NOISE IN OSCILLATORS WITH TWO ASYNCHRONOUS OSCILLATIONS

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ABSTRACT

This paper describes noise phenomena in oscillators with two degrees of freedom, sustaining two-frequency asynchronous oscillations. The oscillator is comprised of two parallel high-Q-resonant, RLC circuits connected in series to an active voltage-controlled one-port device with a symmetric volt-ampere characteristic having a nonlinearly described by a function arctg(x). The oscillator under analysis is isochronous, as it uses a purely resistive active device and has no additional phase shifts in the positive feedback loops. Phase noise at the two frequencies due to white noise sources is uncorrelated, while amplitude noise shows some mutual correlation. The main features of the noise characteristics arise due to the interaction of the two asynchronous oscillations via the common bias.

1. INTRODUCTION

Asynchronous oscillation in dynamic systems with two degrees of freedom is a classical problem of nonlinear oscillation theory [1]. Aside from pure scientific interest, such oscillations are worthy of our attention, since they can be used to improve frequency stability of precision quartz oscillators [2-7] and can cause distortions in microwave and other sources [8].

A typical arrangement of the oscillator under analysis (Fig. 1) is comprised of two parallel high-Q-resonant RLC circuits connected in series to an active voltage-controlled non-linear one-port device. If the tank circuits resonant frequencies ν_1 and ν_2 are incommensurable and sufficiently separated, the total voltage waveform across the active device (AD) in a steady-state regime u(t) consists of a bias voltage u_o and one or two fundamental voltage components $u_1(t)$ and $u_2(t)$:

$$u(t) = u_1 + u_2 + u_o = U_1 \cos(2\pi\nu_1 t + \phi_1) + U_2 \cos(2\pi\nu_2 t + \phi_2) + u_o. \quad (1)$$

In general, the dc term u_o is not necessarily identical to the applied bias voltage U_{00} since there may be some rectification of the rf voltage components.

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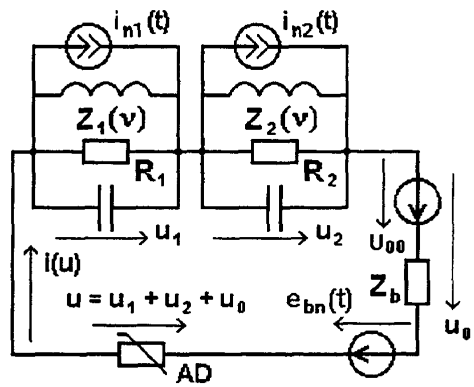

Figure 1. An equivalent network of an oscillator sustaining two-frequency asynchronous oscillations with added noise sources.

The kind of oscillation which actually occurs in the network depends, first of all, on the particular AD non-linearity. Van der Pol came to the conclusion that stable oscillation at two asynchronous frequencies simultaneously could not occur if the AD volt-ampere characteristic (VAC), i(u), was represented by a cubic relation [1]. Instead, the device oscillates at either one or the other frequency, depending on initial conditions. It was soon was discovered that this result is only a consequence of the specific form of VAC and that for an appropriate VAC, simultaneous oscillation at two frequencies is possible. Apparently, the first time this was studied was by Chikhachev [9]. Twelve years later Skinner [10] showed that for stable asynchronous oscillations to occur, there must be at least a fifth-order term in the power-series expansion of VAC, that is

$$i(u) = a_o + a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4 + a_5 u^5, \quad (2)$$

where the parallel resonant (antiresonant) impedances of the two tank circuits, R_1 and R_2, be not too unequal. However, as pointed out by Schaffner [11] and Anisimov [9], these conditions for oscillations are not self-starting because the effective resistance of the system is positive for small oscillations. A 5th-order polynomial VAC, which lead to sharply excited asynchronous oscillations, can only have about the same values for the stationary amplitudes U_1 and U_2. This restriction, however, does not apply if VAC is represented by a 7th-order polynomial [8].

The common peculiarity of analysis in [8, 9, 11] is a suggestion that the AD output current responds to an instantaneous value of the applied voltage while parameters of the VAC and an operating point stay unchanged. Disman and Edson [10] checked the last condition and found that replacement of a constant bias by an automatic one, when the bias is derived by rectification of the rf voltage applied to AD, changes the oscillator properties drastically. In particular, mild excitation of stable asynchronous oscillations are now possible. Additional publications in this area revealed interesting details about asynchronous oscillations in different cases but did not change the understanding.

