Interpreting Anomalously Low Voltage Noise in Two-Channel Measurement Systems

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***Abstract*—In this work we 1) analyze and give a theoretical explanation for the anomalously low cross-spectral density of voltage fluctuations that is observed when two thermal noise sources with matched intensities are coupled to the inputs of two-channel phase modulation (PM) or amplitude modulation (AM) noise measurement systems (NMS), 2) empirically evaluate spectral resolutions of different types of measurement systems, and 3) discuss noise measurement techniques involving cross-correlation signal processing. Our work shows that the statistical uncertainty, which sets the ultimate spectral resolution in the thermal noise limited regime, is approximately the same for both systems. However, in practical terms, the non-stationary nature of the noise, the temporal separation of calibration and measurement, and the difficulty of reproducing the calibrations for two measurements make it extremely difficult to resolve noise that is more than 10 dB below the noise floor in a single channel NMS. In a two-channel NMS, however, the calibrations of the two channels are carried out simultaneously, and one can take full advantage of a large number of averages and make reproducible noise measurements with resolution 10 dB below the noise floor of a single channel NMS.**

I. Introduction

Cross-correlation noise measurements date back to the mid 1970s. At that time, cross-correlation signal processing was proposed as a radical way to improve the sensitivity of the conventional phase bridge NMS by approximately 25 dB [1]. This improvement is achieved by splitting the noise in the device under test (DUT) between two nominally identical single channel phase bridges and cross-correlating the voltage fluctuations of their outputs. As a result of such signal processing, the high intensity spurious noise fluctuations that originate in non-linear mixing stages of the phase bridges and in the post amplifiers are significantly cancelled with only 3 dB loss of the useful signal. The reduction in the noise floor relative to a single channel system of the same type is approximately $\sqrt{N_{avg}}$, where N_{avg} is the total number of averages taken by the Fast Fourier Transform (FFT) spectrum analyzer. For $N_{avg} = 10^5$, this improvement can be as large as 25 dB. The drawback of using cross-correlation measurement systems is that a long measurement time is required to obtain a significant reduction in the noise floor, especially when analyzing noise spectrum close to the carrier.

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Another breakthrough in the field of precision noise measurements at microwave frequencies occurred in the mid 1990s with the development of interferometric measurement techniques [2], [3]. Modern interferometric systems can make measurements almost in real time with an uncertainty limited primarily by the thermal noise in the lossy components of the microwave interferometer. The spectral density of intrinsic phase fluctuations of a 9-GHz interferometric NMS was measured to be 10^{-19} rad^2/Hz for Fourier frequencies > 1 kHz. This corresponds to a single sideband PM noise power 193 dB below the carrier in a 1-Hz bandwidth (−193 dBc/Hz), which is only 4 dB above the limit imposed by the thermal noise at an input carrier power to the DUT of 20 dB above 1 mW (dBm). With the advent of the interferometric NMS, it became possible to study noise phenomena in the quietest components, which were previously thought to be noise free. In particular, the interferometric NMS produced the first experimental evidence of intrinsic 1/f PM noise in microwave circulators [3].

With the two types of measurement systems now coexisting, one may wonder whether it is worth complementing the interferometric measurement system with cross-correlation signal processing. In other words, is it worth sacrificing the simplicity of a single channel NMS for a two-channel NMS to achieve a possible improvement in the measurement system's sensitivity? This question was addressed in [4], [5], where a two-channel interferometric NMS with cross-correlation signal processing, labeled the "double interferometer," was assembled and tested. According to [4], [5], the white phase noise floor of the double interferometer was measured to be 10 to 12 dB lower than the thermal PM noise floor. This has led to extensive discussions regarding the definition of the phase noise floor of a two-channel NMS and the uncertainty of spectral measurements that can be potentially achieved with such systems. Responding to such discussions, we have conducted our own study aimed at understanding the properties of two-channel noise measurement systems.

In this work, we 1) specify the conditions under which the cross-spectral density of voltage fluctuations at the output of two-channel measurement systems is anomalously low, 2) analyze statistical uncertainties in spectral measurements and compare spectral resolutions of single and two-channel measurement systems, and 3) discuss noise measurement techniques that involve cross-correlation signal processing.

