

# The Performance and Capability of Cesium Beam Frequency Standards at the National Bureau of Standards

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**Abstract**—NBS II, the older of the two cesium atomic beam frequency standards which are used alternatively as the United States Frequency Standard, has been operating for more than five years. The contribution to inaccuracy produced by uncertainties in the *C* field has been reduced by a factor of 30 to  $\pm 2 \times 10^{-13}$ . The average precision of measurement (standard deviation of the mean) has been demonstrated to be  $1 \times 10^{-12}$  for averaging times of 1 hour and  $2 \times 10^{-13}$  for 12 hours. The overall accuracy is considered to be  $\pm 8 \times 10^{-13}\sigma$ . A new cesium standard, NBS III with an interaction length of 3.66 meters is in operation and has demonstrated an improved precision of  $1 \times 10^{-13}$  over 2 hours and an accuracy of  $\pm 5 \times 10^{-13}\sigma$ . The *C* field contributions to inaccuracy in this machine have been reduced to  $\pm 1 \times 10^{-13}$ . Considerable effort has been devoted to the detection and elimination of small frequency shifts produced by various electronic components of the excitation systems. In spite of the various improvements effected, a small unexplained difference in frequency of about  $1 \times 10^{-12}$  continues to exist between the standards. The extremely high stability of the difference frequency, however, suggests that resolution of the difficulties should result in an accuracy capability of perhaps  $\pm 1 \times 10^{-13}\sigma$ .

## I. INTRODUCTION

THE PROGRAM at the National Bureau of Standards for the development, evaluation, and provision of atomic frequency and time standards has led to the development of two operating cesium standards and one thallium standard at the present time. In addition to these, the laboratory has two ammonia ( $N^{15}$ ) masers which are used primarily as test instruments in the evaluation of atomic time and frequency standards. Also, a hydrogen maser has been operating since August, 1964.

It is the purpose of this report to discuss the progress made in this NBS program over the past three years, particularly that part of the program concerned with the evaluation and performance characteristics of the cesium atomic beam standards. The two cesium beams are used alternatively as the United States Frequency Standard. They also comprise the frequency element of the NBS Time Standard.

The three existing beam standards are designated NBS I, NBS II, and NBS III. The first of these, NBS I, is the oldest machine. It was originally operated using cesium but was converted in 1962 from cesium to thallium. NBS I is the shortest of the three standards,

providing a spectral line width of about 300 Hz for cesium. NBS III is the newest and longest machine and provides a spectral line width of 48 Hz, while the line width for NBS II is 110 Hz.

The accuracy<sup>1</sup> of the cesium beams is considered to be  $\pm 1 \times 10^{-11}$  for NBS I,  $\pm 8 \times 10^{-12}$  for NBS II, and  $\pm 5 \times 10^{-12}$  for NBS III. These figures are determined from an analysis of various auxiliary experiments and tests on a given machine and are confirmed by comparisons of the three independent machines. NBS II and NBS III have also been compared with the cesium standard at Neuchatel, Switzerland, by making use of radio signals [1] and portable atomic standards [2], [3]. The radio comparison data averaged over 1.3 years (1962.2–1963.5) indicated agreement to within  $1 \times 10^{-11}$  for that period. The more recent clock-carrying experiments gave the same result. The precision<sup>1</sup> associated with the NBS standards is at best  $1 \times 10^{-13}$  for two-hour averaging periods. More typically, values range from  $0.5 \times 10^{-12}$  to  $1 \times 10^{-12}$  for the one-half hour averaging periods normally employed in most measurements.

It appears that electronic problems and phase shifts in the resonant cavity are presently the most severe sources of uncertainty and that close attention must be given to them if accuracy is to be increased. Although time consuming, the solutions of these problems do not appear prohibitively difficult, at least for an improvement in accuracy of up to one order of magnitude. It is perhaps not too optimistic to expect an accuracy of  $\pm 1 \times 10^{-13}\sigma$  with the present standards within the next two years.

Some of the details of the analyses and characteristics of the NBS cesium beam standards are discussed in Section II.

## II. THE NBS CESIUM BEAM STANDARDS

### A. Comparison of NBS I and NBS II

NBS I was first placed into regular operation in the spring of 1959 as a cesium beam. It became the United

<sup>1</sup> Accuracy and precision have the following meaning in this manuscript: 1. accuracy refers to the fractional uncertainty in determining an atomic state separation of the free atom and is expressed by  $3\sigma$  limits for statistically determined quantities and by estimated extreme limits for other quantities; 2. precision refers to the fractional uncertainty within which a given machine provides a reproducible measurement and is expressed by  $1\sigma$  limits unless otherwise noted.

