

# PSEUDO-RANDOM CODE CORRELATOR TIMING ERRORS DUE TO MULTIPLE REFLECTIONS IN TRANSMISSION LINES

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## Abstract

*Multiple reflections in transmission lines can cause an amplification of timing errors associated with the delay sensitivity to environmental effects of the transmission lines. This multiplicative effect arises because the timing error in an early/late code correlator is a function of the relative RF carrier phase, the relative power levels, and the relative time delay between the direct and reflected signal.*

## INTRODUCTION

The theoretical analysis of the timing errors introduced by the correlator sensitivity to multipath signals is well known [1,2]; however, the correlator timing errors due to multiple signal propagation in transmission lines has not been fully appreciated by the timing community. Multiple reflections of a spread-spectrum-modulated signal in a cable can cause large biases in the correlation function. These biases are tracked by the delay-locked loop (DLL), resulting in timing errors. Multiple reflections of a sinusoidal signal can be characterized by the voltage standing wave ratio (VSWR). Destructive or constructive interference may occur depending on the length of the transmission line, the source and load terminations, and the frequency of the signal. In a similar manner a spread spectrum signal will be influenced by the presence of a reflected signal that has an arbitrary delay, phase, and amplitude with respect to the direct signal.

## MULTIPLE REFLECTIONS IN TRANSMISSION LINES

Multiple reflections in transmission lines can occur when there is a mismatch in impedance and a signal reflects off the load back toward the source. If the backward propagating signal is reflected forward again, it will become an interfering reflected signal that is smaller in amplitude, delayed in time, and arbitrary in phase as compared to the direct signal. If the reflections occur at the ends of the transmission line, the multipath signal is delayed from the direct signal by twice the propagation delay of the transmission line. For simplicity we will assume that the reflections occur at the ends of the transmission line and only one interfering signal is dominant. This is not necessarily

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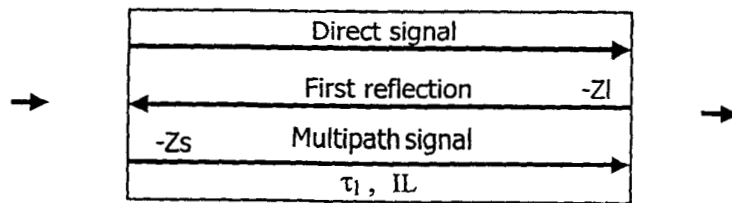


Figure 1. Graphical representation of a cable.

true because damage or dielectric imperfections along the length of the transmission line may cause an impedance change that results in multiple reflections. Figure 1 is a graphical representation of a cable, with a direct signal and an interfering signal that results from reflections due to an impedance mismatch at the ends of the cable.  $Z_s$  is the return loss of the source and  $Z_l$  is the return loss of the load. The power of the direct signal is  $P$  and the power of the reflected interfering signal is  $P_m$ . The cable has an electrical length  $\tau_1$  and the insertion loss is  $IL$ . The relation between the power level of the direct signal and the level of the reflected signal is given by

$$P_m = P - 2 \cdot IL - Z_s - Z_l. \quad (1)$$

## EARLY/LATE CODE CORRELATOR

A simplified model of a noncoherent early/late (E/L) correlator is described below. The complete derivations of these equations are presented in references [1,2]. In Figure 2, the E/L code loop detector is displayed in its simplest form. The input signal  $Y(t)$  is split and is the input to the early and the late correlator channels. The reference pseudo-random code sequence (PN) to the early and late correlators is spaced by a chip. The time variable is  $t$ , the reference sequence time estimate is  $T$ , and  $T_c$  is the chipping period. In both the early and late correlation channels, the output of the correlator is band-pass filtered and squared. The code detector error signal  $S(\epsilon)$  is the difference of the early and late correlation channel outputs. The error signal  $S(\epsilon)$  is driven to zero by the DLL in normal tracking operation. The tracking loop error is  $(t-T)$  and is given by  $\epsilon$ .