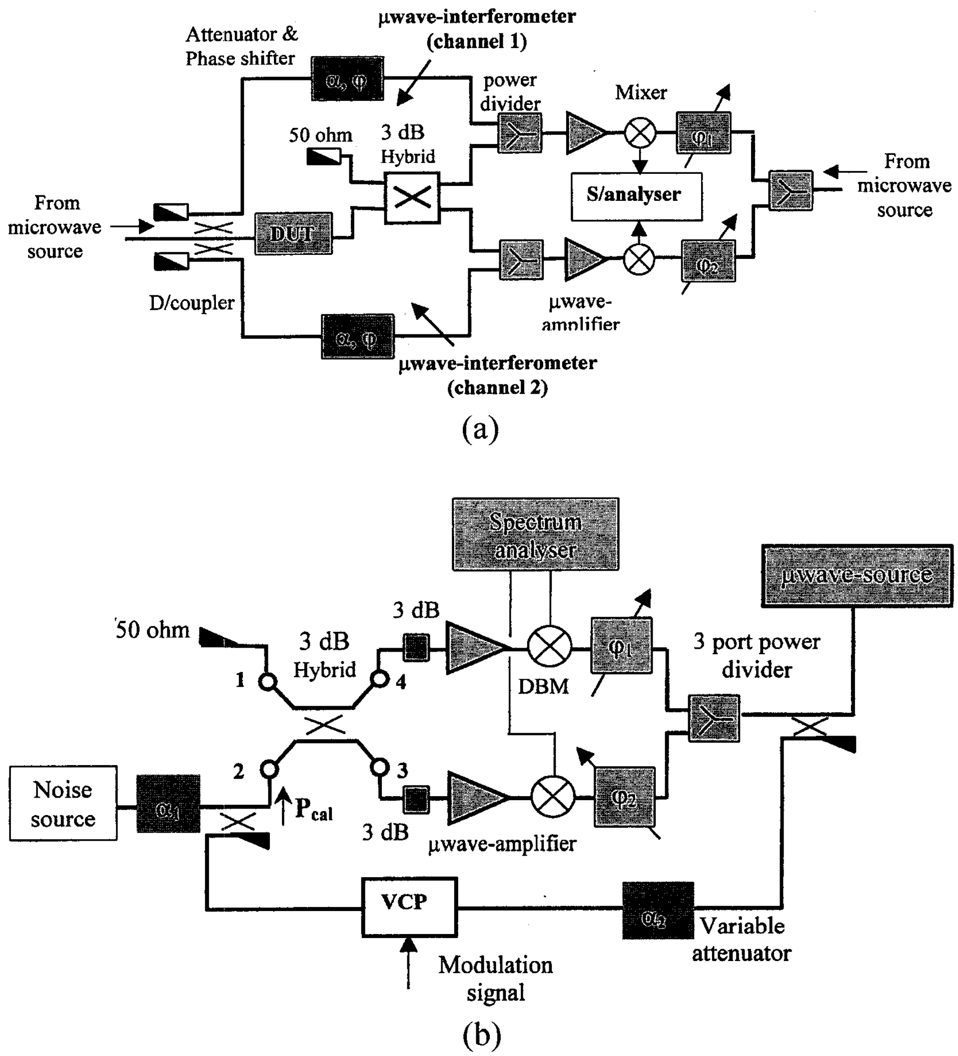

Fig. 1. a) Two-channel interferometric noise measurement system. DUT = device under test, SA = spectrum analyzer, DBM = double-balanced mixer, and $\varphi_{1,2}$ are phase shifters. b) Experimental setup for studying the basic properties of a cross-correlation noise measurement system. VCP = voltage-controlled phase shifter.

II. Experimental Study of Thermal Noise Cancellation Effect in Two-Channel Noise Measurement Systems

In this section, we report on experiments that establish the relationship between the effective temperature of the noise at the input of the cross-correlation NMS and cross-spectral density of voltage fluctuations at its output.

Fig. 1(a) shows a two-channel (double interferometric) NMS. Fig. 1(b) shows the readout portion (low noise homodyne down-converter) of this system that was used to study the noise processes in a two-channel NMS [3]. Two 3-dB pads simulate the loss in the power combiners used in the double interferometer for carrier suppression. The experimental setup also contains two microwave amplifiers and two double-balanced mixers. Phase shifters φ_1 and φ_2 are used for optimizing the sensitivities of both channels with respect to either phase or amplitude variations of the incoming signal. The latter is derived from the same source that drives the mixer's local oscillator (LO) ports and enables the calibration of the readout system. A noise source with the effective temperature T_{ns} is attached to one of the inputs of the measurement system (port 2 of the 3-dB hybrid). Variable attenuator α_1 allows the effective temperature of the input noise, T_{inp}, to be varied from the highest value of T_{ns} to the level of ambient temperature, T_o. Another variable attenuator α_2 in the path of the calibration signal sets the carrier levels at the microwave amplifiers' inputs similar to those that would exist under normal, carrier-suppressed conditions. In the course of the experiments, both channels of the cross-correlation NMS were tuned to be maximally phase sensitive. Their sensi-

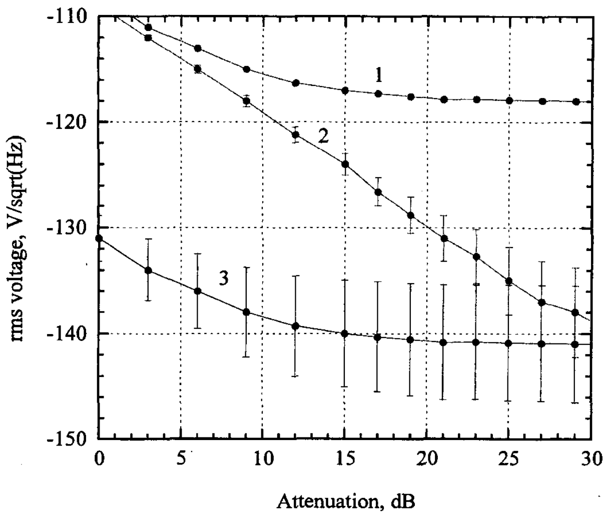

Fig. 2. Voltage fluctuations (rms) at the output of a cross-correlation measurement system as a function of intensity of the input noise at a carrier frequency of 9 GHz and a Fourier frequency of 35 kHz. Curve 1 is the single channel voltage noise spectral density, curve 2 is the cross-channel voltage noise spectral density with two identically tuned channels, and curve 3 is the cross-channel voltage noise spectral density with the two channels tuned in quadrature.

tivities were also made approximately equal by adjusting the gain of one of the channels.