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States Frequency Standard (USFS) in the fall of that year. NBS II has been in use since 1960, and until the newest cesium beam, NBS III, was developed, NBS I and NBS II provided the USFS—each was a check on the other. Currently, NBS II and NBS III provide the USFS.<sup>2</sup>

NBS I and NBS II were intercompared for a period of three years [4]–[6]. During this period, practically all of the electronics and a number of the internal components of the beam apparatus were replaced, including deflecting magnets, *C*-field structures, and the waveguide excitation structure of NBS I. Also, both machines were partially disassembled and moved to a neighboring laboratory. With all these changes, the relative frequency difference between them remained fixed at  $1.6 \times 10^{-11}$  within a measurement uncertainty of  $\pm 2 \times 10^{-12}$ . The average precision of frequency comparison (standard deviation of the mean) was  $2 \times 10^{-12}$  for an averaging time of one-half hour. The best precision that was attained was  $2 \times 10^{-13}$  (requiring an averaging time of 10 hours). The statistical behavior was good;  $\chi^2$  tests demonstrated a Gaussian distribution of the data [7]. The accuracy for each machine was considered to be  $\pm 1 \times 10^{-11}$ . This figure was based on the measured frequency difference between the two standards and on the results of a group of auxiliary experiments designed to measure the *C*-field intensity and uniformity, the phase shift between oscillating field regions, the power spectrum of the exciting radiation, effects of neighboring resonances, cavity pulling, and the variation of frequency with power level. These tests were performed for each machine separately. The fixed frequency difference of  $1.6 \times 10^{-11}$  was never adequately explained and implies the existence of a systematic error which still has not been definitely identified.

The comparison of the NBS I and NBS II cesium beams was terminated in 1962 when NBS I was converted to a thallium beam.

### B. Comparison of NBS II and NBS III

The newest cesium standard, NBS III, has been in operation since the summer of 1963. It represents an attempt to improve on previous standards with respect to both precision and accuracy by reducing random errors due to beam fluctuations and certain systematic errors that are line-width dependent. The separations between the oscillating field regions are 164 cm for NBS II and 366 cm for NBS III, with the corresponding spectral line widths being 110 Hz and 48 Hz. Figures 1 and 2 are photographs of these two instruments.

The procedure used in the evaluation of NBS III

<sup>2</sup> The unit of frequency is defined by the frequency separation of the  $F=4$  and  $F=3$  hyperfine structure levels in the free cesium atom. This separation is defined to be exactly 9192631770 Hz. The machines themselves provide the means of measuring frequency in terms of this separation and are designated as standards. The unit of frequency provided by the standard can approach the idealized unit with a certain uncertainty determined by the particular standard and its analysis. Consequently, accuracy limits must be specified.

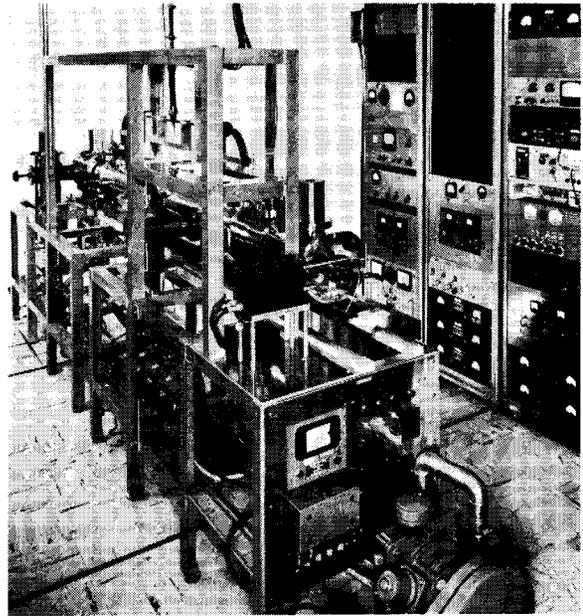


Fig. 1. Cesium atomic beam frequency standard, NBS II.

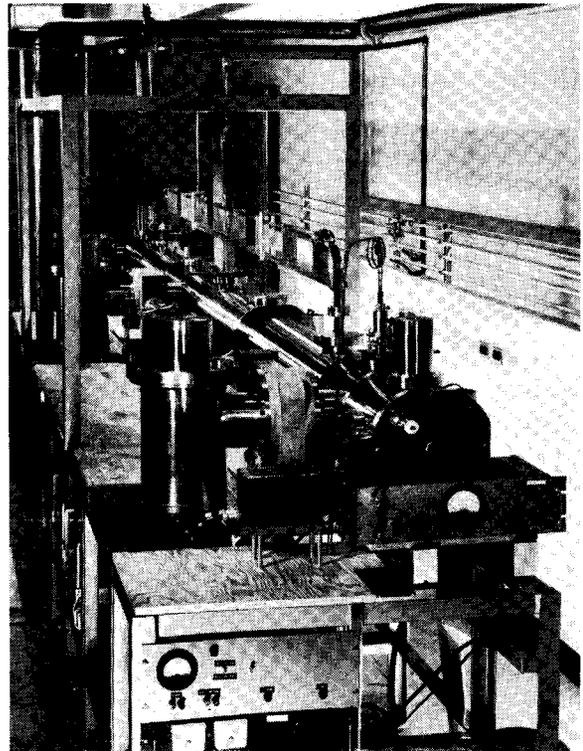


Fig. 2. Cesium atomic beam frequency standard, NBS III.

