Systematic Errors in Cesium Beam Frequency Standards Introduced by Digital Control of the Microwave Excitation

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Abstract— **As part of the accuracy evaluation of the NIST-7 primary frequency standard we have investigated potential sources of frequency offsets due to the electronic servo system. We will present results from investigations of three major sources of frequency bias in slow square wave FM servo systems: RF spectral purity, synthesizer switching transients, and phase-locked-loop settling time.**

INTRODUCTION

The Ramsey technique of separated oscillating fields is widely used in the field of atomic frequency standards. We have investigated potential sources of frequency shifts associated with the microwave radiation interrogating the atoms when using the Ramsey technique. We first measured the frequency bias that results from an interrogation signal whose spectrum contains imbalanced sidebands. We also investigated the frequency offsets that arise in determining the center of the Ramsey fringe using slow square wave digital servos. These servos include a blanking interval to allow the atomic beam tube to reach steady state during the frequency modulation cycle. In addition to atomic beam transit time, there are additional time delays that should be included in the servo's blanking interval. These delays are caused by anomolous switching transients in direct digital frequency synthesizers, and the settling time of phase-locked loops in the microwave synthesis chain.

RF SIDEBAND PULLING

The presence of spurious, discrete sidebands in the spectrum of the microwave radiation can lead to errors in the determination of the center of the Ramsey peak. While these effects are not unique to slow square-wave digital frequency servos, they may be more pronounced in systems employing direct digital synthesis (DDS) due to phase accumulator truncation [1], and electromagnetic interference from high speed digital electronics.

The frequency shift due to a discrete sideband has been calculated theoretically [2]. An independent calculation by one of us (J.S.) has confirmed their result, except that in their equation (9) the drift time T should be replaced by $T+\tau$, where τ is the transit time through one interaction region. For sideband detunings of the order of the Ramsey linewidth, we obtain shifts like those shown by the solid line in figure 2 when we average over the velocity distribution. The curve is antisymmetric about zero detuning; hence symmetric sidebands produce shifts that cancel each other.

The calculations were verified experimentally using the United States primary frequency standard NIST-7 by intentionally modulating the microwave signal in both amplitude and phase to produce a well defined spectrum. Pure amplitude modulation (AM) generates symmetric sidebands about the carrier and does not lead to a frequency shift. Similarly, neither does pure phase modulation (PM). The amplitude and phase of the carrier must be modulated coherently to yield imbalanced sidebands. A carrier wave that is modulated in both amplitude and phase can be represented by

$$E(t) = \mathrm{Re}\left\{Ae^{i\theta}[1+\epsilon\cos(\Omega t+\phi)]e^{i\eta\sin(\Omega t+\psi)}e^{i\omega t}\right\}, \tag{1}$$

where A is the carrier wave amplitude, ω is the carrier frequency, θ is the carrier phase, ϵ is the amplitude modulation index ($\epsilon \leq 1$), Ω is both the amplitude and phase modulation frequency, ϕ is the phase of the amplitude modulation, η is the phase modulation index, and ψ is the phase of the phase modulation. If $\epsilon \ll 1$ and $\eta \ll 1$ this can be rewritten in three terms representing the carrier and the upper sideband and lower sideband, respectively:

$$\begin{aligned} E(t) &= \mathrm{Re}\{Ae^{i\theta}e^{i\omega t} \qquad (2)\\ &+ \frac{1}{2}Ae^{i\theta}(\epsilon e^{i\phi}+\eta e^{i\psi})e^{i(\omega+\Omega)t}\\ &+ \frac{1}{2}Ae^{i\theta}(\epsilon e^{-i\phi}-\eta e^{-i\psi})e^{i(\omega-\Omega)t}\}. \end{aligned}$$

