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EXPERIMENTAL STUDY OF NOISE PROPERTIES OF A TI-SAPPHIRE MODE-LOCKED LASER

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1. Introduction

A femtosecond Ti:sapphire laser is widely recognised as a major research tool in the field of precision optical frequency measurements. Such a laser is capable of producing a comb of equidistant optical frequencies with spectral width of many tens of THz [1]. By broadening the spectrum of an optical comb (in a piece of a microstructure fiber), one can bridge the gap between the harmonics of an optical frequency standard and stabilise absolute frequencies of all spectral components of the comb. Frequency stabilised femtosecond combs can link the optical and microwave domains and permit a coherent frequency transfer from optical to microwave clocks and vise versa [2]. The goal of this work was to study the limitations imposed on frequency stability of synthesised optical signal by intrinsic fluctuations of the femtosecond laser as well as fluctuations in the optical readout system.

2. Technical Fluctuations of Mode-Locked Laser

Parameters of optical pulses (repetition rate, amplitude and carrier frequency) produced by a free-running femtosecond laser fluctuate due to various noise sources, both technical and fundamental. The former include fluctuations of ambient temperature, vibration and acoustic noise. The latter are related to the graininess of light (shot noise) and spontaneous emission fluctuations.

First, we studied the effect of technical noise sources on pulse repetition rate of a free-running femtosecond laser. Noise measurements were accomplished by making use of a conventional two-oscillator measurement technique. Following this technique pulsed laser light was converted into a sequence of ultra-short video pulses with a high-speed photodetector. One of the harmonics of pulse repetition rate was selected by band-pass filtering the photodetector output. The filtered signal was frequency downconverted by mixing it with a signal from a low-noise RF frequency synthesiser. The beat note at the output of the mixer was examined with a FFT spectrum analyser. Knowing the phase to voltage conversion efficiency of the mixing process, the spectral density of pulse repetition rate fluctuations was inferred from the spectrum of the voltage noise.

The schematic diagram of two-oscillator noise measurement system is shown in Fig. 1. Femtosecond light pulses were generated by a Ti:sapphire laser pumped by a frequency doubled $5\,W$ Nd^{+3}:YAG laser. Duration of the light pulses was close to $10\,fs$ and pulse repetition rate, f_R, was of the order of $100\,MHz$. The 9th harmonic of pulse repetition rate was phase locked to the RF synthesiser. Another identical RF synthesiser was used as a frequency reference against which both phase and amplitude fluctuations of a harmonic signal were measured.

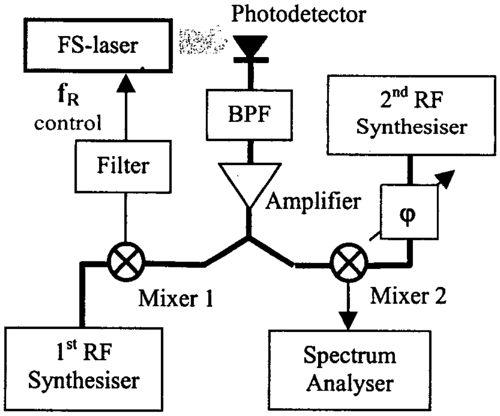

Fig. 1. Experimental setup for measurement of pulse repetition rate fluctuations

2.1. Pulse Repetition Rate Stabilisation.

Stabilisation of pulse repetition rate was achieved by controlling the length of the laser cavity with two end mirrors attached to piezo electric transducers (PZT). The dependence of pulse repetition rate on the PZT bias voltage, U_{pzt}, was almost linear with a gradient $df_R/dU_{pzt} \approx 0.63\ Hz/V$. The tuning range, Δf_R^{max}, was close to $250\,Hz$. By superimposing a broadband voltage noise on the PZT bias, we measured the voltage-to-frequency conversion df_R/dU_{pzt} as a function of the Fourier frequency. This enabled us to find spurious piezo resonances in the mirror mounts.

For example, the lowest piezo resonance was at the frequency $2.3\,kHz$. To ensure a stable operation of the phase lock loop (PLL) its bandwidth was chosen to be a few times narrower than the frequency of the lowest mechanical resonance.