followed closely that previously used and reported in connection with NBS I and II [4]–[7]. Evaluation with respect to accuracy involved the study of each possible source of error in order to determine its maximum contribution to the overall measurement uncertainty. In many of the experiments, NBS II was used to provide the stable reference frequency for direct comparison with NBS III while the particular param-

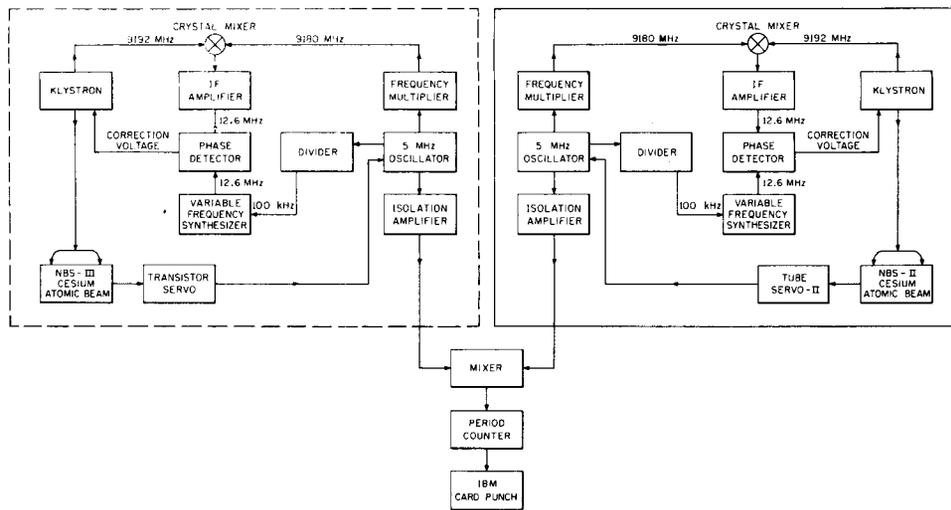


Fig. 3. Block diagram of measurement system for direct comparison of NBS II and NBS III.

eter being studied in the NBS III system was systematically varied. Figure 3 is a block diagram showing the actual method of comparison. Each standard controls a crystal oscillator by means of appropriate servo electronics, and the period of the beat frequency between these two oscillators is measured.

Data from two comparisons of this type are shown in Figs. 4 and 5. Each plotted point, representing the difference frequency averaged over one hour, is the mean of 20 three-minute measurements. The half-length of the vertical bar drawn at each point in Fig. 4 represents the computed standard error of the one-hour observation (standard deviation of the mean of the 20 three-minute averages) and is thus an estimate of the relative precision of this measurement process. This precision figure is typically  $7 \times 10^{-13}$  for these data. When the measurement averaging time of interest is one hour or less, somewhat better precision is usually obtained for comparisons between very high quality quartz crystal oscillators and NBS III than for the direct NBS II-NBS III measurements.

A second estimate of the frequency fluctuations appropriate for one-hour averaging times can be obtained from the data in Fig. 4 by computing the standard deviation of the 48 one-hour averages. This value for the NBS II-NBS III data is  $1.0 \times 10^{-12}$ . It can easily be shown that if the measurement process is in statistical control, i.e., the individual three-minute averages behave as independent samples from a stable probability distribution, these two estimates should be equal. In view of the reasonably good agreement observed, we can predict, in a probability sense, that for any similar one-hour frequency measurement the uncertainty in the result produced by the random variations would be about  $\pm 1 \times 10^{-12}$ . Analysis of a vast amount of comparison data accumulated over several years involving NBS II, NBS III, and many other frequency sources shows that the measurement precision for NBS III is usually two or three times better than for NBS II. Al-

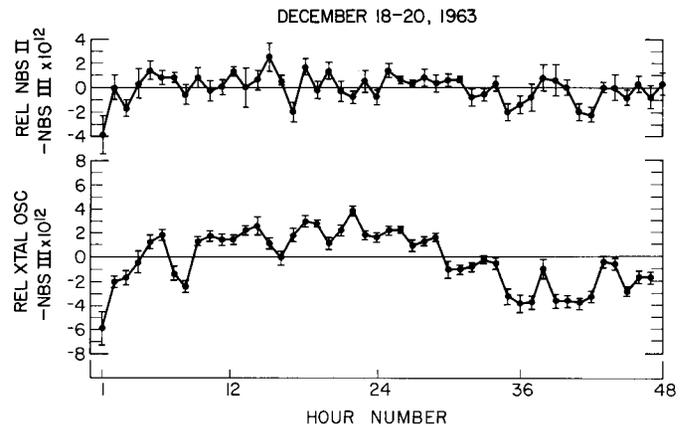


Fig. 4. 48-hour stability comparisons of NBS II, NBS III, and a crystal oscillator.