The spectral densities of various voltage fluctuations measured with the above setup as a function of the intensity of the input noise are shown in Fig. 2. For example, curve 1 shows the dependence of the rms voltage noise at the output of a single channel NMS on attenuation α_1. Curves 2 and 3 show the dependence of cross-channel rms voltage on α_1. The data given by curve 2 were acquired when both channels were identically tuned (either phase or amplitude sensitive). Curve 3 corresponds to the case when the channels were in quadrature; that is, one channel was phase sensitive, while the other was amplitude sensitive. These data were collected at a Fourier frequency of 35 kHz, which was chosen to minimize the effect of the microwave amplifiers' flicker noise on the voltage noise floor of the NMS (see the noise spectra in Fig. 4). The flicker noise was caused by noise processes in the input of the microwave amplifiers in the presence of a carrier signal. The carrier level was approximately -47 dB below 1 mW (-47 dBm), which was typical for our operating conditions. Finally, low noise audio amplifiers with $G_{LNA} = 36$ dB were used before the spectrum analyzer to amplify the mixer signals.

As seen in Fig. 2, curves 1 and 3 follow each other with an almost constant offset of 23 dB. This offset is set by the noise in the uncorrelated single channel measurements reduced by $\sqrt{N_{avg}}$, where $N_{avg} = 10^5$ because of the cross-correlation signal processing. For this reason, curve 3 of Fig. 2 represents a cross-spectrum voltage noise floor for $N_{avg} = 10^5$.

The fractional one-sigma statistical uncertainty of the measurement in curve 1 of Fig. 2 is $\pm 1/\sqrt{N_{avg}}$, where N_{avg} is the number of averages. The fractional statistical uncertainty of the cross-correlated two-channel measurement in curve 3 of Fig. 2 is approximately 1 when effective temperature of the input noise, T_{inp}, is close to the ambient temperature. This is because the mean value of the voltage noise cross-spectral density in such a regime is equal to its rms deviation and reduces as $1/\sqrt{N_{avg}}$. The error bars in Fig. 2 show the uncertainty in the three measurements.

From Fig. 2, it follows that the spectral density of cross-channel voltage fluctuations, $S_{u1,2}$, depends almost linearly on the intensity of the input noise and, therefore, can be related to the spectral density of input phase noise, S_φ^{inp}, as

$$S_{u1,2} = S_{PD}^2 S_\varphi^{inp} \tag{1}$$

where S_{PD} is the efficiency of phase-to-voltage conversion. The latter is found by calibrating the measurement system. This involves applying voltage noise with a white spectrum to the voltage-controlled phase (VCP) shifter (see Fig. 1) and measuring the following parameters: 1) cross-spectral density of output voltage fluctuations, $S_{u1,2}^{cal}$; 2) power of the microwave signal at the input to the 3-dB hybrid, P_{cal}; 3) spectral density of voltage noise source, S_u^{source}; and 4) voltage to phase conversion ratio of the VCP, $d\varphi/du$, at a given bias voltage. Parameter S_{PD} is calculated from

$$S_{PD}^2 = \frac{S_{u1,2}^{cal}}{S_u^{source}} \frac{1}{(d\varphi/du)^2}. \tag{2}$$

The equivalent phase noise spectral density S_φ^{inp} in (1) is defined as

$$S_\varphi^{inp} = \frac{k_B \hat{T}_{inp}}{P_{cal}} \tag{3}$$

where k_B is Boltzmann's constant and $\hat{T}_{inp}$ is an estimate of the effective input noise temperature.

A true value of the effective input noise temperature, T_{inp}, is calculated from

$$T_{inp} = T_{ns}\alpha_1 + T_o(1 - \alpha_1) \tag{4}$$

where the effective temperature of the noise source, T_{ns}, was $\approx 12\,000$ K and $T_o \approx 300$ K.

Applying (1)–(3) to the experimental data enables the effective noise temperature, $\hat{T}_{inp}$, to be estimated. This is shown by curve 1 in Fig. 3. Curve 2 is a graphical solution of (4), which corresponds to the true value of effective noise temperature, T_{inp}. Curve 3 shows another estimate of the effective noise temperature, which is obtained from (1) by replacing $S_{u1,2}$ with the voltage noise spectral density of a single channel readout system. This results in the effective noise temperature of the microwave amplifier: $T_{amp} \approx 1300$ K.

Comparing experimental and predicted results (curves 1 and 3), the relationship between $\hat{T}_{inp}$ and T_{inp} is obtained:

$$\hat{T}_{inp} \approx T_{inp} - T_o. \tag{5}$$

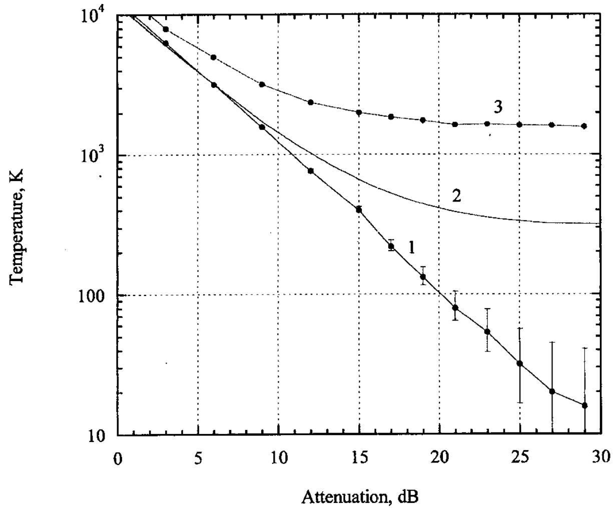

Fig. 3. Effective temperature of input noise as a function of its intensity for a carrier frequency of 9 GHz and a Fourier frequency of 35 kHz. Curve 1 is the inferred value of the T_{inp} from cross-spectral density of voltage fluctuations, curve 2 is true value of effective noise temperature, and curve 3 is the inferred value of the T_{inp} from single channel voltage noise spectral density.