The upper and lower sideband relative amplitudes can then be written

$$S_+ = \frac{1}{2}(\epsilon e^{i\phi}+\eta e^{i\psi}), \tag{3}$$

$$S_- = \frac{1}{2}(\epsilon e^{-i\phi}-\eta e^{-i\psi}). \tag{4}$$

If we choose $\epsilon = \eta$, then the relative sideband intensities are

$$|S_+|^2 = \frac{1}{2}\epsilon^2[1+\cos(\phi-\psi)], \tag{5}$$

$$|S_-|^2 = \frac{1}{2}\epsilon^2[1-\cos(\phi-\psi)]. \tag{6}$$

The upper and lower sidebands have equal intensity for

$$(\phi-\psi) = \frac{\pi}{2} \pm n\pi, \text{ for } n = 0, \pm1, \pm2, \ldots \tag{7}$$

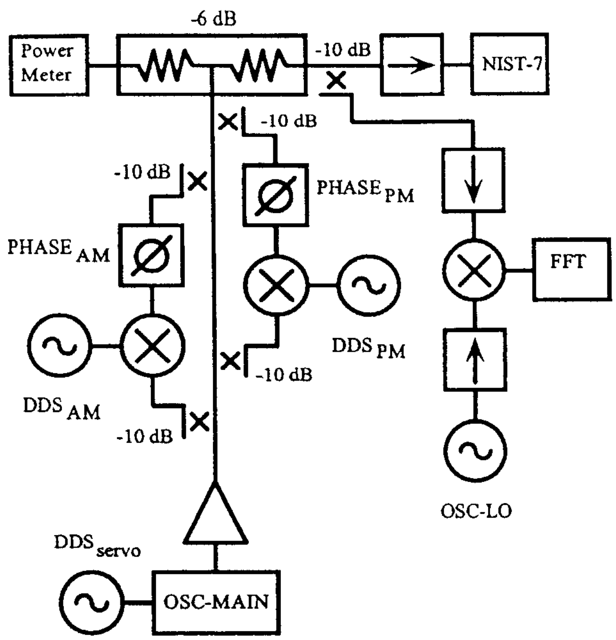

Figure 1: Single Sideband Modulator.

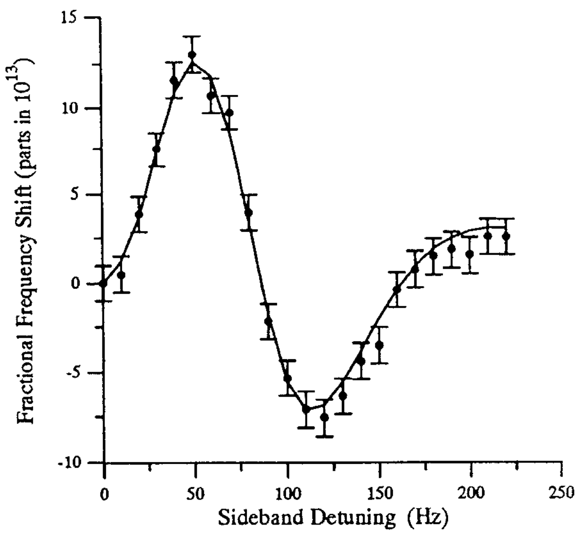

Figure 2: Frequency Shift Due to Sideband Pulling. Solid line is the theoretical calculation. Dots are the experimental measurements. Carrier power: -2.5 dB relative to optimum power; lower sideband: -29 dBc; upper sideband: -66 dBc. Experiment duration: 300 seconds per point.

To measure the frequency pulling with high accuracy and within a short time, we chose to introduce the maximum sideband asymmetry (a single sideband). This is achieved by setting

$$(\phi - \psi) = 0 \pm n\pi, \text{ for } n = 0, \pm 1, \pm 2, \ldots \quad (8)$$

Such a single sideband modulator is realized with the circuit of figure 1. A low phase noise, 9 GHz microwave source (OSC-MAIN) is modulated in both amplitude and phase by two independent modulators of the type described in [4]. One modulator is optimized to produce high purity AM (by tuning $\mathrm{PHASE_{AM}}$), while the other is optimized to generate only PM (by tuning $\mathrm{PHASE_{PM}}$). Two phase-locked direct digital synthesizers ($\mathrm{DDS_{AM}}$ and $\mathrm{DDS_{PM}}$) provide the audio frequency modulation signals ($\cos(\Omega t+\phi)$ and $\sin(\Omega t+\psi)$) to two mixers. The synthesizers are adjusted (under computer control) so that $\epsilon = \eta$ and $\phi = \psi + \pi$. This is verified by monitoring the microwave spectrum with a spectrum analyzer (FFT) at an intermediate frequency of 50 kHz, using a local oscillator (OSC-LO). The amplitudes of the carrier, lower, and upper sidebands are then recorded by the computer.