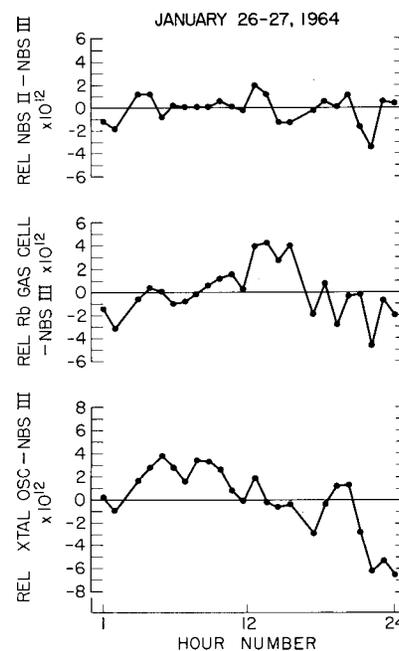


Fig. 5. 24-hour stability comparison of NBS II, NBS III, a crystal oscillator, and a Rb gas cell.

though precisions of  $5 \times 10^{-13}$  and  $1 \times 10^{-12}$  for NBS III and NBS II, respectively, are typical for one-hour averaging times, values of  $2 \times 10^{-13}$  have been obtained consistently during recent comparisons between NBS-III and the NBS hydrogen maser. No attempt has yet been made to determine the individual contributions of NBS III and the maser to this value.

In addition to these random measurement errors, a great variety of systematic error sources may exist which must be considered in determining an overall accuracy figure to be associated with a particular standard. Those sources of error which have been found to be most significant in the case of NBS II and NBS III include errors due to uncertainties associated with the uniform magnetic  $C$  field, phase shifts in the resonant cavity, the presence of unwanted sidebands in the spectrum of the microwave frequency exciting the cesium resonance, and certain other imperfections in the electronics system.

The cesium frequency for the  $(F=4, m_F=0) \leftrightarrow (F=3, m_F=0)$  transition depends on the magnitude of the  $C$  field according to

$$\nu = \nu_0 + 427 \overline{H^2(x)}, \quad (1)$$

where  $\nu$  is the transition frequency (in Hz) in the magnetic field used,  $\nu_0$  is the cesium frequency (in Hz) in zero field,  $H(x)$  is the magnitude of the  $C$  field (in oersteds), and  $\overline{H^2(x)}$  is a spatial average over the length between the two oscillating field regions. Any uncertainties in our knowledge of  $\overline{H^2(x)}$  will thus lead to corresponding errors in frequency measurements referred to  $\nu_0$ .

In actual practice, since  $\overline{H^2(x)}$  is difficult to determine directly, the procedure normally followed is to determine first the quantity  $\overline{H(x)}$  from relatively simple measurements of various microwave transitions which depend linearly on the field, square the result to obtain  $[\overline{H(x)}]^2$ , and then compute the correction  $427 [\overline{H(x)}]^2$  for application to all measurements. Frequency uncertainties resulting from the use of this procedure arise, therefore, from two sources:

- 1) uncertainties in the value of  $\overline{H(x)}$ , and
- 2) uncertainties due to the use of  $[\overline{H(x)}]^2$  for  $\overline{H^2(x)}$ .

The frequency uncertainty  $\Delta\nu$  resulting from an uncertainty  $\Delta H$ , independent of  $x$ , in  $\overline{H(x)}$  or  $H(x)$  can easily be computed from (1). Thus,

$$\nu + \Delta\nu = \nu_0 + 427 [\overline{H(x)} + \Delta H]^2 \quad (2)$$

$$= \nu_0 + 427 [\overline{H^2(x)} + 2\overline{H(x)}\Delta H + (\Delta H)^2]. \quad (3)$$

Neglecting the small term  $(\Delta H)^2$  and subtracting  $\nu$  from both sides, again making use of (1), gives

$$\Delta\nu = 854\overline{H(x)}\Delta H. \quad (4)$$

This uncertainty was evaluated in the following way for both NBS standards. The frequencies of each of the  $(4, -1) \leftrightarrow (3, -1)$ ,  $(4, 1) \leftrightarrow (3, 1)$ ,  $(4, 2) \leftrightarrow (3, 2)$ , and