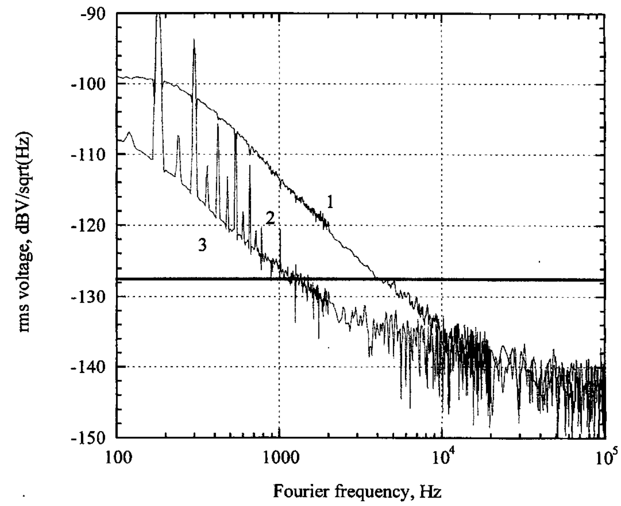

Fig. 4. Voltage noise cross-spectral densities as functions of Fourier frequency at the output of a 9-GHz, two-channel measurement system. Curve 1 is the measured rms cross-spectral voltage noise at $P_{inp} = -47$ dBm, curve 2 is the measured rms cross-spectral voltage noise at $P_{inp} = -70$ dBm, and horizontal line 3 is the calculated rms voltage noise of a single channel measurement system caused by ambient temperature fluctuations.

Remembering that the effective noise temperature at the second port of the measurement system is equal to T_o (because of 50-Ω termination), one can conclude that the two-channel measurement system is sensitive to the difference between the effective temperatures of the two input noise processes. This property is studied in detail in the experiments with two noise sources described subsequently.

Terminating both inputs of the two-channel measurement system with 50-Ω resistors allows the noise floor of the two-channel NMS to be measured as a function of Fourier frequency. The results are shown in Fig. 4 for different levels of input power, P_{cal}. The straight line (curve 3) in Fig. 4 shows the thermal noise floor of a single channel measurement system, $u_{rms}^{thermal}$, calculated from

$$u_{rms}^{thermal} = \chi\sqrt{k_B T_o K_{amp}} G_{LNA} \quad (6)$$

where χ is the mixer power to voltage conversion ratio and K_{amp} is the power gain of the microwave amplifier [3]. Substituting the experimentally measured values of χ, K_{amp}, and G_{LNA} into (6) results in $u_{rms}^{thermal} \approx 127$ dB below 1 V/Hz (−127 dBV/Hz).

The data in Fig. 4 show that at Fourier frequencies greater than a few kilohertz, the mean value of cross-spectral voltage noise is 12 to 15 dB below the thermal noise floor of a single channel system. This is a consequence of a two-channel NMS being primarily sensitive to the temperature difference of two input noise processes and the large number of averages taken to reduce the single channel noise contributions. Similar effects of thermal noise suppression were observed in earlier experiments with conventional two-channel measurement systems, but at much higher Fourier frequencies, typically closer to 10 MHz [1]. The reasons for that were a high level of flicker noise exhibited by the microwave mixers, the finite isolation between the two channels, and the finite number of averages. Introducing a high gain microwave amplifier in front of the mixer effectively overrides its high effective noise temperature and enables the thermal noise cancellation to be observed at relatively low Fourier frequencies. Lowering the power at the input of the microwave amplifier further extends the range of Fourier frequencies in which the effect of anomalously low voltage noise can be studied (compare curves 1 and 2 in Fig. 4).

The greater uncertainty in voltage noise cross-spectral density at larger Fourier frequencies (see Fig. 4) indicates that the noise originates from the single channel noise reduced by the cross-correlation signal processing. This residual noise sets an upper limit to the smallest variations in the measurement system noise floor (or DUT) that can be resolved. We compare the spectral resolutions of a single and a two-channel NMS by considering their voltage noise floors in curves 1 and 2, respectively (Fig. 5). Each noise floor looks like a 'fuzzy' trace because of the scatter of experimental data. By measuring the width of such a trace in the vicinity of a given Fourier frequency, one can empirically estimate the spectral resolution of the measurement system. The geometrical interpretation of this approach is given in Fig. 5, where intervals σ_1 and σ_{12} characterize the spectral resolutions of the single and two-channel measurement systems, respectively. At Fourier frequencies > 35 kHz, $\sigma_1 \approx 7 \times 10^{-8} V/\sqrt{Hz}$ and $\sigma_{12} \approx 1.2\times10^{-7} V/\sqrt{Hz}$ for $N_{avg} = 10^5$. These results are consistent with the fluctuations in both noise measurements as they decrease as $1/\sqrt{N_{avg}}$. For the FFT spec-

