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CORRELATION BETWEEN UPPER AND LOWER NOISE SIDEBANDS

F. L. Walls

National Institute of Standards and Technology
325 Broadway, Boulder CO 80303, USA

Abstract

Experimental measurements, supported by a simple model, are used to show that the upper and lower phase modulation (PM) noise sidebands are always equal and 100 % correlated, independent of whether the noise power originates from multiplicative or additive processes. Similarly we show that the upper and lower amplitude modulation (AM) noise sidebands are also equal and 100 % correlated, independent of whether the noise power originates from multiplicative or additive processes. Moreover the single sideband AM (PM) noise is always equal to ½ the total AM (PM) noise. Although the upper and lower PM (AM) noise sidebands are equal and correlated for broadband additive noise, the phase between the AM and the PM sidebands varies randomly with time. These conclusions still hold even when the RF noise sidebands are not symmetric about the carrier.

Introduction

Figure 1 shows a typical spectrum of a noisy signal. The upper and lower sidebands contain power from both phase modulation PM and amplitude modulation (AM). Equation (1) is used to describe the waveform when the total AM and PM noise power is small compared to the power in the carrier [1].

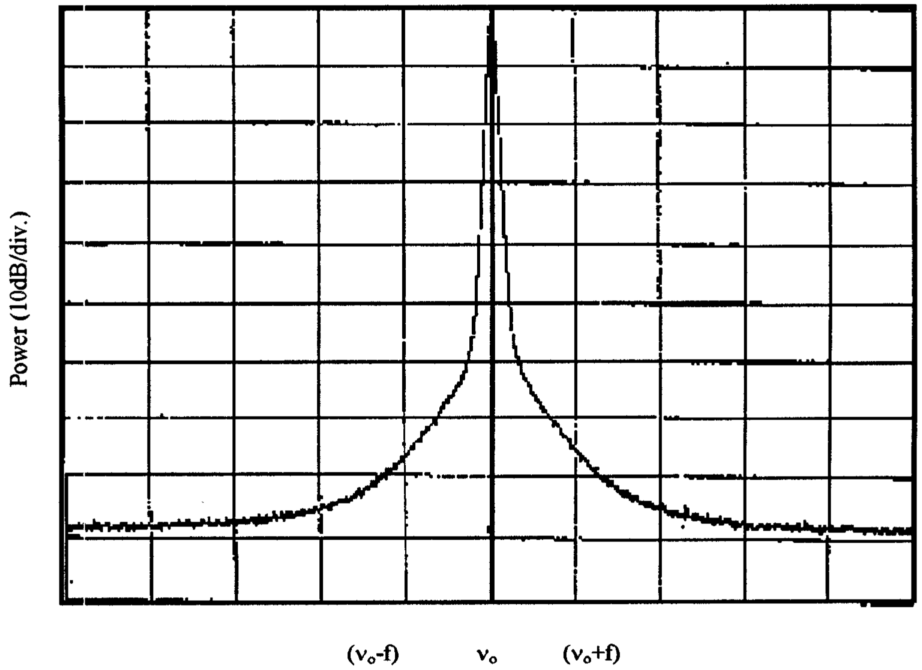

Figure 1. Typical power spectrum of a noisy signal. The noise sidebands contain power from both AM and PM noise.

$$V(t) = [V_o + \varepsilon(t)] \mathrm{Cos}\,(2\pi\upsilon_o t + \phi\,(t)), \qquad (1)$$

where $\varepsilon(t)$ is the fluctuations in the amplitude from the average amplitude V_o, $\phi\,(t)$ is the phase fluctuations about the average phase $2\pi\upsilon_o t$. The average carrier frequency is υ_o. From these definitions, the average of $\varepsilon(t)$ and $\phi(t)$ is approximately 0. AM and PM noise are defined by

$$S_a(f)=\left(\frac{\varepsilon(f)}{V_0}\right)\phi^2(f)\frac{1}{BW}, \qquad (2)$$

$$S_\phi(f)=2L(f)=\phi^2(f)\frac{1}{BW}. \qquad (3)$$

The PM noise and AM noise in most devices have two distinct regions. Close to the carrier the noise originates from multiplicative processes and is roughly independent of carrier power [2]. Far from the carrier the addition of uncorrelated noise results in an equal amount of PM and AM noise which scales as the inverse of the carrier power. These two basic processes are illustrated in Fig. 2a and 2b.