2. STEADY STATE REGIMES

In contrast to other work, the AD described here has a symmetric VAC, which in our opinion, is a better match to many practical cases. A specific form of the VAC nonlinearity is represented by the function

$$i(x) = A[(2/\pi)\,\mathrm{arctg}(x) + 1], \qquad (3)$$

where $x = C_n \times u$ is the normalized total instantaneous voltage across the one-port AD, and C_n is the normalizing coefficient. The VAC chosen has odd symmetry relative to the argument value $x = 0$.

The analysis is based on the method of symbolic shortened equations (SSE method) developed by Evtyanov [12]. The partial oscillations without noise components are written as

$$u_k(t) = U_k\cos\Psi_k(t) = U_k\cos[2\pi\nu_k t + \phi_k(t)]. \qquad (4)$$

Bearing in mind that $Q_{1,2} \gg 1$ for resonators of interest, the amplitudes $U_{1,2}(t)$ and the phases $\phi_{1,2}(t)$ can be considered slowly varying functions of time. This permits us to replace instantaneous values of variables by their envelopes and thus to lower the order of the initial differential equations. Using the SSE method to derive simplified differential equations, requires us to approximate the circuit impedance in the vicinity of all operating frequencies and use the expression obtained as differential operators acting on the complex amplitudes $\mathbf{U}_k(t) = U_k(t)\exp[\phi_k(t)]$.

The main equations describing behavior of the oscillator can be obtained from its equivalent circuit in Fig. 1. For single tank circuits, the approximated impedances, $\mathbf{Z}_1$ and $\mathbf{Z}_2(\mathbf{p})$, have the form

$$\mathbf{Z}_i = R_i/(1 + T_i\mathbf{p}), \qquad (5)$$

where $R_k = c^2_k\,\rho_k\,Q_k$ is the parallel resonant impedance, c_k is the coupling coefficient, $\rho_k = (L_k/C_k)^{1/2}$ is the characteristic impedance, Q_k is the loaded Q-factor, $T_k = 2Q_k/\omega_{0k}$ is the resonator time constant, $\mathbf{p} = d/dt$ is the differential operator with respect to time, $k = 1, 2$. In a steady-state regime, $\mathbf{p} = j\Omega$, where $\Omega = 2\pi f = \omega - \omega_{0k}$ is the angular offset frequency,
$\omega_{0k} = 2\pi\nu_{0k} = (L_kC_k)^{-1/2}$ is a self-resonant angular frequency, and $f = \nu - \nu_{0k}$ is the frequency shift in hertz with respect to the current frequency ν and also a Fourier frequency in the noise analysis.

The full set of the equations with the noise sources consists of two equations describing the asynchronous carriers, two equations for the frequency shifts $\dot\phi_{fk}$ due to noise and the relationship for the bias circuit

$$T_k\dot X_k = (G_kR_{kn} - 1)X_k + I_{\|k};$$

$$\dot\phi_{fk} = I_{\perp k}/I_{1k}T_k = I_{\perp k}/G_k(X_k)X_kT_k; \qquad (6)$$

$$T_b\dot X_0 = X_{00} - X_0 - R_{bn}I_0 + X_{bn}; \quad (k = 1,2)$$

where $X_k = C_n\times U_k$, $X_{0(0)} = C_n\times U_{0(0)}$ is the normalized values of partial oscillations and bias, X_{00} is the constant part of the resulting bias voltage X_0. $G_k = I_{1k}/X_k$ is the normalized averaged partial transconductances, $_{1k}(X_1,X_2,X_0)$ is the amplitudes of partial fundamental rf currents, and $k = 1, 2$.

The operating frequencies ω_k have no regular shifts regarding ω_{0k} ($\dot\phi_k = \omega_k - \omega_{0k} = 0$) since both partial currents $\mathbf{I}_{1k}$ are in-phase with the voltages $\mathbf{U}_k$.

In Eq. (6) $R_{bn} = C_n\times R_b$ and T_b arise from the expression for bias impedance $\mathbf{Z}_b(\mathbf{p}) = R_b/(1+\mathbf{p}T_b)$ which describes the inertial properties of the bias network.

