

A REVIEW OF SENSOR SENSITIVITY AND STABILITY

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Introduction

Acoustic sensor sensitivity is often reported in units of Hz per unit of measurand. It has been frequently stated or implied in the sensor literature that higher-frequency sensors are preferable to lower frequency sensors because the frequency change per unit of measurand is higher, i.e., the higher-frequency sensors are more "sensitive."

At the paper selection meeting of the 2000 IEEE International Frequency Control Symposium Technical Program Committee, it was noted that an otherwise good paper submission claimed, once again, that higher-frequency sensors are superior because they are more sensitive. The program committee felt that an invited review paper that discusses the advantages and disadvantages of higher-frequency sensors was needed (in spite of a previous correspondence on the subject [1]).

Although it is true that a higher-frequency resonator will generally produce a larger frequency change per unit of measurand, it is also true that a higher-frequency can result in lower accuracy and in a lesser ability to resolve small changes in the measurand. The reasons are that higher-frequency resonators of a given material and manufacturing technology are inherently noisier [2,3], and, at least in the case of quartz resonators, low frequency (e.g., SC-cut) resonators can be made more temperature stable than high frequency (e.g., SAW) resonators. Other disadvantages of using higher frequencies are that higher-frequency resonators have higher aging rates, and higher-frequency digital electronics require more power. An advantage of higher-frequency resonators is that such resonators can be made smaller than low frequency resonators.

This paper will review the resonator instabilities that lead to the conclusions that, 1) sensitivity expressed as Hz per unit of measurand is not a good measure of sensor quality, and 2) when compared on the bases of accuracy, reproducibility and resolution capability, "good" low frequency sensors are often superior to "good" high frequency ones.

Resonator Stability

Brief summaries of the most sensor-relevant instabilities follow. More details can be found in the references.

Noise

Noise limits the accuracy with which one can determine the frequency of an oscillator. Similarly, in a sensor, noise limits the measurement resolution, i.e., it limits the minimum quantity of a measurand that can be measured with a specified uncertainty.

As discussed in reference [2] (and its references), the known causes of short-term instabilities are:

- Johnson noise due to the motional resistance of the resonator and within the oscillator circuitry,
- Phonon scattering noise in the resonator,
- Changes in external load,
- Thermal response of resonator - static and dynamic,
- Temperature fluctuations - an activity dip at the measurement temperature can greatly magnify the frequency fluctuations caused by temperature fluctuations,
- Random vibration,
- Fluctuations in the number of adsorbed molecules,
- Stress relief and fluctuations at interfaces (between the quartz plate and its electrodes and its mounting and bonding structure).

The standard measure of random fractional frequency fluctuations (also called short-term instabilities, or noise) in the time domain is the two-sample deviation, $\sigma_y(\tau)$, also called Allan deviation [4]. In the region of sampling time τ where $\sigma_y(\tau)$ versus τ is minimum, $\sigma_y(\tau)$ is inversely proportional to resonator Q . This is the typical region where sensor frequencies are measured (~0.1s to 10s). The empirical relationship between resonator Q and noise floor, as measured by $\sigma_y(\tau)$, is

$$\sigma_y(\tau) \geq \frac{1.0 \times 10^{-7}}{Q}$$

Moreover, for a given cut of resonator, the maximum achievable Q times the frequency is a constant, i.e., $Q_{\max}f = \text{constant}$. For the commonly used bulk acoustic wave (BAW), and surface acoustic wave (SAW) cuts,

$$(Q_{\max}f)_{\text{BAW}} = 1.6 \times 10^{13} \text{ Hz, for AT- and SC-cuts [5,6]}$$

$$(Q_{\max}f)_{\text{SAW}} = 1.1 \times 10^{13} \text{ Hz, for ST-cut [7,8]}$$

Therefore, for a given resonator type, the minimum attainable $\sigma_y(\tau)$, is proportional to f :

$$\sigma_y(\tau, f) \geq 6 \times 10^{-21} f, \text{ when } f \text{ is in Hz (BAW), and}$$

$$\sigma_y(\tau, f) \geq 9 \times 10^{-21} f, \text{ when } f \text{ is in Hz (SAW)}$$

i.e., the higher the frequency, the higher the noise floor.