Another criteria, which was taken into account in designing the loop filter, was related to PLL's ability to cope with a large scale time drift of pulse repetition rate. Such a drift was caused by variations of ambient temperature and was of the order $120\ Hz/hour$. Introducing an integrator into the feedback loop enabled us to keep the pulse repetition rate locked for almost an hour under usual operating conditions in the laboratory. This was sufficiently long to carry out noise measurements at low Fourier frequencies close to a fraction of a Hz.

2.2. Fluctuations of Pulse Repetition Rate

The spectral density of pulse repetition rate fluctuations, S_φ^{rep}, was inferred from the results of voltage noise measurements in accordance with

$$S_\varphi^{rep}(f) \approx \frac{|1+\gamma|^2}{n^2\, S_{PD}^2}\, S_u(f), \qquad (1)$$

where $S_u(f)$ is a spectral density of the voltage noise at the output of the noise measurement system (output of the 2nd mixer), n is a number of the selected harmonic of pulse repetition rate, S_{PD} is a phase-to-voltage conversion efficiency, γ is an open loop gain of the control system and f is a Fourier frequency.

The range of Fourier frequencies, within which (1) is applicable, is limited by the measurement system voltage noise floor. The latter represents a joint contribution of two identical RF synthesisers and is given by

$$S_u^{n/f} \approx 2\, S_{PD}^2\, S_\varphi^{synth}, \qquad (2)$$

where S_φ^{synth} is a phase noise spectral density of RF synthesiser.

Requiring that $S_u(f) \geq (3...5) S_u^{n/f}(f)$ gives the frequency range, within which the phase noise calculations can be performed in accordance with (1). Attempts to estimate S_φ^{rep} outside the allowed frequency range (when measured noise is close to the measurement system noise floor) may result in significant errors because of the limited measurement time and non-stationary nature of noise.

Results of noise measurements are presented in Fig. 2. Here, spectrum 1 shows the phase noise floor of the two-oscillator measurement system. Curve 2 is a spectrum of phase fluctuations of 9th harmonic of pulse repetition rate of a phase locked femtosecond laser including the noise contribution of the 2nd RF synthesiser. Curve 3 is a noise suppression factor $|1+\gamma|^{-2}$ of the PLL. Curve 4 is the reconstructed phase noise spectrum of a 9th harmonics of pulse repetition rate, S_φ^n, of a free-running femtosecond laser.

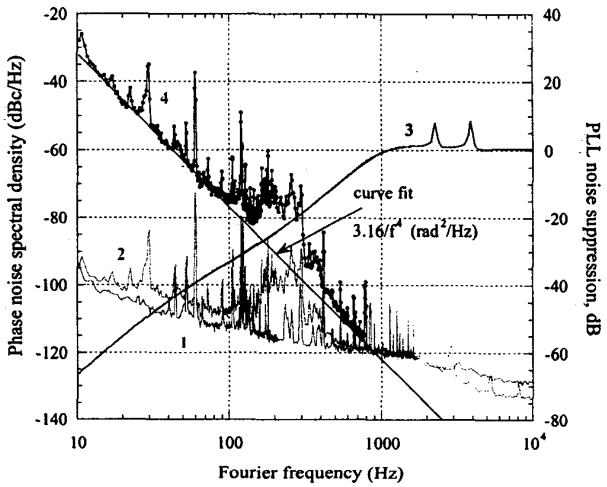

Fig. 2. Noise floor of the measurement system (curve 1) and phase noise of a free-running femtosecond laser (curve 4).

Ignoring the excess noise at frequencies $(150...300)\,Hz$ caused by the vibration sensitivity of the laser resonator, the spectrum S_φ^n can be approximated as $3.16/f^4\ (rad^2/Hz)$.

Taking into account that spectra S_φ^n and S_φ^{rep} are related via the law of ideal frequency multiplication : $S_\varphi^n = n^2 S_\varphi^{rep}$, results in

$$S_\varphi^{rep}(f) \approx 3.9 \cdot 10^{-2}/f^4\ (rad^2/Hz) \qquad (3)$$

This spectrum corresponds to a 'random walk' noise, which is typically associated with temperature induced fluctuations of the laser resonator.