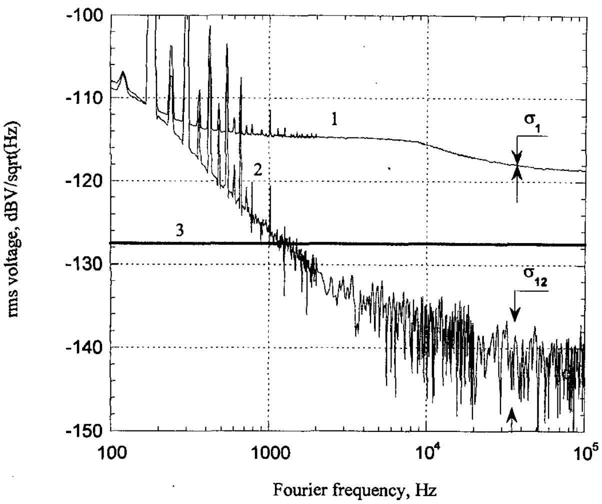

Fig. 5. Voltage noise floors of different 10-GHz noise measurement systems. Curve 1 is the measured rms voltage noise floor of single channel NMS, curve 2 is the measured rms voltage noise floor of a two-channel NMS, and horizontal line 3 is the calculated rms voltage noise of a single channel measurement system caused by ambient temperature fluctuations.

trum analyzer used here, the overall measurement time required to complete 10^5 averages for Fourier offsets from 100 Hz to 100 kHz was approximately 30 min.

Knowing the spectral resolutions σ_1 and σ_{12} enables us to evaluate the smallest changes in the effective temperature of the input noise that can be detected. We provide examples of such evaluations for both types of measurement systems in question.

First, the rms voltage noise at the output of a single channel NMS in Fig. 1 is given by

$$u_{rms} = \chi\sqrt{k_B\left(T_o + T_{amp} + T_{dut}/4\right)K_{amp}G_{LNA}} \tag{7}$$

where T_{dut} is the effective temperature of the DUT excess noise. It is assumed here that the effective noise temperature of the DUT noise is equal to $T_\Sigma = T_o + T_{dut}$. For example, if the DUT is an attenuator at ambient temperature, then $T_\Sigma = T_o$ and, therefore, $T_{dut} = 0$. In such a case, (7) gives the analytical expression for the voltage noise floor of a single channel NMS. The factor of 4 in (7) is due to the insertion loss in the path between the DUT and input of microwave amplifier [Fig. 1(a)]. Finding the variation in the level of output voltage noise caused by the DUT, δu_{rms}, and equating it to σ_1, the analytical expression for the minimum detectable value of T_{dut} is obtained:

$$\frac{T_{dut}}{T_o} \approx \frac{8\sigma_1}{u_{rms}^{thermal}}\sqrt{1 + T_{amp}/T_o}, \tag{8}$$

which results in $T_{dut}/T_o \approx 0.4$. This means that the single channel measurement system can resolve changes in the input noise effective temperature that are lower than the ambient temperature. The confidence of such measurements can be estimated by making certain assumptions regarding the nature of the voltage noise in question. For example, considering u_{rms} as a normally distributed random variable with the rms deviation $\sigma_u = \sigma_1/2$, the probability of detecting the additive noise in the DUT with effective temperature $T_{dut} \geq 0.4\,T_o$ exceeds 95% [6].

Assuming a perfect cancellation of the thermal noise and taking into account (5), the rms voltage noise at the output of the two-channel measurement system is given by

$$u_{rms} = \chi\sqrt{k_B\,\frac{T_{dut}}{4}\,K_{amp}G_{LNA}}. \tag{9}$$

Satisfying the equality $u_{rms} = \sigma_{12}$, the fractional temperature resolution of the two-channel measurement system is obtained:

$$\frac{T_{dut}}{T_o} \approx \left(\frac{2\sigma_{12}}{u_{rms}^{thermal}}\right)^2. \tag{10}$$

Applying (10) to experimental data results in $T_{dut}/T_o \approx 0.5$, which is comparable with the fractional temperature resolution of a single channel measurement system.

Several conclusions follow from this analysis. First, a two-channel measurement system does not, in principle, offer any advantage over a single channel one in its ability to resolve small changes in the effective temperature of the input noise. Second, variations in the input noise's intensity that are smaller than the thermal noise floor can be detected with both single and two-channel measurement systems. This is because the accuracy of spectral measurements improves with the observation time or the number of averages taken [6], [7].

Following [7], one can find a temperature resolution of a single channel measurement system because of statistical errors in measuring the white voltage noise spectral density with 95% confidence:

$$T_{dut} \approx 8\left(T_o + T_{amp}\right)/\sqrt{N_{avg}}. \tag{11}$$

Making use of (11), a single channel interferometric NMS with $T_{amp} \approx 60\ K$ [3] should, in principle, be capable of detecting changes in the effective temperature of the input noise, which are on the order of 10 K with $N_{avg} = 10^5$. However, the non-stationary nature of noise, the temporal separation of calibration and measurement, and the difficulty of reproducing the calibrations make it almost impossible to resolve the excess noise in the DUT, which is 10 dB below the noise floor of a single channel measurement system. By contrast, one of the main limitations imposed on the sensitivity of a two-channel NMS, as pointed out by De Marchi [8], results from the lack of isolation between the channels. It is expected that by decoupling the channels and making them almost identical, the spectral resolution of a two-channel NMS potentially can be improved beyond that of a single channel one.