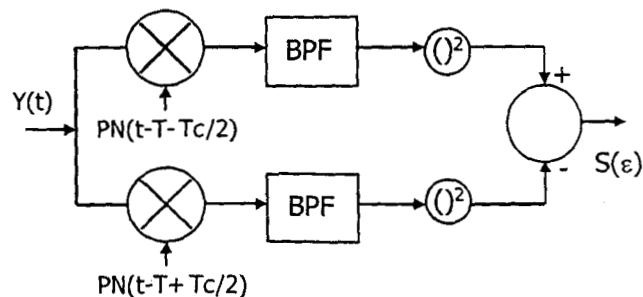


Figure 2. Code loop detector model.

The input signal  $Y(t)$  is the sum of the direct signal and one interfering signal resulting from a reflection off the ends of a cable as described in the preceding section.  $Y(t)$  is given by

$$Y(t) = \sqrt{2P} \cdot PN(t-T) \cdot \cos(2\pi f \cdot t + \phi) + \sqrt{2P_m} \cdot PN(t-T-\tau_m) \cdot \cos(2\pi f \cdot t + \phi + \phi_m) \quad (2)$$

where  $f$  is the direct signal carrier frequency,  $\phi$  is the carrier phase of the direct signal,  $\tau_m$  is the time delay of the reflected interfering signal with respect to the direct signal, and  $\phi_m$  is the relative carrier phase between the direct and the interfering signal.

The error signal from the code loop detector  $S(\epsilon)$  is given by

$$S(\epsilon) = P \cdot R_{PN}^2(\epsilon - \frac{T_c}{2}) + P_m \cdot R_{PN}^2(\epsilon + \tau_m - \frac{T_c}{2}) + 2P \cdot R_{PN}(\epsilon - \frac{T_c}{2}) \cdot P_m R_{PN}(\epsilon + \tau_m - \frac{T_c}{2}) \cdot \cos(\phi_m) - P \cdot R_{PN}^2(\epsilon + \frac{T_c}{2}) - P_m \cdot R_{PN}^2(\epsilon + \tau_m + \frac{T_c}{2}) - 2P \cdot R_{PN}(\epsilon + \frac{T_c}{2}) \cdot P_m R_{PN}(\epsilon + \tau_m + \frac{T_c}{2}) \cdot \cos(\phi_m) \quad (3)$$

when the input to the system is  $Y(t)$  as described in equation 2. The function  $R_{PN}(\epsilon)$  is the auto-correlation function of  $PN(t-T)$ . The magnitude of the timing error is a function of  $\phi_m$ , RF carrier phase between direct and reflected signals,  $\tau_m$  the time delay between the direct and reflected signal, and  $P_m/P$  the ratio of reflected signal power level to direct signal power level.

During tracking mode the delay lock loop adjusts the time estimate of the reference,  $T$ , such that  $S(\epsilon)$  is zero. A plot of the correlator loop detector error is shown in Figure 3 for the case where there is no interfering signal present. A plot of the correlator loop detector error is shown in Figure 4 for the case where a reflected interfering signal is present. Note that the  $S(\epsilon)$  function is distorted and that there is a timing error bias. The timing error during tracking is given by solving for  $\epsilon$  for the condition  $S(\epsilon)=0$ .

Since the magnitude of the timing error is a function of  $\phi_m$ ,  $\tau_m$ , and  $P_m/P$ , the

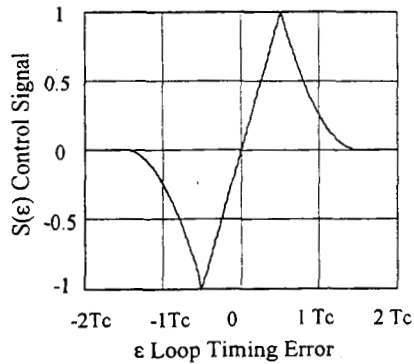


Figure 3.  $S(\epsilon)$  without multipath.