Making the measurement system in Fig. 1 sensitive to power fluctuations of the input signal (by adjusting the reference phase shift, φ) amplitude noise of 9th harmonic of pulse repetition rate was measured. The spectral density of amplitude noise was found to be much less than that of a phase noise. For example, at Fourier frequencies below $100\,Hz$ the difference between two noise spectra was more than $40\,dB$.

We've also found a strong correlation between power fluctuations of the femtosecond laser and its pump laser. In those experiments one photodetector was demodulating power fluctuations of the pump laser, while another photodetector was subjected to the light from the femtosecond laser. Output of each photodetector was low-pass filtered and cross-spectral density of voltage fluctuations was measured. Results of these measurements indicate that power fluctuations of the femtosecond laser are caused mainly by power fluctuations of the pump laser at least within the interval of Fourier frequencies from $400\,Hz$ to $20\,kHz$

3. Shot Noise Measurements and Discussion

Intensity of the shot noise at the output of a photodetector defines the ultimate accuracy with which the frequency stability of an optical 'clock' can be transferred to the microwave frequency range. This can be understood by considering the spectrum of a microwave signal at the output of a photodetector subjected to ultra-short light pulses. Such a spectrum consists of discreet spectral lines at harmonics of pulse repetition rate and a broadband pedestal due to the random arrival of photons. Suppressing technical fluctuations of the laser (by stabilising pulse repetition rate, offset frequency of the comb and power of the pump laser) reduces the broadening of the spectral lines, but does not affect the intensity of the background noise (height of the broadband pedestal).

A schematic diagram of a shot noise measurement system is shown in Fig. 3. It represents a two-channel frequency down-converter, where a signal from the photodetector is mixed with that from the RF synthesiser and the Fourier spectra of two beat notes are examined. Frequency of the RF synthsesiser, f_{synth}, is chosen to be in the middle between two adjacent harmonics of pulse repetition rate to ensure that no technical noise sources contribute to voltage fluctuations at the mixer output.

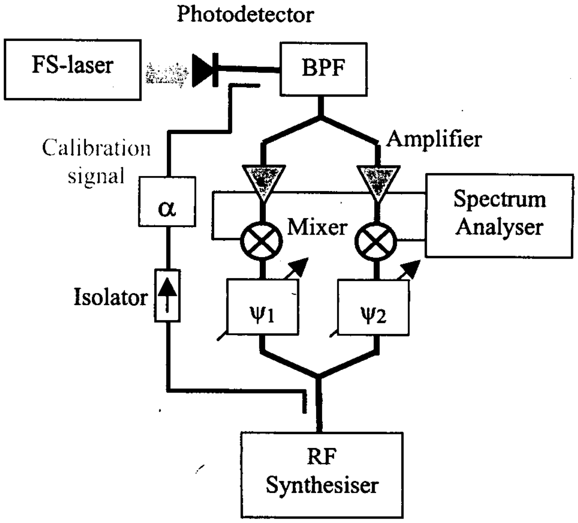

Fig. 3. Experimental setup for shot noise measurement

To improve the sensitivity of noise measurements, the signal from the photodetector is filtered with a band-pass filter tuned at RF synthesiser's frequency, f_{synth}. This eliminates carriers from the spectrum of the input signal and allows a low-noise amplifier to be introduced in front of the mixer. High gain amplification of the residual noise before its demodulation overrides the high effective noise temperature of the non-linear demodulator so that a thermal noise limited sensitivity can be achieved [3].

Having two-channel measurement system allows a cross-spectral density of voltage fluctuations to be calculated. This minimises the effect of uncorrelated noise sources in each channel on resolution of spectral measurements. Two-channel measurement system is also immune to the thermal noise and its noise floor can be $10...15\,dB$ below than the standard thermal noise limit, provided that fluctuations are stationary, and the measurement time is long enough [4].