III. Cross-Spectral Density of Voltage Fluctuations in Thermal Noise Suppression Regime

The complex amplitudes of the output signals of an ideal 3-dB hybrid coupler are given by

$$\begin{aligned} U_3 &= \left(U_1 e^{j\varphi H} + U_2\right)/\sqrt{2} \\ U_4 &= \left(U_1 + U_2 e^{j\varphi H}\right)/\sqrt{2} \end{aligned} \tag{12}$$

where φ_H is a hybrid differential phase shift and U_1 and U_2 are the complex amplitudes of the input signals (Fig. 1). The differential phase shift in (12) is equal to $\pi/2$ to satisfy the energy conservation conditions.

If the hybrid is lossy, (12) is no longer valid, and a complete set of S-parameters is required to describe the relationship between the complex amplitudes of input and output signals [8], [9]. Apart from that, the differential phase shift in a real hybrid coupler is frequency dependent. For example, in commonly used coplanar stripline couplers, φ_H varies by $\pm 7^o$ with respect to $\pi/2$ over the operating frequency range. In the following analysis, we assume the previously given description of an ideal 3-dB coupler with $\varphi_H = \pi/2$.

Considering a noise source with a white power spectrum at the input of the measurement system (port 1; Fig. 1), we can write analytical expressions for the cross-spectral densities of the output voltage fluctuations that are a result of both PM and AM components.

The voltage noise cross-spectral density caused by PM components of input noise is given by

$$\begin{aligned} S_{u1,2}^{(1)PM} &= \chi^2 \frac{k_B T_{inp1}}{8} K_{amp} G_{LNA} \\ &\left\{ \cos\left(\varphi_H - \Delta_{21}\right) - \cos\left(\varphi_H - \Sigma_{21}\right) \right\} \end{aligned} \tag{13a}$$

where T_{inp1} is the effective temperature of the noise entering port 1 and Δ_{21} and Σ_{21} are phase angles calculated as $\Delta_{21} = \varphi_2 - \varphi_1$ and $\Sigma_{21} = \varphi_2 + \varphi_1$, respectively.

The voltage noise cross-spectral density caused by AM components of input noise is

$$\begin{aligned} S_{u1,2}^{(1)AM} &= \chi^2 \frac{k_B T_{inp1}}{8} K_{amp} G_{LNA} \\ &\left\{ \cos\left(\varphi_H - \Delta_{21}\right) + \cos\left(\varphi_H - \Sigma_{21}\right) \right\}. \end{aligned} \tag{13b}$$

Combining (13a) and (13b), the cross-spectral density of output voltage fluctuations caused by the first noise source is obtained:

$$S_{u1,2}^{(1)} = \chi^2 \frac{k_B T_{inp1}}{4} K_{amp} G_{LNA} \cos\left(\varphi_H - \Delta_{21}\right). \tag{13c}$$

By analogy, the cross-spectral density of voltage fluctuations caused by the second noise source attached to port 2 in Fig. 1 is

$$S_{u1,2}^{(2)} = \chi^2 \frac{k_B T_{inp2}}{4} K_{amp} G_{LNA} \cos\left(\varphi_H + \Delta_{21}\right) \tag{13d}$$

where T_{inp2} is an effective temperature of the second noise source.

Combining (13c) and (13d) results in the total cross-spectral density of voltage fluctuations at the output of the NMS:

$$\begin{aligned} S_{u1,2} = \chi^2 \frac{k_B K_{amp}}{4} G_{LNA} \{ & T_{inp1} \cos\left(\varphi_H - \Delta_{21}\right) \\ & + T_{inp2} \cos\left(\varphi_H + \Delta_{21}\right) \}, \end{aligned} \tag{13e}$$

which at $\varphi_H = \pi/2$ becomes

$$S_{u1,2} = \chi^2 \frac{k_B K_{amp}}{4} G_{LNA} \{T_{inp1} - T_{inp2}\} \sin \Delta_{21}. \tag{13f}$$

The phase angle Δ_{21} in (13f) is close to $\pi/2$, provided that both channels of the measurement system are identically tuned (either phase or amplitude sensitive).

This result confirms that the two-channel NMS performs a differential temperature measurement. From previous discussions, it also follows that the effect of anomalously low voltage noise cross-spectral density must be observed with arbitrary noise sources no matter how high their intensities, provided that the noise sources are stationary, and that their intensities are matched.

If a three-port Wilkinson power divider is used to split the power of a single noise source between the channels of the NMS, the analytical expression for the voltage noise cross-spectral density is given by

$$S_{u1,2} = \chi^2 \frac{k_B K_{amp}}{4} G_{LNA} \{T_{inp} - T_o\} \sin \Delta_{21} \tag{13g}$$

where T_{inp} is the effective temperature of the input noise source. In such a case, the effect of anomalously low voltage noise would be observed only at T_{inp} approaching T_o.

