Laser-Induced Wakes in Ion Crystals[†]

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Abstract. Wakes in a Coulomb crystal are produced by "pushing" with radiation pressure on a rotating spheroidal cloud of laser-cooled $^9Be^+$ ions. The wakes are stationary in the lab frame and are caused by the interference of "drum-head" type oscillations. Velocity images of these wakes are obtained directly through the dependence of the ion fluorescence on Doppler shifts, and new analytical calculations accurately reproduce these experimental wake images. The technique demonstrates a way to excite and study modes, that were not accessible with previous techniques.
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INTRODUCTION

Mode studies [1–3] provide an excellent way to diagnose, control, and probe cold ion clouds. Previously at NIST-Boulder, electric fields (due to either applied voltages or trap imperfections) acting on the entire cloud were used to excite modes of relatively long wavelength on the order of the cloud's size [2,3]. In this article, we present results on a novel way to locally excite a spectrum of waves with relatively short wavelength by "pushing" with radiation pressure on a crystallized ion cloud. In the experiments shown here, the waves interfere "downstream" from the push to produce a wake pattern that is stationary in the lab frame, analogous to the wake behind a ship moving in deep water [4]. The technique offers a way to locally probe and diagnose ion clouds and demonstrates a method for studying waves that were not accessible with previous techniques. In addition, wakes in Coulomb crystals are also of current theoretical interest [5,6], primarily due to recent experiments in two-dimensional (2D) Dusty Plasma crystals [7,8].

EXPERIMENTAL SETUP

A schematic of the experimental setup [9] is shown in Fig. 1(a). We use a cylindrical Penning-Malmberg trap to confine $N_{tot} = 15\,000$ to $45\,000$ $^9Be^+$ ions
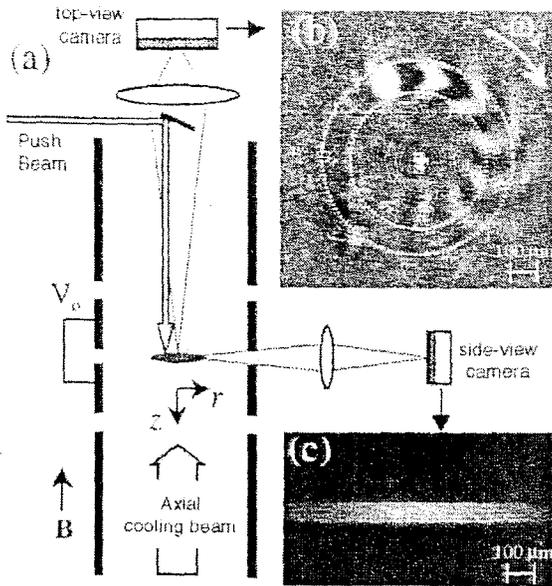
FIGURE 1. (a) Schematic of the cylindrical Penning-Malmberg trap and imaging diagnostics. (b) Top-view image of the differential fluorescence $\Delta I(r, \theta)$ for an $\alpha = 0.042$ Be^+ ion crystal. The annular region defined by the dotted circles is analyzed to obtain dispersion relationship data. (c) Side view image of an $\alpha = 0.042$ crystal.

at a density of $n \sim 2 \times 10^8 \, cm^{-3}$ with an axial magnetic field of $B = 4.465 \, T$ (giving a cyclotron frequency of $\Omega_c/2\pi = 7.6 \, MHz$) and an electrostatic potential of $V_o = -1000 \, V$ (giving an axial frequency of $\omega_z/2\pi = 800 \, kHz$). The radial extent of our clouds varies from $R_o = 0.3$ to $2 \, mm$, which is much smaller than the radius of the trap walls $R_w = 2.0 \, cm$. The ion clouds are spheroidal in shape (i.e. an ellipse of revolution) described by an aspect ratio $\alpha \equiv Z_o/R_o$, where $2Z_o$ is the axial extent of the cloud. The aspect ratio is directly related to the rotation frequency ω_r, which we control with a dipole "rotating-wall" perturbation [10].

Thus, we are able to experimentally set the cloud shape α by using the rotating wall to set ω_r. Here, $\omega_r/2\pi$ ranges from $42.5 \, kHz$ to $128 \, kHz$ with a respective range in aspect ratio from $\alpha = 0.005$ (corresponding to a 2D single plane) to $\alpha = 1.0$ (corresponding to a 3D spherical-shaped cloud). For the images in Fig. 1, $\alpha = 0.042$ with $\omega_r/2\pi = 45 \, kHz$, and in both images the cloud extended slightly beyond the aperture limit of the cameras.

The ions are laser-cooled to temperatures $T \lesssim 5 \, mK$ (giving coupling parameters of $\Gamma \gtrsim 200$) using a tunable laser set slightly to the red of a strong atomic transition in $^9Be^+$ ions [9]. The fluorescence due to the axial cooling beam (shown in Fig. 1 pointing in the $-\hat{z}$ direction) provides our primary diagnostic. The resolution of the optical systems is $\sim 4 \, \mu m$, whereas, the inter-particle spacing is $\sim 10 \, \mu m$. Unlike

previous mode studies [3], we do not strobe the camera; instead, we simply collect the fluorescence continuously for 30 s to 120 s to generate an image.

