# Quantum Computation, Spectroscopy of Trapped Ions, and Schrödinger's Cat *

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We summarize efforts at NIST to implement quantum computation using trapped ions, based on a scheme proposed by J.I. Cirac and P. Zoller (Innsbruck University). The use of quantum logic to create entangled states, which can maximize the quantum-limited signal-to-noise ratio in spectroscopy, is discussed.

## 1. INTRODUCTION

The invention by Peter Shor [1] of a quantum algorithm for factorizing large numbers has stimulated a host of theoretical and experimental investigations in the field of quantum information [2]. In the area of quantum computation, various schemes have been proposed to realize experimentally a model quantum computer [2]. In the ion storage group at NIST, we are trying to realize such a device based on the proposal by Cirac and Zoller [3].

In the Cirac-Zoller scheme, qubits are formed from two internal energy states, labeled $|\downarrow\rangle$ and $|\uparrow\rangle$, of trapped atomic ions. If the ions are laser cooled in the same trap, they form a crystalline array whose vibrations can be described in terms of normal modes. The ground and first excited states of a selected mode can also form a qubit. This qubit can serve as a data bus, since the normal modes are a *shared* property of the ions. An individual ion in the array can be coherently manipulated and coupled to the selected normal mode by using focused laser beams [3]. A universal logic operation, such as a controlled-not (CN) logic gate between ion qubit $i$ and ion qubit $j$, is accomplished by (1) mapping the internal state of qubit $i$ onto the selected motional qubit, (2) performing a CN between the motional qubit and qubit $j$, and (3) mapping the motional qubit state back onto qubit $i$. Each of these steps has been accomplished in the NIST experiments with a single ion [4,5]. We are currently devoting efforts to: (1) scaling quantum logic operations to two or more ions (Sec. 5), (2) applying quantum logic to study fundamental measurement problems on EPR and GHZ-like states, and (3) applying quantum logic to fundamentally improve the signal-to-noise ratio (SNR) in spectroscopy and atomic clocks. In this paper we briefly discuss this last application. We are aware of similar efforts to implement trapped-ion quantum logic at IBM, Almaden; Innsbruck University; Los Alamos National Laboratory; Max Planck Institute, Garching; and Oxford University.

## 2. ENTANGLED STATES FOR SPECTROSCOPY

A collection of atoms (neutral or charged) whose internal states are entangled in a specific way can improve the quantum-limited SNR in spectroscopy. This application of quantum logic to form entanglement is useful with a relatively small number of atoms and logic operations. For example, for high-accuracy, ion-based frequency standards [6], a relatively small number of trapped ions ($L \leq 100$) appears optimum due to various experimental constraints; with $L = 10 - 100$, a significant improvement in performance in atomic clocks could be expected. In contrast, factoring a number which cannot easily be factored on a classical computer would require considerably more ions and operations.

In spectroscopy experiments on $L$ atoms, in which the observable is atomic population, we can view the problem in the following way using the spin-1/2 analog for two-level atoms. The total angular momentum of the system is given by $\mathbf{J} = \sum_{i=1}^{L} \mathbf{S}_i$, where $\mathbf{S}_i$ is the spin of the $i$th atom ($S_i = 1/2$). The task is to measure $\omega_0$, the frequency of transitions between the $|\downarrow\rangle$ and $|\uparrow\rangle$ states, relative to the frequency $\omega_R$ of a reference oscillator. We first prepare an initial state for the spins. Typically, spectroscopy is performed by applying (classical) fields of frequency $\omega_R$ for a time $T_R$ according to the method of separated fields by Ramsey [7]. We assume the same field amplitude is applied to all atoms (the phases might be different) and that the maximum value of $T_R$ is fixed by experimental constraints (Sec. 3). After applying these fields, we measure the final state populations; for example, the number of atoms $L_\downarrow$ in the $|\downarrow\rangle$ state. In trapped-ion experiments, this has been accomplished through laser fluorescence detection with nearly 100% efficiency, which we assume here (see the discussion and references in Ref. [5]). In the spin-1/2 analog, measuring $L_\downarrow$ is equivalent to measuring the operator $J_z$, since $L_\downarrow = J\mathbb{I} - J_z$ where $\mathbb{I}$ is the identity operator. The SNR (for repeated measurements) is fundamentally limited by the quantum fluctuations in the number of atoms which are observed to be in the $|\downarrow\rangle$ state. These fluctuations can be called quantum projection noise [8]. Spectroscopy is typically performed on $L$ initially unentangled atoms (for example, $\Psi(t=0) = \prod_{i=1}^{L} |\downarrow\rangle_i$) which remain unentangled after the application of the Ramsey fields. For this case, the imprecision in a determination of the frequency of the transition is limited by projection noise to the "shot noise" limit $(\Delta\omega)_{meas} = 1/\sqrt{LT_R\tau}$ where $\tau \gg T_R$ is the total averaging time [8]. If the atoms can be prepared initially in particular entangled states, it is possible to achieve $(\Delta\omega)_{meas} < 1/\sqrt{LT_R\tau}$.