The maximum Q allowed by the material is rarely realized in sensors, especially when the crystals are not hermetically sealed (causing air loading) or when added mass produces damping. It should also be noted that the maximum Qf product is dependent on crystal cut. In quartz BAW resonators, for example, whereas the typical maximum Qf is 1.6×10^{13} Hz for AT-cut and SC-cut resonators, it is 3.8×10^{13} Hz for BT-cut devices.

Frequency vs. temperature stability

In nearly all cases, frequency instabilities due to temperature instabilities in the sensor's environment will exceed the resonator's inherent frequency fluctuations. There are two ways to reduce the frequency versus temperature (f vs. T) instability of a resonator. When a resonator's temperature is controlled, the oscillator's f vs. T instability can be minimized by maintaining the temperature of the resonator at the point where the f vs. T characteristic's slope is zero. Since this method tends to interfere with the sensor's interaction with the environment, it is not one that can be easily applied to some types of sensors.

Compensation, as is used in temperature compensated crystal oscillators (TCXO) and microcomputer compensated crystal oscillators (MCXO), can be readily applied to sensors [9,10]. The major limitation on the attainable TCXO or MCXO stabilities is hysteresis [11]. The same hysteresis will limit the accuracy and reproducibility attainable with sensors. For repeated cyclings between -55°C and $+85^{\circ}\text{C}$, the best hysteresis observed in 10 MHz/3.3 MHz dual mode MCXO resonators has been in the parts in 10^9 range. No such stability has been reported for high frequency resonators, either BAW or SAW. The hysteresis observed in TCXOs is typically in the parts in 10^7 range.

Hysteresis is not well understood, however, mechanisms that can cause hysteresis, such as contamination transfer to and from the resonators' surfaces and stress relief, will cause larger effects in higher-frequency resonators. The thinner the resonator, the larger will be the frequency change caused by a given amount of contamination. Similarly, stress relief in the electrodes will cause a larger frequency change in a thinner resonator. (Reducing the electrode thickness to compensate for the thinner resonator is usually not possible because the thicknesses of resonator electrodes are determined by factors such as the need for film continuity and low resistivity, and energy trapping.)

Other types of hysteresis can also limit the inherent accuracy of sensors, e.g., pressure versus frequency hysteresis limits pressure sensors' accuracy [12].

Aging

Aging can affect the absolute accuracy and calibration of a sensor. The frequency dependence of aging was one of the topics examined in a paper reviewing resonator aging [13]. It was found that "For a given fabrication process, the aging rate tends to scale with the volume-to-surface ratio of the resonator's active area, i.e., with the frequency of the plate. The scaling with frequency appears to apply to SAW devices too; e.g., the aging rates of "good" 500-MHz SAW resonators are typically on the order of 100 times higher than the aging rates of "good" 5-MHz bulk acoustic wave resonators."

Frequency Measurement Accuracy

One advantage that has occasionally been claimed for higher-frequency resonators is that, since a higher-frequency resonator will generally produce a larger frequency change per unit of measurand, the frequency change can be measured faster and with higher accuracy. This may be true for conventional counters that measure the number of cycles in a given gate time, but it is not true for modern, reciprocal counters.

A reciprocal counter measures the time interval for some integer number of input cycles, then computes frequency by dividing the number of cycles by the time interval. Since no fractional cycle measurements are involved (as is the case for conventional counters that measure the number of cycles in a fixed time interval), extremely high frequency resolution can be achieved. The frequency measurement accuracy is determined primarily by the time interval measurement accuracy, independent of the frequency being measured (within the instrument's frequency range). The resolution capability is determined primarily by the stability of the time-base oscillator in the counter. For example, one manufacturer claims an up to 11 digit resolution for a one second gate time, up to 1.3 GHz [14].