Effect of thermal noise suppression in a two-channel noise measurement system is illustrated in Fig. 6. Curve 1 shows the calculated rms voltage noise at the output of a single channel measurement system as a function of input noise intensity. Curve 2 shows the cross-channel rms voltage noise calculated with only one noise source turned on [T_{inp2} is replaced with T_o in (13f)]. Curve 3 corresponds to the case of two noise sources with effective temperatures $T_{inp2} = 1.05 \times T_{inp1}$. This is a close approximation of the real situation where T_{inp1} and T_{inp2} are varied synchronously, although always remaining slightly different. Experimental data are shown in Fig. 6 as "fat" dots. In plotting these data points, only the mean values of spectral densities were used with no error bars shown. Both experimental data and calculations indicate almost 20 dB of thermal noise suppression when a second noise source with $T_{inp2} \approx T_{inp1}$ is switched on.

In the previous analysis, the noise sources were assumed to have a white power spectrum. The same approach can be extended to study the noise in an arbitrary DUT with different levels of phase and amplitude fluctuations. For example, assuming that both channels are phase sensitive,

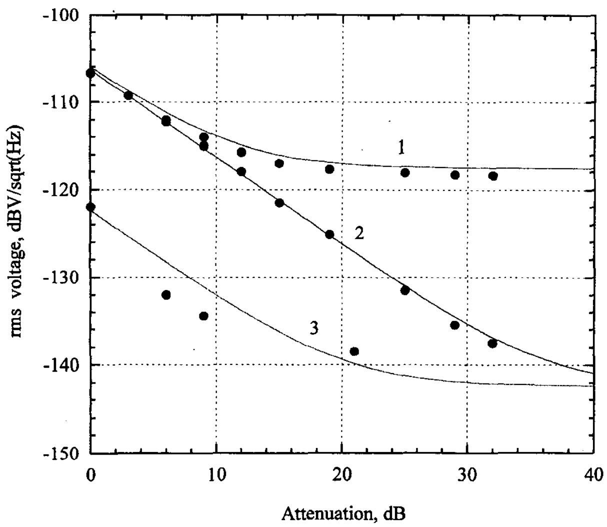

Fig. 6. Effect of thermal noise suppression in a two-channel noise measurement system. Curve 1 is the rms spectral density of voltage fluctuations at the output of a single channel NMS, curve 2 is the rms cross-spectral density of voltage fluctuations with one input noise source, and curve 3 is the rms cross-spectral density of voltage fluctuations with two input noise sources of similar intensities.

the cross-spectral density of output voltage fluctuations is given by

$$S_{u1,2} = S_{PD}^2 S_\varphi^{dut} + S_{u1,2}^{n/f} \tag{14}$$

where S_φ^{dut} is spectral density of PM fluctuations in the DUT and $S_{u1,2}^{n/f}$ is the measurement system voltage noise floor. Various mechanisms contribute to $S_{u1,2}^{n/f}$. In particular, the thermal noise component of the voltage noise floor is given by

$$S_{u1,2}^{n/f} = \frac{1}{2}\chi^2 k_B T_o K_{amp} G_{LNA} \cos^2(\varphi_H). \tag{15}$$

The latter is obtained from (13e) by replacing T_{inp1} and T_{inp2} with T_o and taking into account the identical tuning of both channels ($\Delta_{21} \approx \pi m - \varphi_H$). The differential phase shift in (15) is deliberately retained to allow for imperfections in the real 3-dB hybrid couplers. In this respect, the parameter $\delta = \cos^2(\varphi_H)$ can be introduced to characterize the degree of asymmetry between the two channels of the measurement system.

IV. Measurement of Differential Phase Delay in Hybrid Couplers

This technique for measuring the voltage noise cross-spectral densities when either one or two noise sources are acting at the input of the measurement system can be applied to the evaluation of differential phase delay in hybrid couplers. The key element of this technique is to achieve a high degree of cross-channel voltage noise cancellation

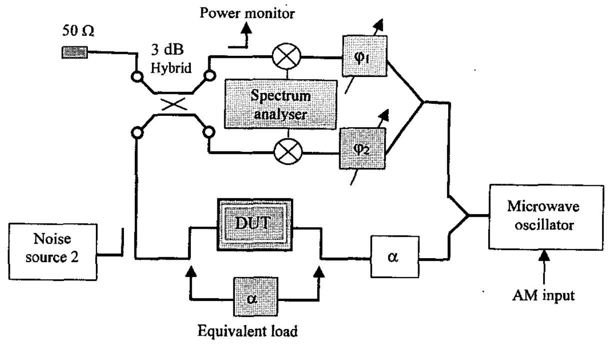

Fig. 7. Conventional cross-correlation noise measurement system. Equivalent load is an attenuator with insertion loss matched to that of the DUT.

by equalizing the effective input noise temperatures T_{inp1} and T_{inp2}. This process involves switching on one noise source at a time, measuring the cross-spectral densities of voltage fluctuations, and making them equal to within the resolution limit of the spectrum analyzer.

Assuming that two noise sources with equal effective temperatures ($T_{inp1} = T_{inp2} = T_{inp} \gg T_o$) are switched on, from (13e) it follows that

$$S_{u1,2}^{dual} = \chi^2 \frac{k_B K_{amp}}{2} G_{LNA} T_{inp} \cos^2(\varphi_H). \tag{16a}$$

If only one noise source is on, the cross-spectral density of voltage fluctuations is given by

$$S_{u1,2}^{sngl} = \chi^2 \frac{k_B K_{amp}}{4} G_{LNA} T_{inp}. \tag{16b}$$

Combining (16a) and (16b) results in

$$\varphi_H = \cos^{-1}\sqrt{0.5\, S_{u1,2}^{dual}/S_{u1,2}^{sngl}}. \tag{17}$$

Measurements of φ_H were performed in a frequency range from 7.5 to 11.5 GHz, with the mean value of φ_H being close to 85°. A few degrees of phase uncertainty in these measurements were due to the lack of noise temperature balance and limited integration time.