In optics, squeezed states have been shown to improve the SNR in interferometers beyond the shot noise limit [9,10]. In 1986, Yurke [11] showed how particular entangled states, if they could be created, could be used as inputs to Mach-Zehnder interferometers to approach the Heisenberg limit of SNR. In 1991, Kitegawa and Ueda [12] showed how the Coulomb interaction between electrons in the two arms of an electron interferometer might be used to improve the SNR beyond the shot-noise limit. Because of the formal identity of Mach-Zehnder interferometers and Ramsey spectroscopy [13], similar ideas might be applied to the spectroscopy problem. Reference [13] showed how a Jaynes-Cummings-type coupling between trapped-ion internal states and a normal mode could be used to improve the SNR in spectroscopy beyond the shot-noise limit. The scheme in Ref. [13] has the advantage that the appropriate states can be generated by acting on all the ions at once (thus not requiring focused laser beams), but has the disadvantage that these states

are entangled with the motion, thereby requiring small motional decoherence. Reference [14] investigated the use of the generalized GHZ state, sometimes called the maximally entangled state, in spectroscopy. This state has the form

$$\psi_{max} = \frac{1}{\sqrt{2}}\Big(|\downarrow\rangle_1|\downarrow\rangle_2\cdots|\downarrow\rangle_L + e^{i\phi(t)}|\uparrow\rangle_1|\uparrow\rangle_2\cdots|\uparrow\rangle_L\Big), \tag{1}$$

where $\phi(t) = \phi_0 - L\omega_0 t$. After application of the Ramsey radiation, we measure the operator $\tilde{O} \equiv \prod_{i=1}^{L} S_{zi}$. The resulting signal gives the exact Heisenberg limit of SNR ($(\Delta\omega)_{meas} = 1/L\sqrt{T_R\tau}$ where $\tau \gg T_R$) in spectroscopy (and interferometry).

The state $\psi_{max}$ can be generated in a straightforward way by the application of $L$ CN gates [3]. An alternative method was suggested in Ref. [14] and in Refs. [5] and [15] methods to generate $\psi_{max}$ with a fixed number of steps (independent of $L$) are discussed. For all of these methods, the the motion is entangled with internal states during the creation of $\psi_{max}$, but is not entangled afterwards. Therefore, once $\psi_{max}$ is created, the motion can lose coherence without affecting the entanglement of the internal states.

### 2.1. Schrödinger's Cat

As $L$ becomes large and more macroscopic, states like $\psi_{max}$ become more like Schrödinger's cat in that they represent coherent superpositions between widely separate regions of a large Hilbert space; for example, $|\uparrow\rangle_1|\uparrow\rangle_2\cdots|\uparrow\rangle_L \Longleftrightarrow$ "live cat;" $|\downarrow\rangle_1|\downarrow\rangle_2\cdots|\downarrow\rangle_L \Longleftrightarrow$ "dead cat". As has been emphasized in many discussions, as $L$ becomes large the coherence between the two components of the cat becomes harder and harder to preserve [16]. This is apparent in Eq. (1) because if, for example, $\omega_0$ fluctuates randomly, the two components of $\psi_{max}$ will decohere relative to each other $L$ times faster than for one ion ($\psi_{max}$ for $L = 1$). Trapped ions are interesting because it may be possible to make $L$ very large without significant decoherence. This is the same property that makes trapped ions interesting as possible frequency standards. For example, in Refs. [17] and [18], coherence times for individual ions ($L = 1$) exceeding 10 minutes were obtained.