The frequency bias introduced by this single sideband is determined by measuring the center of the Ramsey fringe using a slow square-wave frequency servo (computer controlled by synthesizer $\mathrm{DDS_{SERVO}}$). Figure 2 shows the measured frequency pulling (dots) as a function of the sideband detuning Ω. The error bars represent $\pm 1\sigma$ statistical uncertainty. Since the experimental velocity distribution [3], sideband intensities, and detunings were used in calculating the theoretical curve, no free parameters were adjusted to obtain the agreement shown in figure 2. We have obtained similar results for detunings comparable to the width of the Rabi pedestal.

Switching Transients in Slow Square Wave FM Servos

The use of direct digital synthesis in microwave synthesis chains has grown in recent years. This is due to improvements in the frequency resolution and agility of such synthesizers. Slow square-wave frequency servos are readily implemented when DDS instruments are computer controlled. The task of evaluating the frequency biases in such a servo is greatly simplified if the beam tube and electronics are allowed to reach steady state before the clock signal is measured. Thus, after a frequency step command is sent to the DDS, a blanking interval τ_b is established, during which the clock signal is discarded. After the blanking interval, the clock signal is recorded during the acquisition interval τ_a. It is desireable to have τ_a occupy a large fraction of the modulation period in order to enhance frequency stability. Simply reducing the modulation frequency is an unacceptable approach to increasing the ratio τ_a/τ_b, because it increases the system's sensitivity to noise that exhibits $1/f$ power spectra. The blanking interval can be written as the sum

$$\tau_b = \tau_r + \tau_\phi + \tau_{pll} + \tau_\rho, \quad (9)$$

where τ_r (10 ms) is the response time of the DDS, that is, the time from the transmission of a frequency step command to the change in frequency at the DDS output. τ_ϕ is phase-settling time of the DDS, τ_{pll} is the settling time of the phase-locked loop (PLL), and τ_ρ (25 ms) is the transit time of the slowest atoms of interest across the Ramsey cavity and to the detection region. The PLL is shown in detail in figure 3. There is

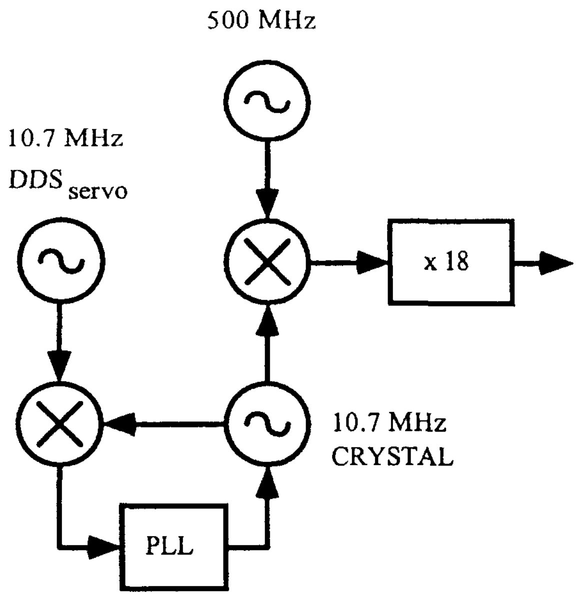

Figure 3: Details of Microwave Synthesis Chain. A 10.7 MHz "clean-up" oscillator is phase locked to the DDS unit in order to reduce the levels of spurious sidebands.