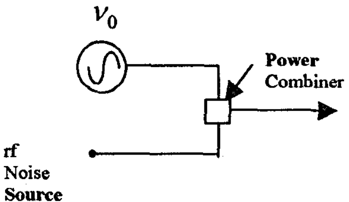

Figure 2a

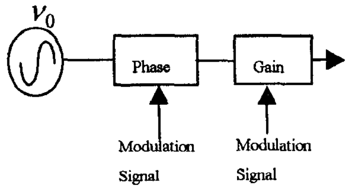

Figure 2b

Figure 2a. Schematic representation for additive noise that produces equal AM and PM. Figure 2b. Schematic representation of mulplicative AM and PM noise

A question arises on the details of the conversion of the PM noise about the carrier into the detected baseband signal. Namely, is there a difference between the detection of the obviously correlated noise sidebands from the multiplicative noise and the sidebands generated from the uncorrelated noise? If so, one might expect a 3 dB difference between the single sideband noise viewed on a spectrum analyzer (SA) and that detected on a linear detector. To answer this question we have performed a number of detailed tests using coherent AM, PM, and single sideband modulation of the carrier. From the results of these test we have constructed a simple model that shows that the upper and lower PM sidebands are always equal and 100 % correlated whether the modulation originates from multiplicative or additive noise processes. This is true even when the rf spectrum about the carrier is not symmetric. The same is true for the AM noise sidebands.

Experimental measurements

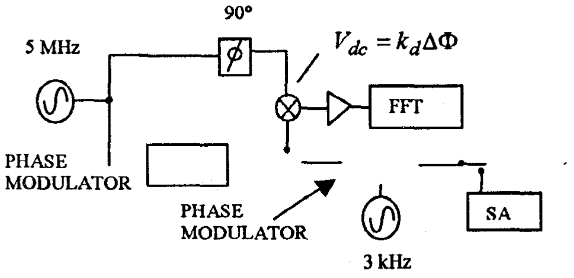

Figure 3. Block diagram of the experimental setup to create and observe coherent PM. The spectrum can be observed on the spectrum analyzer (SA) or with a linear phase detector and a FFT spectrum analyzer.

Figure 3 shows the experimental setup for the measurements of coherent PM. The switch could be changed to observe the spectrum on a traditional swept spectrum analyzer (SA), or to detect the PM with a phase detector. The output of the phase detector was calibrated in volts/radian using the NIST PM/AM noise standard [3]. The uncertainty of this calibration was ± 0.3 dB. The modulation frequency was chosen to be far enough from the carrier that the sidebands were readily separated from the noise of the source. Figure 4 depicts the observed spectrum. The SA indicated that the upper and lower sidebands were 31 dB below the carrier (-31 dBc) and equal to within ± 0.2 dB. The phase detector indicated that 1/2 $S_\phi(f)$= L(f) was - 31 dBc with an uncertainty of ± 0.3 dB. This measurement showed that coherent PM is detected with the same sensitivity as the broadband Gaussian noise used for the calibration.

Figure 5 shows the experimental setup for the measurements of coherent AM. The switch could be changed to observe the spectrum on a traditional swept SA or to detect the AM with a diode detector. The amplitude detector

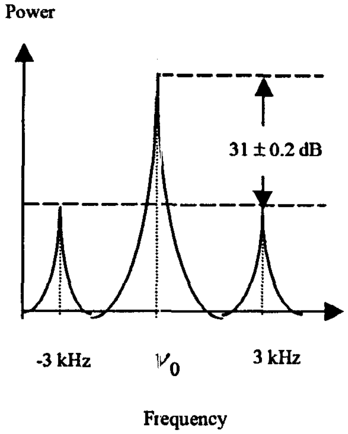

Figure 4. Spectrum observed using the PM setup of Fig. 3 and also the AM setup of Fig. 5.

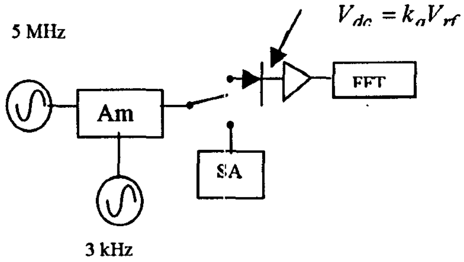

Figure 5. Block diagram of the experimental setup to create and observe coherent AM. The spectrum can be observed on the spectrum analyzer (SA) or with an amplitude detector and a FFT spectrum analyzer.

was calibrated using the NIST PM/AM noise standard [3]. The uncertainty of this calibration was ± 0.3 dB. The modulation frequency was chosen to be far enough from the carrier that the sidebands were readily separated from the noise of the source. The SA indicated that the upper and lower sidebands were -31 dBc and equal to within ± 0.2 dB. The observed spectrum is identical to that of Figure 4. The phase detector indicated that 1/2 $S_a(f)$ = - 31 dBc with an uncertainty of ± 0.5 dB. This measurement showed that coherent AM is detected with the same sensitivity as the broadband additive Gaussian noise used for the calibration.