V. Precision Measurements with Two-Channel Measurement Systems

In this section, we discuss the calibration and measurement procedure for the conventional two-channel measurement system shown in Fig. 7.

At the first stage of the calibration procedure an equivalent load (attenuator), with an insertion loss closely matched to that of the DUT, is introduced into the measurement system. Carrier power at the input of the DUT is set to be equal to its operational value, P_{inp}, and phase sensitivities of both channels are optimized. An effective way of maximizing the phase sensitivity of the measurement system relies on the different treatment of PM and

AM signals by the phase bridge. For instance, when the phase bridge is AM sensitive, its PM sensitivity is minimal, and vice versa. For this reason, by applying an AM signal to the input of the measurement system and minimizing the responses of both channels (by adjusting phase shifts φ_1 and φ_2), the phase sensitivities of both channels are maximized. This phase tuning procedure has a clear advantage over placing a VCP shifter inside the phase bridge and generating a PM signal because the intrinsic fluctuations in the phase shifter increase the measurement system's PM noise floor.

At the next stage, the noise source with effective temperature T_{ns} is turned on at the input of the measurement system, and the cross-spectral density of output voltage fluctuations, $S_{u1,2}^{cal}$, is measured. This allows the phase-to-voltage conversion coefficient of measurement system to be determined in accordance with NIST-developed calibration procedures [10]:

$$S_{PD}^2 = S_{u1,2}^{cal} P_{inp} L_{dut} / (k_B T_{ns}) \tag{18}$$

where P_{inp} is the power at the input of equivalent load, L_{dut} is the equivalent load insertion loss, and $T_{ns} \gg T_o$.

The noise source is switched off and the voltage noise floor, $S_{u1,2}^{n/f}$, is measured. The latter contains a flicker and a white noise component as seen from Fig. 4:

$$S_{u1,2}^{n/f}(f) \cong \frac{\gamma}{f} + \varepsilon \tag{19}$$

where coefficients γ and ε characterize intensities of flicker and white noise processes, respectively. At $\varphi_H \approx \pi/2$, the white noise term in (19) coincides with the spectrum analyzer's noise floor and, as was experimentally found, is a factor of $\sqrt{N_{avg}}$ smaller than the voltage noise floor of a single channel NMS.

The DUT is inserted into the measurement system, and the phase sensitivities of both channels are maximized. The noise source is turned on again, and the cross-spectral density of output voltage fluctuations, $S_{u1,2}^{max}$, is measured. By adjusting P_{inp}, spectral densities $S_{u1,2}^{max}$ and $S_{u1,2}^{cal}$ can be made equal to each other to ensure that the calibration and measurements are performed under similar conditions.

The noise source is turned off, and the cross-spectral density of output voltage fluctuations with the DUT are installed; $S_{u1,2}^{total}$ is measured. The phase noise of the DUT is inferred from

$$S_{\varphi}^{DUT} = \frac{S_{u1,2}^{total} - S_{u1,2}^{n/f}}{S_{PD}^2} \tag{20}$$

The spectral resolution of such measurements is evaluated following the procedure described in Section II.

VI. Conclusion

The effect of anomalously low cross-spectrum voltage noise density in a two-channel noise measurement system has been studied both experimentally and theoretically. We have shown that this effect occurs when two thermal noise sources with matched intensities act at the inputs of a two-channel NMS. A simple phenomenological model of the thermal noise cancellation effect in a two-channel NMS has been developed. According to such a model, a two-channel NMS is sensitive to the difference of effective temperatures of stationary noise processes at its inputs.

We showed that the thermal phase (amplitude) noise floor of a single channel interferometric NMS for either PM or AM noise was given by $2k_B T_o/(P_{inp} L_{dut})$, where k_B is Boltzmann's constant, T_o is the ambient temperature, and P_{inp} and L_{dut} are power at the input of the device under test and its insertion loss, respectively. The thermal phase (amplitude) noise floor of two-channel NMS was found to be equal to $4k_B T_o/(P_{inp} L_{dut} \sqrt{N_{avg}}) + 2k_B T_o \delta/(P_{inp} L_{dut})$, where δ is the parameter ($|\delta| \ll 1$) characterizing the asymmetry between the two channels (or lack of isolation between them) and N_{avg} is the number of averages. The statistical uncertainty, which sets the ultimate spectral resolution in the thermal noise limited regime, is approximately $4k_B T_o/(P_{inp} L_{dut} \sqrt{N_{avg}})$ for both systems.

Our results demonstrate that both single and two-channel measurement systems are capable of measuring the DUT additive noise with an effective temperature much less than the ambient temperature. As far as the resolution of spectral measurements is concerned, the two-channel NMS was found to offer no advantage over a single channel one. Nevertheless, it is expected that the spectral resolution of a two-channel NMS could be improved by decoupling the channels and making them almost identical, provided that measurements are not significantly affected by the flicker noise. The latter was found to be weakly suppressed by the cross-correlation signal processing.

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