The relative intensity of the fluorescence is highly sensitive to an ion's axial velocity since the cooling laser is tuned to a relatively steep part of the transition curve. Ions that are moving towards the axial cooling beam ($v_z > 0$) are Doppler-shifted closer to the resonance peak, hence they scatter more photons and fluoresce more strongly; conversely, those moving away from the beam ($v_z < 0$) are Doppler-shifted further from resonance and fluoresce less strongly. For random thermal motion the effects average out. However, for coherent ion motion the changes in fluorescence enable the identification and measurement of axial oscillations using this so-called "Doppler velocimetry" diagnostic [3].

For the present experiments we excite waves in our ion crystals by pushing on them with laser radiation. The "push beam" is derived from the same near-resonant laser used for cooling; however, it has a relatively narrow waist ($\sim 50\,\mu m$) and is offset from the rotation axis (center of the cloud) by $R_{pb} = 155\,\mu m$ to $450\,\mu m$.

EXPERIMENTAL RESULTS

Fig. 1(b) shows an example of Doppler velocimetry for a laser-induced wake. Here we show the change in fluorescence $\Delta I(r, \theta) \equiv I(r, \theta) - I_o(r, \theta)$, where we subtract off a "background" top-view image $I_o(r, \theta)$ taken without the push beam from a top-view image $I(r, \theta)$ taken with the push beam. The large white spot is due to the push beam located at a distance $R_{pb} \approx 320\,\mu m$, but the alternating dark and light arcs are variations in fluorescence intensity due to coherent ion motion [11]. We estimate [12] that in this case the peak change in fluorescence corresponds to a change in velocity by $\Delta v_z \sim \pm 1\,m/s$. We further estimate [13] that this Δv_z corresponds to a displacement of $\Delta z \sim \pm 0.3\,\mu m$, which is much less than the inter-particle spacing. In other words, we are able to detect oscillations of very small amplitude due to the extreme sensitivity available with our Doppler velocimetry diagnostic.

These laser-induced wakes are analogous to wakes behind a ship moving in deep water [4]. Due to the radiation pressure, ions receive a downward "kick" as they rotate through the push beam, similar to the kick that water experiences as it passes under a moving ship. In both situations, the push excites a large spectrum of waves traveling in all possible directions at speeds described by a dispersion relationship. The observed wakes are stationary in the frame of the source and are due to the constructive interference of waves that satisfy the stationary phase condition [4]. Along the direction of motion, this condition is satisfied by transverse waves with a phase velocity ω/k that matches the relative velocity v of the source. For our rotating ion crystals, the motion is along a circle of radius R_{rp} with $v = \omega_r R_{rp}$.

We obtain dispersion relationship data $\omega(k)$ by assuming that the mode pattern in an annular region directly behind the push is due primarily to transverse waves; we fit a damped [14] sinusoid to the average change in intensity in this region to
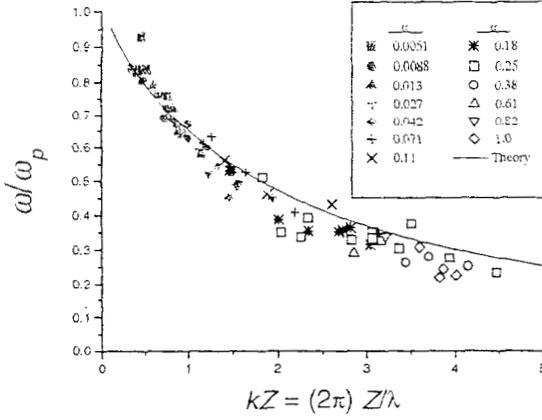
ω/ω_p

kZ = (2π) Z/λ

	κ		sc
⊠	0.0051	*	0.18
◉	0.0088	□	0.25
▲	0.013	○	0.38
▽	0.027	△	0.61
◄	0.042	▽	0.82
+	0.071	◇	1.0
X	0.11	——	Theory

FIGURE 2. Measured dispersion relationship for clouds of various aspect ratios compared to a theoretical dispersion relationship for drum-head oscillations on an infinite planar slab of thickness $2Z$.

obtain $k = 2\pi/\lambda$; and use the relationship $\omega/k = \omega_r R_{pb}$ to obtain ω. For example, for the annular region defined by the dotted circles in Fig. 1(b), the distance between peaks gives $\lambda \approx 185\,\mu m$, which in turn gives $\omega/2\pi \approx 490\,\text{kHz}$ using the calculated velocity $\omega_r R_{pb} \approx 90\,\text{m/s}$.

Dispersion relationship data obtained in this manner is shown in Fig. 2, where the different symbols correspond to different aspect ratios. Here the wave frequency is scaled by the plasma frequency $\omega_p = [2\omega_r(\Omega_c - \omega_r)]^{1/2}$; and the wavenumber is multiplied by half the cloud thickness Z at the radial position of the push beam. For a single aspect ratio, different wake patterns are generated by changing R_{pb}, which effectively changes the relative velocity of the push beam.