Figure 6 shows the experimental setup for the measurements of coherent single sideband (SSB) modulation. The switch could be changed to observe the spectrum on a traditional swept SA or to detect the PM with a phase detector. The output of the phase detector was calibrated in volts/radian using the NIST PM/AM noise standard [3]. The uncertainty of this calibration was ± 0.3 dB. Figure 7 depicts the observed spectrum. The SA indicated that the upper sideband was -31 dBc. The lower sideband was -61 dBc. The phase detector indicated that L(f) was - 37 dBc with an uncertainty of ± 0.3 dB.

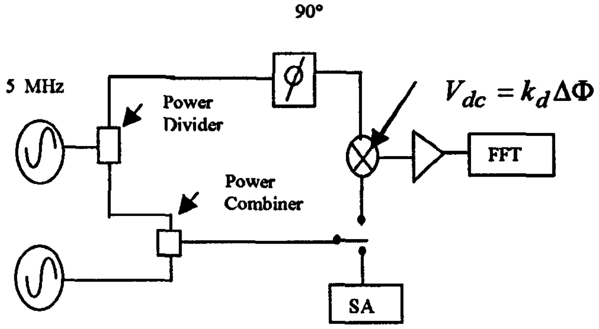

Figure 6. Block diagram of the experimental setup to create and observe coherent single sideband (SSB) modulation. The spectrum can be observed on the spectrum analyzer (SA) or with a linear phase detector and a FFT spectrum analyzer.

Figure 8 shows the spectrum that results by following the SSB generation of Fig. 7 by a limiter that removes the AM. L(f) was still - 37 dBc. The explanation for these results is that the SSB results from a special phase relationship between equal upper and lower PM and AM sidebands. Since the power must be divided equally into 4 signals, L(f)= 1/2 $S_a(f)$ = - 31 - 6= - 37 dBc.

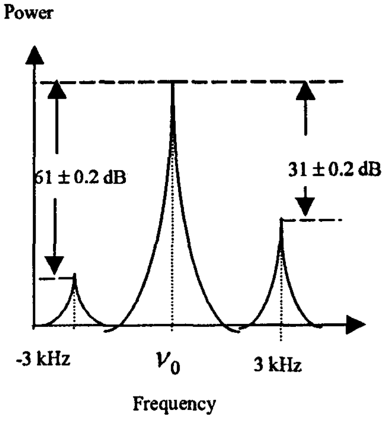

Figure 7. Spectrum observed on the spectrum analyzer for the SSB setup of Fig. 6.

This is further explored in Figs. 9 and 10 where we see the upper and lower PM and AM sidebands depicted as vectors rotating about the carrier signal at a rate $\Omega=2\,\pi(3$

kHz) radians/s. The upper sideband rotates clockwise (CW) while the lower sideband rotates counter clock wise (CCW). Figure 9 shows the condition where the SSB signal is producing PM but no AM. Figure 10 shows the vectors $\pi/2\Omega$ s (90°) later. In this case the SSB signal is producing AM but no PM. In both Figs. 9 and 10, summing the AM and PM vectors results in a cancellation of the lower (CCW) vector.

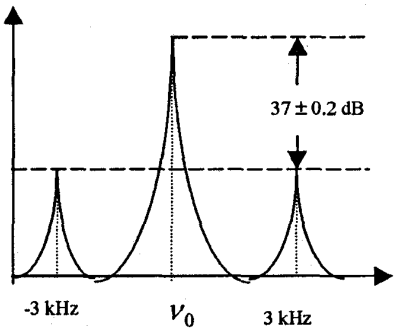

Figure 8. Spectrum observed on the spectrum analyzer when an AM limiter is added after the power combiner of the SSB setup of Fig. 6.

The results of the SSB example can now be applied to additive noise. Consider an upper noise sideband at the carrier + f, a width 1 Hz, and power $-\beta$ dBc. It will produce equal upper and lower PM sidebands and upper and lower AM noise sidebands that are $-\beta$-6 dBc. If there is an equal amount of noise at the lower sideband at the carrier - f, with a width 1 Hz, and power $-\beta$ dBc, then the combined PM and AM are $L(f) = 1/2\ S_a(f) = -\beta-3$ dBc. The powers add because the upper sideband and lower sideband noise is uncorrelated. Although the PM and AM noise sidebands are equal, the phase between them varies with time in a random way.