## 3. Applicability

In the above, we have assumed that $T_R$ is fixed, limited by some independent experimental factor. This assumption is warranted in many trapped-ion atomic clock experiments, where, for example, we want to limit the heating that takes place with laser cooling radiation absent. (During application of the Ramsey fields the cooling radiation must be removed to avoid perturbing the clock states.) Additionally, we may want to lock a local oscillator to the atomic reference in a practical time [6,19], thereby limiting $T_R$.

However, the use of entangled states may not be advantageous, given other conditions. For example, Huelga, *et al.* [20] assume that the ions are subject to a certain dephasing decoherence rate (decoherence time less than the total observation time). In this case, there is no advantage of using maximally entangled states over unentangled states. The reason is that since the maximally entangled state decoheres $L$ times faster than the states of individual atoms, when we use the maximally entangled state, $T_R$ must be reduced by a factor of $L$ for optimum performance. Therefore, the gain from using the maximally entangled state is offset by the required reduced value of $T_R$.

Reference [5] discusses another case of practical interest. In atomic clocks, the frequency of an imperfect "local" oscillator, whose radiation drives the atomic transition, is controlled by the atom's absorption resonance. Depending on the spectrum of this oscillator's frequency fluctuations (when not controlled) the use of entangled states may or may not be beneficial.

## 4. Implementations

If we are able to create, with good fidelity, the state $\psi_{max}$ (Eq. (1)), how do we perform spectroscopy? First, we note that $\psi_{max}$ is the state we want *after* the first Ramsey $\pi/2$ pulse. Therefore, if we were to follow as closely as possible the Ramsey technique, we would take $\psi_{max}$ and apply a $\pi/2$ pulse of radiation at frequency $\omega_0$ to make the input state for the Ramsey radiation. However the first Ramsey $\pi/2$ pulse would only reverse this step; therefore, it is advantageous to take the creation of $\psi_{max}$ as the first Ramsey $\pi/2$ pulse. The second Ramsey pulse (after time $T_R$) can be applied directly with radiation at frequency $\omega_R$. The phase of this pulse (on each ion) must be fixed relative to the phases of the radiation used to create $\psi_{max}$. In general, the relation between these phases and $\phi_0$ (Eq. (1)) will depend on the relative phases of the fields at the positions of each of the ions [5,21]. This will lead to a signal $S = \langle\tilde{O}\rangle \propto cos(L\Delta\omega T_R + \phi_f)$ where $\Delta\omega \equiv \omega_R - \omega_0$ and where $\phi_f$ depends on all of these phases.

We can extract $\omega_0$ (relative to $\omega_R$) by measuring $\langle\tilde{O}\rangle$ as a function of $T_R$, with $\Delta\omega$ fixed. This can be further simplified by measuring the signal for two values of $T_R$, $T_{R2} \gg T_{R1}$, where $\langle\tilde{O}\rangle \simeq 0$. Unfortunately, if the measured signal has a systematic bias as a function of $T_R$, an error in the determination of $\Delta\omega$ will result. This might happen, for example, if the ions heat up during application of the Ramsey radiation and a loss of signal occurs due to a reduced overlap between the ions and the laser used for fluorescence detection of the states. This problem could be overcome by measuring $\langle\tilde{O}\rangle$ for two values of $\omega_R$, $\omega_{R1}$ and $\omega_{R2}$ such that $\omega_{R1} - \omega_0 \simeq -(\omega_{R2} - \omega_0)$ (determined by the above method), and two values of $T_R$, $T_{R1} \ll T_{R2}$. We then iterate the following steps: (1) we make $\langle\tilde{O}((\omega_{R1} - \omega_0)T_{R1})\rangle \simeq \langle\tilde{O}((\omega_{R2} - \omega_0)T_{R1})\rangle$ by adjusting the phase of the final $\pi/2$ pulse to make $\phi_f \to 0$. This will take a negligible amount of time since $T_{R1} \ll T_{R2}$. (2) We make $\langle\tilde{O}((\omega_{R1} - \omega_0)T_{R2})\rangle \simeq \langle\tilde{O}((\omega_{R2} - \omega_0)T_{R2})\rangle$ by adjusting $\omega_{R1}$ and/or $\omega_{R2}$ to force $\omega_{R1} - \omega_0 \to -(\omega_{R2} - \omega_0)$. This gives $\omega_0$ relative to $\omega_R$ even if $\langle\tilde{O}\rangle$ has a systematic bias as a function of $T_R$.