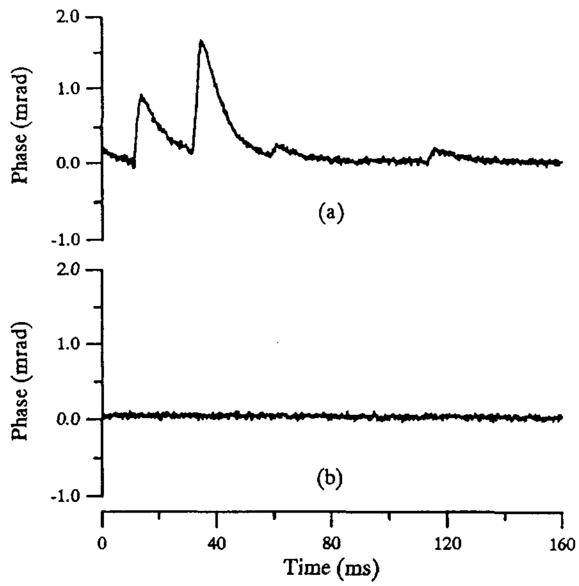

Figure 4: Phase Transients Due to Computer Control. Trace (a): phase transients occuring after frequency control command is sent to synthesizer at $t = 0$ ms. Trace (b): supression of phase transients when phase locked loop is disabled.

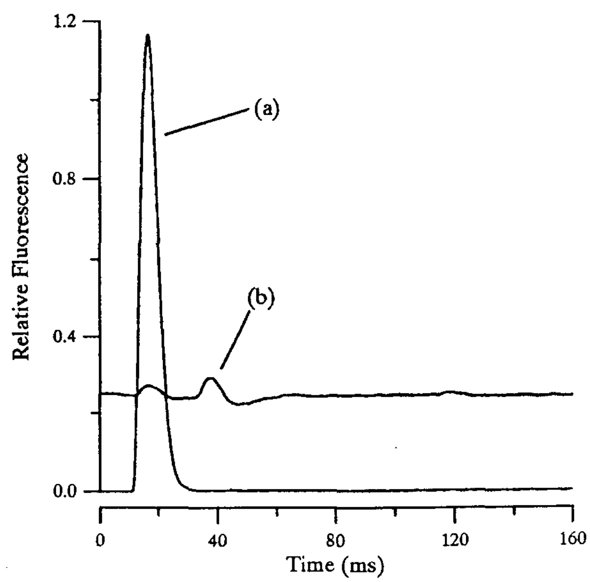

Figure 5: Effect of Synthesizer Phase Transients. Trace (a): Normal slow square wave FM transient due to the atomic transit time distribution. This trace was taken with the frequency servo enabled, thus the steady state fluorescence levels before and after the transient are equal. Trace (b): Effect of DDS phase transients (figure 4(a)) on atomic beam fluorescence. Trace (b) has been offset for clarity.

a lower limit placed on τ_b by τ_r and τ_ρ. We have, however, discovered techniques whereby τ_ϕ may be eliminated, improving frequency stability and eliminating a potential frequency bias.

Synthesizer Switching Transients

We have discovered a potential frequency bias due to phase transients in a commercial direct digital synthesizer. The transients occur whenever the synthesizer is sent frequency control instructions by the computer. These transients are therefore synchronous with the frequency modulation, some occurring tens, and even hundreds of milliseconds after the frequency step is performed. Since the position of the Ramsey fringe is sensitive to the phase difference between the two ends of the cavity as seen by atoms taking several milliseconds to traverse the cavity, such phase transients can give rise to significant frequency offsets. The transients were measured by synchronously recording the relative phase of two synthesizers while one was sent frequency control commands by a computer. To simplify the measurement, the same frequency control command was sent to the synthesizer repeatedly. Figure 4(a) shows the resulting phase transients. The peak phase deviation of 1.7 mrad occurs 35 ms after the instruction is sent to the synthesizer.

When a DDS unit is used in a microwave synthesis chain, it is common to provide the device with an external frequency reference. The DDS then phase-locks its own internal crystal oscillator to the external signal. We think that the "housekeeping" operations of the microprocessor within the DDS are inadvertently coupled to this phase-locked loop, generating phase transients that are synchronous with its operation. We were able to eliminate the phase transients by removing the instrument's internal crystal oscillator and instead injecting a suitable reference frequency derived from an external source. Figure 4(b) shows the dramatic improvement in phase stability.