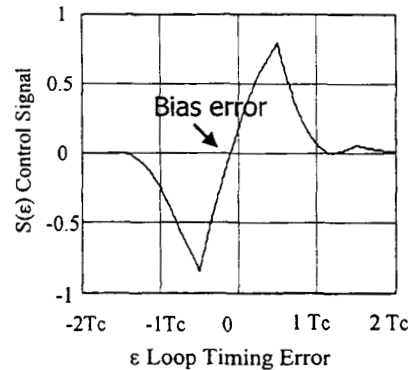


Figure 4.  $S(\epsilon)$  with multipath.

timing error will be a function of the propagation path through the circuit elements as well as a function of the various termination impedances when multiple reflections are present. If the timing error did not change, a simple calibration procedure could be established to eliminate the bias; however, small changes in propagation path or termination impedance may occur due to environmental effects and the calibration procedure would be ineffective. A plot of the timing error as a function of  $\tau_m$  is shown in Figure 5, for the case of an interfering signal that is 30 dB lower in power than the direct signal. The carrier frequency is 70 MHz and the chip rate is 2.5 MHz. Note that a 7 ns propagation delay can cause a 13 ns delay change as measured by the correlator.

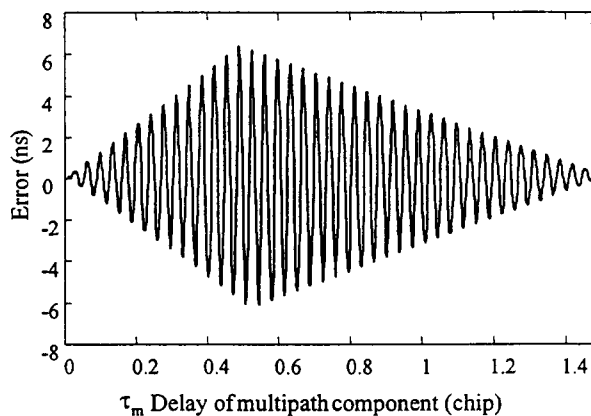


Figure 5. Timing error due to multipath, as a function of the electrical length of a cable.

In addition, you can increase the length of the propagation path, and the correlator, depending on the RF carrier phase, may measure a shorter delay. In the next section a more dramatic numerical example will be presented to illustrate how small physical delay changes due to environmental effects such as the temperature coefficient of a cable can be amplified by the correlator into unexpectedly large timing errors in a GPS receiver.

## A GPS EXAMPLE

The relative RF carrier phase between the direct and reflected signals is a function of the propagation path and the source and load terminations. Aging, temperature, and other environmental effects affect the propagation delay through the cable and give rise to larger than expected timing errors. For example, consider a typical GPS timing receiver system that is composed of an antenna and low noise amplifier, a 30 m low loss cable, and a receiver located indoors. The carrier frequency is 1575 MHz and the C/A code has a chip rate of 1.023 MHz that corresponds to chip period of 978 ns. A good cable may have a delay temperature coefficient of 7 ppm/°C. The propagation delay through the 30m cable is 110 ns and will have variations on the order of 0.8 ps/°C. Assuming that the multi-path signal in the cable is caused by impedance mismatch at the antenna and receiver and that it is 30 dB attenuated with respect to the direct signal, we

can calculate the timing error for a 10 °C temperature rise of the cable. The propagation of the direct signal will change by 8 ps and the propagation of the multi-path signal will change by 16 ps. The correlator, however, will indicate a change of -800 ps. This example does not illustrate a minimum or maximum error. The magnitude of the error for the same cable and temperature excursion will be different for a small increase or decrease in overall length.

## CONCLUSION

Propagation of multiple reflections in transmission lines can cause larger than expected timing errors. These timing errors are a function of the propagation path and are sensitive to environmental effects such as aging, temperature, and humidity. Good matching and care in component selection are essential in trying to minimize multiple reflection signal propagation. Temperature compensation of these timing errors may not be effective, because the temperature coefficient of delay is also a function of the propagation path. In fact, the temperature coefficient of the phase delay through a system is not the same as the temperature coefficient of delay as measured by a PN code correlator.

In some systems, sweeping the carrier frequency can be used to evaluate the magnitude of the multipath environment. In other systems, a careful selection of high phase stability cable and a judicious choice of cable length may minimize the multipath problems.

## REFERENCES

1. Holmes, J. K., *Coherent Spread Spectrum Systems*, John Wiley and Sons, Inc., New York, 1982.
2. Van Dierendonck, A. J., "GPS Receivers," *Global Positioning Systems: Theory and Applications*, Vol. 1, American Institute of Aeronautics and Astronautics, Inc., Washington, 1996.