$(4, 3) \leftrightarrow (3, 3)$  transitions were measured. Theory relates these frequencies to the magnitude of the  $C$  field, and from these relationships  $\overline{H(x)}$  can be obtained [5]. This gives four different measurements of the average field for each setting of the  $C$ -field current. The data so obtained were plotted, and a least-squares line was fitted to the points. The range over which the field was measured was between 0.045 Oe and 0.148 Oe. The field used for normal operation is about 0.048 Oe. This range of field values eliminates overlap effects caused by neighboring transitions and insures good linearity. The standard deviation of a point from this line is 0.000019 Oe. If this is taken to be the uncertainty in the magnitude of the  $C$  field  $\Delta H$ , the corresponding uncertainty in the frequency of the  $(4, 0) \leftrightarrow (3, 0)$  transition is  $\pm 1 \times 10^{-13}$ . These figures apply to NBS III. Similar measurements for NBS II give an uncertainty of  $\pm 0.00004$  Oe, corresponding to an uncertainty of  $\pm 2 \times 10^{-13}$  in the  $(4, 0) \leftrightarrow (3, 0)$  frequency measurement. Previously, this uncertainty had been  $\pm 6 \times 10^{-12}$  and was the largest single contribution to the inaccuracy. More stable power supplies for the  $C$ -field current are now employed and account for the improvement. No changes have been made in the  $C$ -field structure of NBS II.

The uncertainty from the second source mentioned is a consequence of the nonuniformity of the  $C$  field, for in that case,  $\overline{H^2(x)}$  is not equal to  $[\overline{H(x)}]^2$ . The error in Hz which results from using the latter quantity is given by

$$\Delta\nu = 427 [\overline{H^2(x)} - [\overline{H(x)}]^2] \quad (5)$$

If the magnitude of the  $C$  field is expressed as

$$H(x) = H_0(x) + cI, \quad (6)$$

where  $H_0(x)$  is the nonuniform residual field in the drift-space region observed with the current  $I$  producing the  $C$  field turned off, and  $c$  is a constant, it can readily be shown by direct substitution of (6) into (5) that

$$\Delta\nu = 427 [\overline{H_0^2(x)} - [\overline{H(x)}]^2]. \quad (7)$$

It is assumed here that the residual field  $H_0(x)$  does not depend on  $I$ .

In the case of NBS II and NBS III,  $H_0(x)$  has been measured by drawing a small sensitive magnetometer probe through the  $C$ -field region along the beam axis. A continuous plot of this field measurement for NBS II is shown in Fig. 6. The end points of the recording

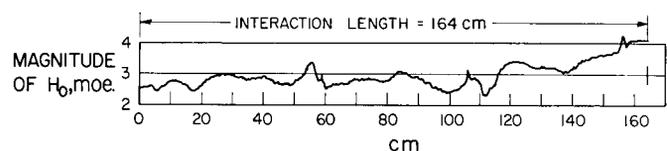


Fig. 6. Variation of magnitude of residual field  $H_0$ , along the length of  $C$ -field region of NBS II.

coincide with the positions of the openings in the magnetic shields where the two ends of the resonant cavity pass through the shields. Similar data for NBS III show slightly better uniformity even though the drift-space region is much longer. The average residual field in NBS II was 0.001 Oe about four years ago and has deteriorated to about 0.003 Oe at the present time.  $\bar{H}_0(x)$  for NBS III is 0.0004 Oe. No degaussing of the magnetic shields has been employed to date. If the error  $\Delta\nu$  is computed from (7) using the residual field data in Fig. 6, a value of less than  $1 \times 10^{-14}$  is obtained. Since it is impractical to measure this uniformity very often, however, it is considered that this effect may contribute a maximum uncertainty of perhaps  $\pm 1 \times 10^{-13}$  to the estimate of inaccuracy.

Certain other types of  $C$ -field nonuniformity can distort the observed cesium resonance asymmetrically, producing a frequency error as a result. Such an effect occurs, for example, if the average field in the region between the oscillating fields differs from the average field within the oscillating field regions. From an examination of the symmetry of the field dependent (4, 1)  $\leftrightarrow$  (3, 1) transition, which provides a sensitive indication of this condition, we conclude that any resultant errors should be less than  $\pm 5 \times 10^{-13}$  for both NBS II and NBS III. Local nonuniformity of the  $C$  field within the oscillating field regions can produce large frequency shifts, which are expected to be strongly dependent on the microwave power level. The absence of a large dependence of frequency on power level for either standard, together with the magnetometer data, suggests the lack of a significant contribution to inaccuracy from this source.

Reversing the polarity of the NBS III  $C$  field produced no frequency shift greater than the measurement precision of  $5 \times 10^{-13}$ .