THEORY

To obtain a theoretical dispersion curve with which to compare to the data we consider fluid-like, drum-head oscillations in an infinite planar slab of thickness $2Z$. Using a analysis similar to that in References [2] and [5], we obtain

$$\tan\left[\frac{kZ}{\sqrt{\omega_p^2/\omega^2 - 1}}\right] = \sqrt{\omega_p^2/\omega^2 - 1}. \tag{1}$$

For a given value of kZ this equation has an infinite number of solutions; however, we consider only the lowest-order mode, for which our diagnostic is most sensitive (for higher-order modes axial variations in fluorescence will tend to average out in our top-view images). The dispersion relationship for these modes is plotted as the
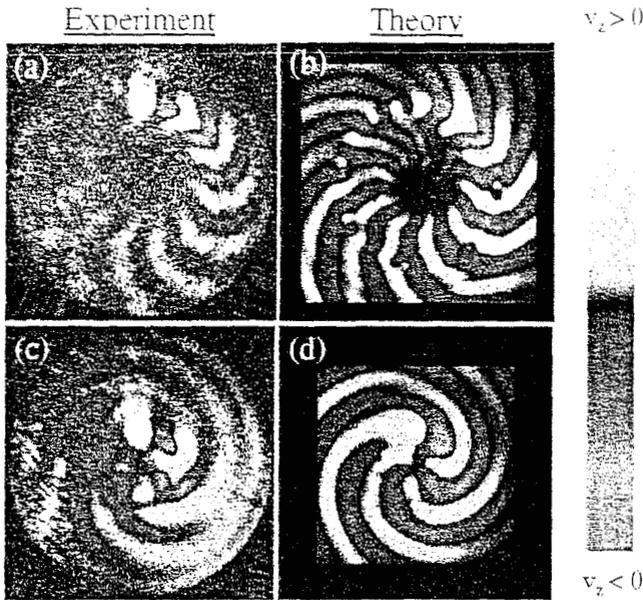
FIGURE 3. Experimental images of wakes for clouds with (a)$\alpha = 0.042$ and (c)$\alpha = 0.25$ shown alongside corresponding theoretical wakes (b) and (d) for the respective experimental parameters. The color scale corresponds to variations in velocity due to coherent motion of the ions.

solid curve in Fig. 2. This theory curve agrees very well with the data, particularly for disk-like clouds with low aspect ratio.

We are also able to theoretically predict the experimental wake patterns as shown by the comparison in Fig. 3. Here, the experimental image in Fig. 3(a) is the same as that shown in Fig. 1(b) for an $\alpha = 0.042$ cloud, and the one in Fig. 3(c) is for a somewhat thicker cloud with $\alpha = 0.25$ and $\omega_r/2\pi = 60\,\text{kHz}$. For the theoretical calculations (b) and (d), we take into account the finite spot size and sum over the excited waves in a rotating slab. These calculations are able to capture the main features of the experimental images very well. The patterns are dominated by arc-shaped transverse wakes; however, lateral wakes also add additional structure.

We do not observe Mach cones in our ion crystals, even under high velocity conditions ($v > 10^4$ m/s). This is because the Mach condition is satisfied only if the disturbance moves faster than *all* of the excited waves. According to the dispersion relationship shown in Eq. 1 and Fig. 2, there is no theoretical limit to the speed of the waves that we consider here, i.e. $\omega/k \to \infty$ as $k \to 0$. Mach cones are also not observed in the case of gravity waves in water for the same reason; however, they do occur for sound waves in air and in dusty plasma crystals [7,8] due to the finite limit on the speed of sound in these systems.

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SUMMARY AND DISCUSSION

In summary, we use radiation pressure to generate stationary wakes in a rotating Coulomb crystal of Be^+ ions. These wakes are theoretically well understood as resulting from the interference of drum-head oscillations. The experiments demonstrate a novel method of exciting waves in spheroidal ion crystals with potentially important applications. For example, by using the dispersion relationship, the method can be turned into a top-view diagnostic for the cloud parameters (i.e. ω_r, N_{tot}, α, etc.). Also, with a setup slightly different from that presented here, it may be possible to study (as yet unobserved) $\mathbf{E} \times \mathbf{B}$ shear modes by modulating the power of a perpendicular push beam. Since these modes occur only in solid-like plasmas, such experiments would provide a new tool with which to study the correlations in a strongly coupled plasma. In addition, these studies should also be useful in efforts to gain precise control over the rotational motion of the ions, which will be necessary for planned experiments on quantum information processing using ion crystals confined in a Penning trap.

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11. The thin concentric dark and light circles are an artifact of the crystal structure.
12. For this estimate we assumed that Doppler broadening was negligible and used the slope of the transition curve at a detuning of 10 MHz.
13. For this estimate we used $\Delta z = \Delta v_z / \omega$ with $\omega = (2\pi)\, 490\, \text{kHz}$.
14. In these experiments the waves are damped primarily by the axial cooling beam.