An alternative solution is suggested by Huelga, *et al.* [20]. After $T_R$, instead of applying a $\pi/2$ pulse of radiation at frequency $\omega_R$, we apply the time-reversed sequence of operations which created $\psi_{max}$. This has the advantage of cancelling out all of the CN phases that contribute to $\phi_0$ and maps the signal ($\propto cos(L\Delta\omega T_R)$) onto a single ion (whereupon $S_z$ is measured for that ion). This also reduces the problem of detection efficiency to one ion rather than $L$ ions. The disadvantage of this technique is that for large values of $T_R$, the motional mode used for logic will, most likely, have to be recooled. This would require sympathetic cooling with the use of an ancillary ion which, to avoid the decohering effects of stray light scattering on the logic ions, might have to be another ion species [5].

A more serious limitation to the accurate determination of $\omega_0$ is that, in practice, $\psi_{max}$ will be realized only approximately and the state produced by the logic operations

will also be composed of states other than the $|\uparrow\rangle_1|\uparrow\rangle_2\cdots|\uparrow\rangle_L$ and $|\downarrow\rangle_1|\downarrow\rangle_2\cdots|\downarrow\rangle_L$ states; these other states will have a definite phase relation to the $|\uparrow\rangle_1|\uparrow\rangle_2\cdots|\uparrow\rangle_L$ and $|\downarrow\rangle_1|\downarrow\rangle_2\cdots|\downarrow\rangle_L$ states. Consequently, in general, the signal produced with either implemenation will be of the form

$$S = \sum_{p=1}^{L} C_p cos(p\Delta\omega T_R + \xi_p). \tag{2}$$

To accurately determine $\Delta\omega$, it will be necessary to Fourier decompose $S$. Since this will take more measurements, the advantages of using entangled states will be reduced.

In spite of this, in some applications, it will be useful to determine changes in $\omega_0$ with respect to some external influence. For example, we might want to detect changes in $\omega_0$ caused by changes in an externally applied field. In this case, as long as $|C_p| \ll 1$, for all $p < L$, we derive the benefits of entangled states (assuming the decoherence time is longer than $T_R/L$) by measuring changes in $S$ for a particular value of $T_R$.

## 5. Experiments

As usual, our enthusiasm for implementing these schemes far exceeds what is accomplished in the laboratory; nevertheless, some encouraging signs are apparent from recent experiments. In Ref. [22], all motional modes for two trapped ions have been cooled to the ground state. The non-center-of-mass modes are observed to be much less susceptible to heating, suggesting the use of these modes in quantum computation or quantum state engineering. In Ref. [21], we describe logic operations which enabled $\psi_{max}$ for $L = 2$ to be generated with modest fidelity ($\simeq 0.7$). For small $L$, it is only necessary to *differentially* address individual ions to create $\psi_{max}$ and for $L = 2$, general logic can be realized even if the laser beams cannot be focused exclusively on the individual ions [21]. For general logic on more than two ions, two avenues are being pursued. For modest numbers of ions in a trap, the Cirac-Zoller scheme of individual addressing with the use of focused laser beams is the most attractive. Current efforts are devoted to obtaining sufficiently strong focusing to achieve individual ion addressing in a relatively strong trap where normal mode frequencies are relatively high ($\simeq$ 10 MHz) in order to maximize operation speed. Alternatively, general logic on many ions could be accomplished by incorporating accumulators [5], and using differential addressing on two ions at a time. This idea might be realized by scaling up a version of a linear ion trap made with lithographically deposited electrodes as we have recently demonstrated [16,23]. Concurrently, efforts are being devoted to the investigation (and hopefully, elimination) of mode heating [5] for different electrode surfaces and dimensions.

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