The microwave cavity has been investigated in order to determine the existing phase difference between the oscillating field regions, dependence of the measured cesium frequency on cavity tuning, and the existence of possible fringing of the electromagnetic field at the beam coupling holes. Of these effects, the last was investigated by placing a horizontal wire grid across each opening in the resonant cavity end sections in an effort to reduce any existing leakage field. Measurements were then performed to determine the presence of a frequency shift, but none was observed. The measurements were obtained with NBS II as a reference.

A detuning of the resonant cavity by 1.3 MHz resulted in a frequency shift of  $3 \times 10^{-12}$ . It is believed, however, that this tuning change does not result in a pulling of the cesium resonance frequency, but rather results in a change in the phase shift between the two oscillating field regions of the resonant cavity. This is thought to be a result of asymmetry of the tuning mechanism. As long as the cavity is tuned to the cesium resonance, the phase difference should remain constant. The only contribution to uncertainty in the frequency

measurements would then be the uncertainty in the phase difference determination.

The accurate determination of the frequency shift caused by phase differences existing in the cavity has proved to be one of the more difficult problems encountered with NBS III. The first method used to measure this effect consisted of measuring the NBS III resonance frequency relative to that of NBS II both before and after the entire NBS III cavity structure was physically rotated by  $180^\circ$ . One half of any frequency shift observed as a result of this operation may be attributed to a phase-difference effect and applied as a correction to all measurements. The physical rotation of this extremely long cavity, however, appears to deform the structure in an unreproducible manner to such an extent that the observed frequency shift is reproducible to only  $5 \times 10^{-12}$ . In an effort to minimize any such effects, a second method has recently been employed in which the direction of traversal of the beam through the cavity is reversed by interchanging the atomic beam oven and detector, while leaving the cavity undisturbed. Although this technique has been found to yield better reproducibility, the contribution from this source to the overall inaccuracy of NBS III is nevertheless considered to be  $\pm 3 \times 10^{-12}$ .

Experiments to determine the existence of frequency shifts due to oven and detector offset in the horizontal plane have shown no net frequency shift greater than  $1 \times 10^{-12}$  for a position change of 1.7 mm. The normal operating point was included in this range. A horizontal misalignment of 0.5 mm would be considered excessive.

Another possible source of systematic frequency shifts may be a first-order Doppler effect. If, due to imperfect alignment, atoms have a component of their velocity parallel to the radiation field and a net traveling wave exists in the cavity, a frequency shift may occur, given approximately by

$$\Delta\nu = \left( \frac{1 - R}{1 + R} \right) \nu_0 \frac{\bar{v}}{c} \sin \alpha,$$

where  $R$  is the power reflection coefficient,  $\bar{v}$  is the mean speed of the atoms in the beam, and  $\alpha$  is the angle between the waveguide and a line normal to the beam. Some measurements made with the NBS.I thallium standard suggested the existence of such a shift in that machine. Similar experiments performed recently, using NBS II, however, failed to disclose any such shifts.

The unavoidable second-order Doppler shift of  $v^2/2c^2$  amounts to  $4 \times 10^{-13}$  for both standards.

The electronics systems for the atomic beam frequency standards have been extensively investigated as possible sources of systematic frequency errors. As the power level of the excitation signal was varied from 3 mW to 12 mW (normal value = 4 mW), the frequency of NBS III shifted  $-5 \times 10^{-12}$ . A resonant cavity with a phase difference between the oscillating field regions usually produces such behavior, i.e., the frequency shift

is generally power dependent. Under these conditions, as long as the power is constant, the phase difference should be fixed, and no additional inaccuracy should exist.

The power spectrum of the frequency multiplier chain used with the cesium standards has been investigated at intervals with the ammonia maser spectrum analyzer [8]. Most recent results show the brightest sidebands to be down about 43 dB at 9180 MHz, which is the 34th harmonic of the 270 MHz output signal of the multiplier chain. Although sidebands down by this amount cannot cause sufficient distortion of the resonance to result in significant frequency shifts, an allowance must be made for the fact that the spectrum may change significantly from time to time. This has been found to be particularly true when the sidebands are due to the power line frequency and its harmonics, making the spectrum sensitive to changes in ground loops and various electrical connections in the laboratory. In arriving at estimates of inaccuracy for the standards, therefore, contributions of  $\pm 2 \times 10^{-12}$  for NBS III and  $\pm 6 \times 10^{-12}$  for NBS II are included for this possible error source. The larger contribution from NBS II reflects the fact that a wide spectral line is more sensitive to such errors than a narrow one. Although it is impractical to examine the spectrum very often using the maser analyzer, it has been found useful to monitor regularly with an oscilloscope the beat frequency at about 100 Hz between the normal cesium excitation signal and a similar signal from another oscillator-multiplier chain system that is known to have a clean spectrum. On several occasions, small spurious modulations have been detected on the beat signal which were causing frequency shifts of only  $1 \times 10^{-11}$ .

