error becomes

$$e_W = \sqrt{e_1^2 + e_2^2 + 4e_3^2}.$$

(Since E enters as E^2 its error is doubled, but since e_2 appears once in the denominator it must be subtracted from $2e_2$ before squaring.)

Since e_3 is ordinarily so small with respect to e_1 and e_2 that it may be considered negligible even to the first order its square is certainly negligible, and e_W determined in either way is approximately the same.

If we adopt a quantity of charge, Q, as an absolute unit the errors are distributed differently. The relationships between the electrical and mechanical units are determined by the same experiments. The standard of current is obtained with practically no error from $I = dQ/dt$, because time can be measured with such high accuracy. We could, of course, adopt either the ohm or the volt also as an absolute unit, just as the practical men did who worked with the telegraph. Then our electrical units would be completely independent of the mechanical system, but we should have to determine some conversion factor to relate electrical power to mechanical power which we might call the "mechanical equivalent of electricity". If we adopt only one electrical unit we are able, by an appropriate determination of μ_0, to make the mechanical equivalent of electricity equal to unity, subject to the limits of experimental accuracy.

If we have chosen our arbitrary standard for Q to have about the same magnitude as the accepted value for the coulomb in our present system, then μ_0 will be about 10^{-7} in units based on the MKS mechanical system. It is not necessary that Q be chosen to have this magnitude since any difference between the arbitrarily chosen Q and our present standard for the coulomb will be taken care of by the value we observe for μ_0.

The best value for μ_0 is obtained from the experi-

ment by equation (1). The error in μ_0 becomes e_1, as given by the equation, remembering that we have assumed I is determined from Q with negligible error. This is exactly twice as great as the error in the standard for current, obtained by assuming a value for μ_0.

Since the value of μ_0 appears in the equation for deriving the standard for the resistance, equation (2), the error in this standard becomes

$$e_R = \sqrt{e_1^2 + e_2^2},$$

assuming independence of e_1 and e_2. The error in the standard for potential given by equation (3) is now

$$e_V = \sqrt{e_1^2 + e_2^2 + e_3^2}.$$

It seems that our errors have been somewhat worsened by our decision to select an arbitrary unit of charge, but this is not really the case. The electrical standards are still as accurately known with respect to each other as before and the electrical watt is as accurately known in terms of the mechanical watt. The electrical watt defined by the relation $I^2R = W$ is still subject to the error

$$e_W = \sqrt{e_1^2 + e_2^2}$$

since I, being arbitrarily defined, is free of error. From the relation $E^2/R = W$ we get

$$e_W = \sqrt{e_1^2 + e_2^2 + 4e_3^2}.$$

(Since e_1 and e_2 appear in double weight in the numerator and in single weight in the denominator, they appear in single weight in the quotient.) Similar considerations are involved in relating forces due to electric currents to mechanical forces.

Now I do not suggest that our definition of the ampere be revised at the present time, for little would be gained thereby. It would have the effect of elevating the standard of electric charge to *absolute* status and this might make some electricians happier. It would also lead to a more simple array of our prototype standards, and would permit us to have a convenient dimensional system like the MKSQ system for electric quantities with the elegant circumstances that each dimension could be identified with an absolute unit, and each absolute unit with a prototype standard. But this is of little importance as long as we understand what we are doing.

There are many ramifications of this interesting subject which one might care to explore for himself. For example, what logical place does the calorie have in our system of units? In preparing this paper, I have tried to emphasize the highly arbitrary nature of our systems of dimensions, units, and standards. These things we often accept as dogma of our science. I know that I have not resolved by this brief paper all the confusion which exists, and that I have not converted everyone to my views. But I do hope I have sowed some seeds of doubt, for doubt leads to inquiry, and inquiry to understanding.