Figure 5 shows the effect of the phase transient upon the atomic beam fluorescence as well as the normal modulation transient. If we were to determine a blanking interval based solely upon the response time of the synthesizer and the atomic beam transit time distribution, the effect of the synthesizer switching transient would still introduce a significant contribution to the measured fluorescence. The size of the frequency bias due to the switching transient was determined by measuring the center of the Ramsey fringe using two different direct digital synthesizers. Both units were identical except that one unit was modified to eliminate the phase transient. With a blanking interval of 40 ms, the fractional frequency bias produced by the transient was $(50 \pm 1) \cdot 10^{-13}$. Although the blanking interval might be extended to avoid the initial switching transients, periodic transients (visible in figure 4(a) at 120 ms) are virtually unavoidable. We emphasize that these phase transients preserve their sign regardless of the sign of a frequency step performed by the DDS. Thus, the resulting bias cannot be eliminated through complex or pseudorandom modulation sequences. By removing these transients, we were able to transfer at least 20 ms from the blanking interval to the acquisition interval in each half of the modulation cycle.

Phase Locked Loop Settling Time

We have also investigated the contribution to the blanking interval of τ_{pll}, the phase-locked loop's settling time.[5] The microwave synthesis chain used in our experiments contains only one PLL that is affected by the slow square-wave frequency modulation. This PLL (figure 3) controls a 10.7 MHz crystal that serves as a "clean-up" oscillator following the DDS. In selecting the loop gain, there is a compromise between the desire for fast settling time (high gain) and good sideband supression (low gain). The settling time of the PLL is show in figure 6. The time constant for the exponential fit in figure 6 is 0.5 ms. Hence τ_{pll} is small compared to τ_{r} and τ_{ρ} in equation (9). Nevertheless, we have used the value $\tau_{\mathrm{pll}} = 5$ ms in arriving at our total blanking time of 40 ms.

Conclusion

We have investigated three sources of bias in digitally controlled microwave synthesis chains: spurious RF sidebands, synthesizer switching transients, and the phase-locked loop's settling time. The experimental results obtained for RF sideband pulling are in excellent agreement with theory. They apply not only to digital servo systems but also to traditional analog servos. While the synthesizer switching transients we observed were unique to a specific model and might be avoided by selecting a different DDS, our modifications yielded a DDS with phase stability superior to other models we have tested. Our analysis of the phase-locked loop transient in the microwave-synthesis chain shows that it settles rapidly compared to the other delay times.

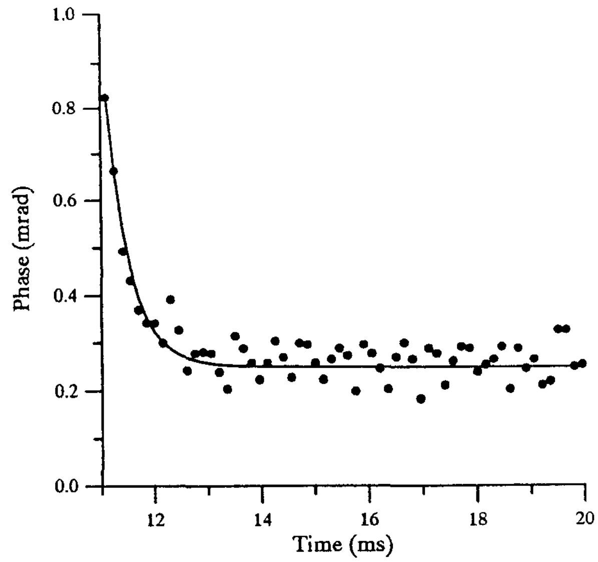

Figure 6: PLL Error Signal During Frequency Step. Dots: Measured phase error after the DDS was instructed to perform a 54 Hz step (the peak to peak modulation amplitude). Solid Line: Exponential fit to the data.

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