Servo system contributions to inaccuracy have been discussed in an earlier report [7]. Some effects have been studied in more detail since that time. Special effort has been made to measure the effects of second harmonic distortion of the 37.5-Hz modulating signal used in the servo systems of both machines. This was accomplished by using, for example, NBS III as a reference while a carefully controlled percentage of second harmonic distortion was added to the NBS II servo system. The frequency of NBS II was observed while the phase of the second harmonic distortion was varied relative to the phase of the 37.5 Hz fundamental. This experiment was performed at 0.4 percent, 1 percent, and 3 percent added second harmonic distortion. The dependence of the frequency of the standard on the second harmonic distortion is basically in accord with the relation  $\Delta\nu = (1/2)Df_a \sin \alpha$  where  $\Delta\nu$  is the shift in the transition frequency,  $D$  is a measure of the second harmonic distortion,  $f_a$  is the peak frequency deviation of the excitation signal due to the modulation, and  $\alpha$  is the phase angle of the second harmonic signal relative to the fundamental. One percent distortion at a relative phase of  $90^\circ$  produced a frequency shift of  $3 \times 10^{-11}$ .

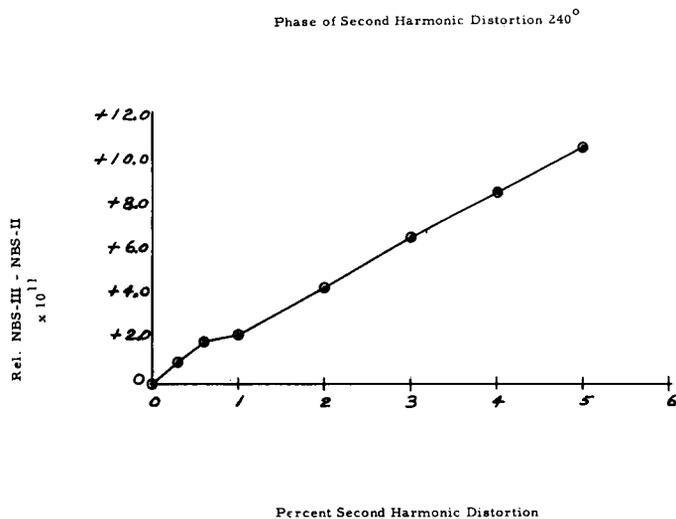


Fig. 7. Relative frequency of NBS II vs. percentage added second harmonic distortion.

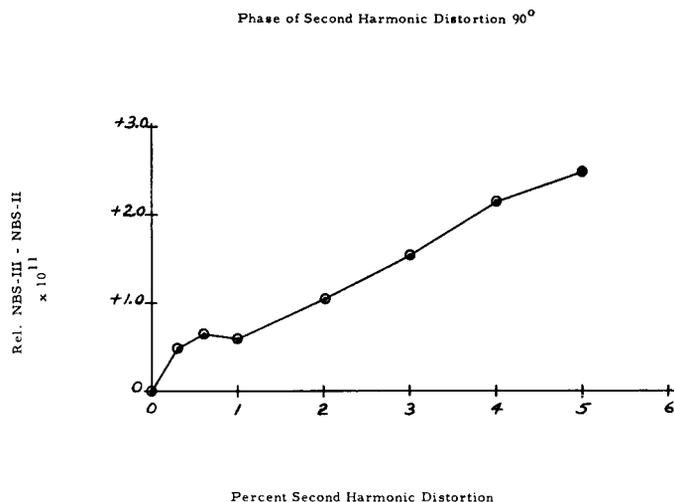


Fig. 8. Relative frequency of NBS III vs. percentage added second harmonic distortion.

The second set of experiments performed on both standards was the determination of the frequency dependence on the percentage of the second harmonic distortion added to the servo system. One standard was the reference in one experiment, while the situation was reversed for the other experiment. The phase of the second harmonic signal relative to the fundamental in each case was adjusted to the value which produced maximum shift of the transition frequency for any particular amount of second harmonic distortion. The results are shown in Figs. 7 and 8. It is apparent that the transition frequency as measured by NBS III is affected by second harmonic distortion less than one half as much as that for NBS II. NBS III therefore has an advantage over NBS II in this respect, due to its narrower line width.

The independent servo systems have been compared

recently via NBS III. Forty-minute measurements of the frequency of a high quality quartz oscillator taken alternately with the two servo systems show agreement to within  $1 \times 10^{-12}$ . The precision of each measurement was  $0.9 \times 10^{-12}$ . Similar results were obtained using NBS II.

Further sources of small errors which have been eliminated are large 60-Hz fields arising from physical layout of power outlets on NBS II and stray field leakage into the chopper demodulator in one of the servo systems. In addition, the frequency multiplier chain used with NBS II has been provided with better shielding, and the input impedance of this chain has been adjusted to  $50 \Omega$ . Possible frequency errors resulting from time dependent phase shifts in the multiplier chain have been considered, and some experimental work in this area is presently underway.