Making noise measurements with two-channel readout system, one has to ensure that both channels are correctly tuned: they must be either phase or amplitude sensitive depending on the type of noise measurements. The tuning was achieved by applying a calibrated AM-modulated signal at frequency $f_{synth} \approx 950\,MHz$ to the input of the band-pass filter via the calibration path and maximising the responses of both channels with phase shifters ψ_1 and ψ_2 (see Fig.3).

By making the power of calibration signal equal to that of 10th harmonic of pulse repetition rate , the amplitude-to-voltage conversion efficiency S_{AD} was measured to be $126\, mV$.

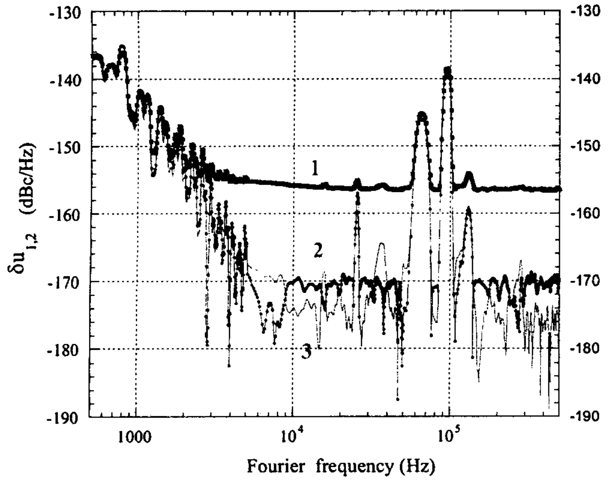

Fig. 4. Voltage noise spectra at the output of a single (curve 1) and two-channel measurement systems (curves 2, 3).

Results of noise measurements are shown in Fig. 4. Curve 1 corresponds to rms voltage fluctuations at the output of a single-channel measurement system, δu_{rms}. Curve 2 corresponds to rms cross-voltage noise, δu_{rms}^{cross}, when the light from a femtosecond laser was incident on the photodetector. Curve 3 shows the noise floor of two-channel measurement system, obtained with a photodetector replaced with a $50\, ohm$ termination. From the comparison of spectra 1 and 2 it is clear, that sensitivity of a single channel readout system is not sufficient to allow accurate measurements of the shot noise.

The noise spectra in Fig. 4 have been measured in the presence of a $950\, MHz$ calibration signal. Such a signal causes flicker noise in front-end amplifiers and degrades the sensitivities of both single and two-channel measurement systems at Fourier frequencies below $10\, kHz$. At Fourier frequencies above $10\, kHz$ there is no excess noise of a technical origin and rms cross-voltage fluctuations were measured: $\delta u_{rms}^{cross} \approx -170\, dBV/\sqrt{Hz}$.

Remembering that at low power levels the phase and amplitude conversion ratios coincide: $S_{AD} = S_{PD}$, the spectral density of an equivalent phase noise was evaluated from

$$S_\varphi^{shot} = \left(\delta u_{rms}^{cross} / S_{AD}\right)^2 \qquad (4)$$

resulting in $S_\varphi^{shot} \approx -152\, dBc/Hz$.

Assuming that 10th harmonic of pulse repetition rate is selected from the microwave frequency comb with a high-Q resonator, its fractional frequency stability can be found from [5]

$$\sigma_y^{shot}(\tau) \approx \frac{\sqrt{3 S_\varphi^{shot} \Delta f}}{2\pi \tau f_o}, \qquad (5)$$

where Δf is a bandwidth of the resonator, τ is an integration time and f_o is a signal frequency ($f_o \approx 1\, GHz$). Substituting the above value of S_φ^{shot} and $\Delta f = 100\, kHz$ (typical bandwidth of a $1\, GHz$ dielectric resonator) into (5) yields $\sigma_y^{shot}(\tau) \approx 5 \cdot 10^{-15}/\tau$.