Definitions of Sensitivity

The IEEE Standard Dictionary of Electrical and Electronics Terms contains 15 definitions of "sensitivity". None are specifically for sensors. Three of the more interesting definitions are:

"• (general comment) Definitions of sensitivity fall into two contrasting categories. In some fields, sensitivity is the ratio of response to cause. Hence increasing sensitivity is denoted by a progressively larger number. In other fields, sensitivity is the ratio of cause to response. Hence increasing sensitivity is denoted by a progressively smaller number."

"• (measuring devices) The ratio of the magnitude of its response to the magnitude of the quantity measured."

"• (radio receiver or similar device) Taken as the minimum input signal required to produce a specified output signal having a specified signal-to-noise ratio."

The second definition is what is commonly used in the sensor field, however, for the reasons discussed above, the third definition, which includes signal to noise ratio in the definition, would be a better indicator of sensor quality. Including the resonator stability [1] or Q-value [15] in the definition of a figure of merit for sensors has been proposed previously.

Comparisons of 10 MHz and 100 MHz Sensors

To compare high and low frequency sensors, consider the following examples. Let us assume that: 1) the sensors are fundamental mode resonators of frequencies 10 MHz and 100 MHz, 2) the noise, as measured by the Allan deviations, $\sigma_y(1s)$, are 10^{-9} and 10^{-8} , respectively, 3) that the hystereses are 10^{-6} and 10^{-5} , respectively, and the aging rates are 10^{-9} per day and 10^{-8} per day, respectively. Furthermore, let us define the resolution capability to be the measurand-induced normalized frequency change that is equal to the noise as measured by $\sigma_y(1s)$, and the measurement uncertainty (reproducibility) after a temperature excursion to be the measurand-induced normalized frequency change that is equal to the hysteresis.

Let sensor type 1 be a Y-cut quartz thermometer with an f vs. T slope = 10^{-4} per K. Then for the 10 MHz and 100 MHz versions, the resolution capabilities are 10^{-5} K and 10^{-4} K, respectively, and the reproducibilities are 10^{-2} K and 10^{-1} K, respectively, i.e., the lower frequency sensor has 10x better resolution, reproducibility, and aging.

Let sensor type 2 be a quartz crystal microbalance onto which a film (of same density as quartz) is deposited the thickness of which is equal to 10^{-6} of the 10 MHz resonator's thickness. The frequency of the 10 MHz resonator then changes by ~ 1 ppm, and the frequency of the 100 MHz resonator changes by ~ 10 ppm. Then, the signal to noise ratios are 10^3 for both, i.e., the resolution capabilities will be the same for the 10 MHz and 100 MHz sensors. Similarly, the reproducibilities will be the same. The aging of the 100 MHz sensor, however, will be 10x worse.

Summary and Conclusions

The consequences of higher-frequency sensors are:

- Larger frequency change per unit of measurand
- Higher noise - offsets the larger frequency change per unit of measurand; can lessen the ability to resolve small changes in the measurand
- Higher hysteresis - offsets the larger frequency change per unit of measurand; can result in poorer reproducibility
- Higher aging - poorer long term stability and accuracy (calibration)
- Smaller resonator
- Higher power (in digital electronics)
- Frequency measurement capability is mostly unaffected if a reciprocal counter is used

Sensitivity, defined as frequency change per unit of measurand, is not a useful measure of sensor quality because sensor instabilities limit the usable sensitivity. A figure of merit should include both the sensitivity and stability, for example, hysteresis divided by sensitivity, and $\sigma_y(1s)$ divided by sensitivity, where sensitivity is the normalized frequency change per unit of measurand. The first is a measure of the sensor's reproducibility, and the second is a measure of its resolution capability.

Standard sensor terms and definitions are needed.

References

(References 1-3, 9-13, 15 are available online in the UFFC-S Digital Archive: <http://www.ieee-uffc.org/archive>.)

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