Conclusion and Discussion

We have shown that the upper and lower PM sidebands must always be correlated and equal, therefore, the detection process is always the same for multiplicative and additive noise. Similarly we have shown that the upper and lower AM sidebands must always be correlated and equal, and therefore, the detection process is always the same for multiplicative and additive noise. Moreover the SSB AM (PM) noise is always equal to ½ the total AM (PM) noise. These conclusions still hold even when the RF noise sidebands are not symmetric about the carrier.

Acknowledgments

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2. See for example F. L. Walls, E. S. Ferre-Pikal, and S. R. Jefferts, "The origin of 1/f PM and AM noise in Bipolar Junction Transistor amplifiers," IEEE Trans. UFFC 44, 326-334, 1997.
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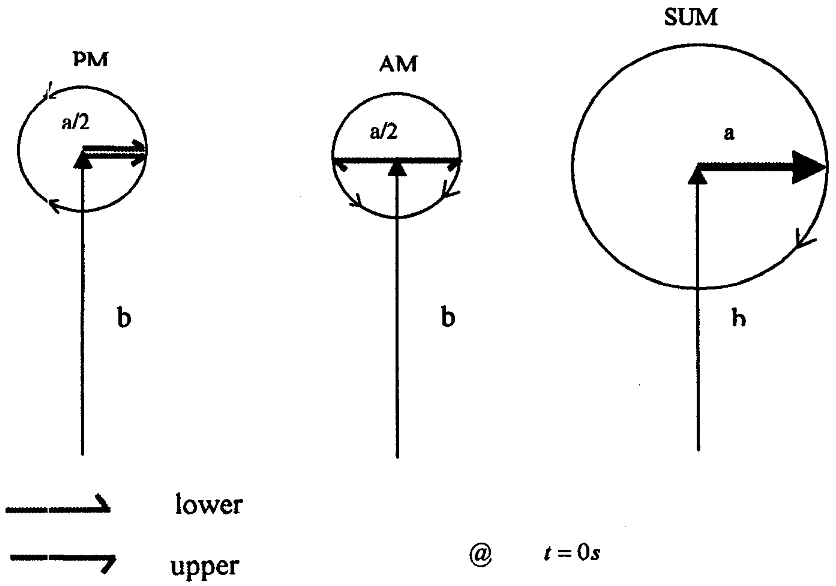

Figure 9. Vector representation of the SSB modulation for Figure 6 and a time when the modulation is causing PM but not AM. Vector b represents the carrier and the small vectors labeled a/2 represent the upper and lower AM and PM components. The sum of the four a/2 vectors is just a.

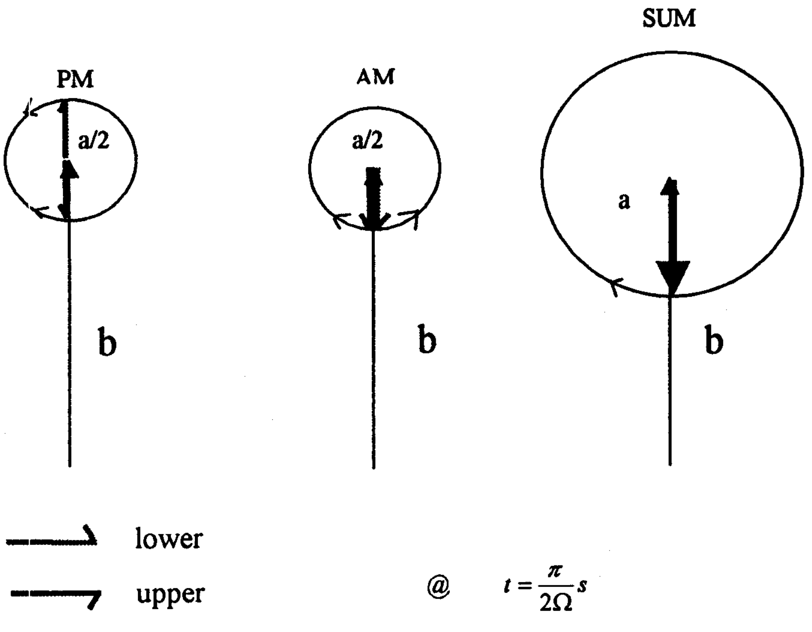

Figure 10. Vector representation of the SSB modulation for Figure 6 and a time when the modulation is causing AM but not PM. Vector b represents the carrier and the small vectors labeled a/2 represent the upper and lower AM and PM components. The sum of the four a/2 vectors is just a.