An overall estimate of inaccuracy for each of the standards may be made by combining in some appropriate manner the individual contributions to uncertainty discussed above. If we make the reasonable assumption that the individual effects are independent of one another, the proper method of combination is to use the square root of the sum of the squares as the overall estimate. Table I is a summary of the estimates of uncertainty due to the individual sources considered for both NBS II and NBS III. The combined total  $3\sigma$  estimates of uncertainty are found to be  $\pm 8.0 \times 10^{-12}$  for NBS II and  $\pm 4.9 \times 10^{-12}$  for NBS III. The individual estimates given in the table are considered to be reasonable limits of error, approximately corresponding to statistically-determined  $3\sigma$  estimates, for those effects not directly amenable to statistical analysis. The values given for Doppler shifts and multiplier-chain transient phase shifts are necessarily tentative at this time, due to a lack of conclusive experimental evidence.

TABLE I  
CONTRIBUTIONS TO INACCURACY FOR NBS II AND NBS III  
Estimated Uncertainty  $\times 10^{12}$

Source of Uncertainty	NBS II	NBS III
1) Random measurement errors (1 hr.) $3\sigma$ limits	$\pm 3.0$	$\pm 1.5$
2) Magnitude of $\overline{H(x)} - 3\sigma$ limits	$\pm 0.6$	$\pm 0.3$
3) Overlap of neighboring transitions	$\pm 2.0$	$\pm 1.0$
4) Use of $\overline{H(x)^2}$ for $\overline{H^2(x)}$	$\pm 0.1$	$\pm 0.1$
5) Distortion effects arising from C field nonuniformity	$\pm 0.5$	$\pm 0.5$
6) C-field polarity effects	$\pm 0.5$	$\pm 0.5$
7) Cavity mistuning	$\pm 0.1$	$\pm 0.1$
8) Uncertainty in magnitude of cavity phase shift	$\pm 2.0$	$\pm 3.0$
9) Doppler shifts	$\pm 1.0$	$\pm 1.0$
10) Microwave power level	$\pm 1.0$	$\pm 1.0$
11) Spectral purity of excitation	$\pm 6.0$	$\pm 2.0$
12) Second harmonic distortion of servo modulation	$\pm 1.5$	$\pm 0.5$
13) Miscellaneous servo system effects	$\pm 2.0$	$\pm 2.0$
14) Multiplier chain transient phase shifts	$\pm 1.0$	$\pm 1.0$
Total $3\sigma$ Estimated Uncertainty (square root of sum of squares)	$\pm 8.0$	$\pm 4.9$

Checks upon the estimates of accuracy are obtained from comparisons of the cesium  $(4, 0) \leftrightarrow (3, 0)$  transition frequency as measured by the two independent standards. The measured frequency difference between NBS II and NBS III contains data accumulated over a long period of time. Many of the data were obtained in the first stages of operation of NBS III when fluctuations were larger than they have been more recently.

Between September 1963 and March 1, 1964, measurements of the frequency difference between NBS III and NBS II were hampered by various electronic problems and a three-year accumulation of dirt and pump oil in the resonant cavity end sections of NBS II. This residue resulted in a large phase difference between the two oscillating field regions. A  $180^\circ$  rotation of the resonant cavity produced a frequency shift of  $2.4 \times 10^{-11}$ . After the cavity was cleaned on March 1, 1964, rotation produced a frequency shift of  $(2.0 \pm 0.4) \times 10^{-12}$ , where the quoted uncertainty is the  $1\sigma$  limit computed from the data. The average frequency difference during this period was  $2 \times 10^{-11}$ .

Between March 1 and May 22, the average frequency difference between the two standards was  $(7 \pm 2) \times 10^{-12}$ . Two changes were made in this period: the input impedance of the frequency multiplier chain used with NBS II was brought down from  $150 \Omega$  to  $50 \Omega$ , and the shielding for this multiplier chain was made more adequate. From May 22, 1964 to June 1, 1965, the average measured difference between the two standards was

$$\text{NBS III} - \text{NBS II} = + (1.4 \pm 1.0) \times 10^{-12},$$

where the quoted uncertainty is once again the computed  $1\sigma$  limit.

## CONCLUSION

Experience gained at the National Bureau of Standards over a period of six years with three independent cesium standards has shown that realistic accuracy figures of  $\pm 5 \times 10^{-12}$  and measurement precisions of  $1 \times 10^{-13}$  in two hours are obtainable with existing standards. By reducing the uncertainties due to cavity phase shifts, random measurement errors, and spectral purity, and by careful attention to the remaining electronic problems, it is believed that an accuracy figure of  $\pm 1 \times 10^{-12} 3\sigma$  may be realized for the NBS standards within the next one or two years.

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