Such a performance is comparable to that which has already been achieved with an optical Ca frequency standard [6]. With the expected improvements in the performance of optical clocks, the shot noise limit may become a serious problem when transferring frequency stability from optical to microwave domain. A possible solution of the 'shot noise problem' could be in increasing the frequency of the output signal, provided that broadband photodetectors are available. For example, by filtering out a 100th harmonic of pulse repetition rate at $f_o \approx 10\, GHz$ and assuming, that $\Delta f = 100\, kHz$ (typical bandwidth of a room temperature sapphire loaded cavity resonator), the shot noise limit can be reduced to $\sigma_y^{shot}(\tau) \approx 5 \cdot 10^{-16}/\tau$.

Having measured the spectral density of a shot noise produced by a femtosecond laser we've verified that its analytical description is very similar to that of a continuous wave laser. In other words, we've found that expressing the rms current fluctuations of the photodetector subjected to ultra-short light pulses in the form:

$$\delta i_{rms} = \sqrt{2 q \eta \overline{P}}, \qquad (6)$$

where q is an elementary electrical charge, η is a responsivity of the photodetector and $\overline{P}$ is an average power of the optical comb, results in a good agreement between the theory and experiment. Such a result was obtained by examining conversion of different types of fluctuations in the two-channel shot noise measurement system in Fig. 3. For such a system, the spectral densities of output voltage fluctuations are given by

$$S_{u1} = \chi^2 \tilde{K}_{amp} \left(k_B T_{RS} + \delta P_{shot} / 2 \right) \qquad (7.1)$$

$$S_{u1,2} = \chi^2 \tilde{K}_{amp} \, \delta P_{shot} / 2 \qquad (7.2)$$

where S_{u1} is a spectral density of voltage fluctuations at the output of a single-channel measurement system, $S_{u1,2}$ is a cross-spectral density of voltage fluctuations, χ is a mixer power-to-voltage conversion ratio, $\tilde{K}_{amp}$ is a total gain of the measurement system (including loss in the band-pass filter), k_B is a Boltzman constant, T_{RS} is an effective noise temperature of measurement system and δP_{shot} is a power spectral density of shot noise at the output of the photodetector. The latter is given by $\delta P_{shot} = \delta i_{rms}^2 R$, where R is a load resistance of the photodetector.

The first term in (7.1) is due to the combined effect of thermal and technical noise sources. It dominates single-channel measurements (spectrum 1in Fig. 4).

By cross-correlating the outputs of two channels the effect of technical and thermal noises sources on the accuracy of shot noise measurements averages out with integration time and can be ignored (spectrum 2 in Fig. 4).

Substituting the experimental data and physical constants in (7.2) results in $\sqrt{S_{u1,2}} = \delta u_{rms}^{cross} \approx -157\,dBc/Hz$, which is $13\,dB$ higher than experimentally measured (see Fig. 4).

The discrepancy between the experimental and predicted results can be explained by the low-pass filtering of the shot noise associated with the stray capacitance of the photodetector. This effect is clearly seen from the spectrum of video pulses at the output of a photodetector (Fig. 5). It is responsible for an additional $10.5\,dB$ attenuation of the shot noise at frequency $f_{synth} = 950\,MHz$, which reduces the above discrepancy to the 'respectable' level of $2.5\,dB$.

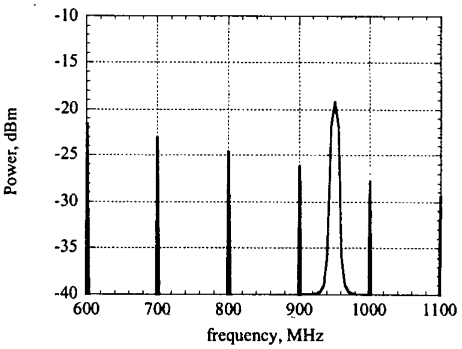

Fig. 5. Spectrum of video pulses at the output of photodetector and transfer function of the band-pass filter

4. Conclusion

Summarising the results of this work:

- the spectral density of pulse repetition rate fluctuations of a free-running femtosecond laser was measured;
- an analytical description for the shot noise produced by a femtosecond laser was obtained;
- the effect of the shot noise on the accuracy of time transfer between optical and microwave frequencies was evaluated.

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