Noise terms in (6) are represented by slowly varying noise currents in vicinity of the operation frequencies

$$\mathbf{I}_{nk}(\omega) = (I_{\|k} + jI_{\perp k}) \times \exp(j\phi_k), \qquad (7)$$

where X_{bn} is the averaged low frequency noise voltage arising from bias self-noise e_{bn} and AD noise current.

It follows from (6) that in the steady-state regime

$$G_{1(2)}(X_1, X_2, X_0)/G_M = G_{1(2)n} = 1/FR_{1(2)}, \qquad (8)$$

where $FR_k = C_n \times G_M \times R_k$ are the partial regeneration factors representing maximal small signal gain of the partial positive feedback loops, $G_M = 2A/\pi$ is the maximal small signal value of G_k which is reached when $X_0 = X_{1,2} = 0$.

The nonlinear functions $G_{1(2)n}(X_1, X_2, X_0)$ define all the main properties of the oscillator. The analysis is based on computer simulation. We find that the oscillator with a constant bias has in general, five stationary points (Fig. 2). Points 2 and 3 describe stable one-frequency regimes while the three points 1, 4, and 5 corresponded to two-frequency solutions. The central point 1 is stable but the biharmonic oscillation is not self-starting (so called *sharp* excitation) since the positive feedback loop

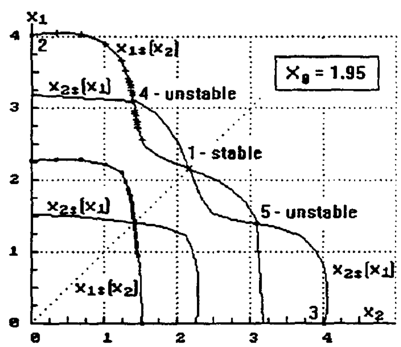

Figure 2. An example of symmetric joint steady-state solutions.

gain is less than unity for small X_1 and X_2. If exited, the two-frequency oscillation can only have small stationary amplitudes X_{1s}, X_{2s}, because the stable point disappears when FR_1 and FR_2 differ too much. If $FR_1 = FR_2 = 3$, a symmetric two-frequency regime can be observed for $|X_0| \in (1.7, 2.03)$ as in Fig. 3.

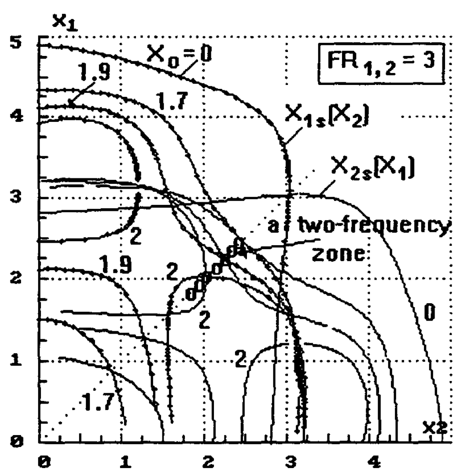

Figure 3. Evolution of the two-frequency oscillator symmetric steady-state regimes with bias change.

In an oscillator with automatic biasing, U_0 depends on a direct current I_0, which in turn, is a function of U_0 and alternating voltages amplitudes. In a steady-state regime the normalized bias is equal to

$$X_0 = X_{00} - R_{bn} \times I_0 (X_1, X_2, X_0). \qquad (9)$$

Fig. 4 illustrates a gradual transition from an unstable to stable joint solution with R_{bn} growth. A symmetric solution is unstable when $R_{bn} = 0$, still unstable for $R_{bn} \le 2.8$, and becomes stable if $2.8 < R_{bn} < 4.46$. The range of mild excitation is narrower and takes place within $2.8 < R_{bn} < 3.6$. If $R_{bn} > 3.6$, in vicinity of the origin there is a zone where regeneration is not sufficient to sustain oscillations. The result is that the excitation is getting sharp. For $R_{bn} > 4.46$ the joint solution completely vanishes.

For the chosen X_{00}, R_{bn}, and FR_1 the joint solution exists in some range of FR_2. If $FR_1 = 3$ and $R_{bn} = 10$, the stable simultaneous asynchronous oscillations exist for $FR_2 \in (2.5, 3.36)$. For even larger FR_2 there is only oscillation at frequency ν_2. If $FR_2 < 2.5$, the only oscillation is at frequency ν_1.

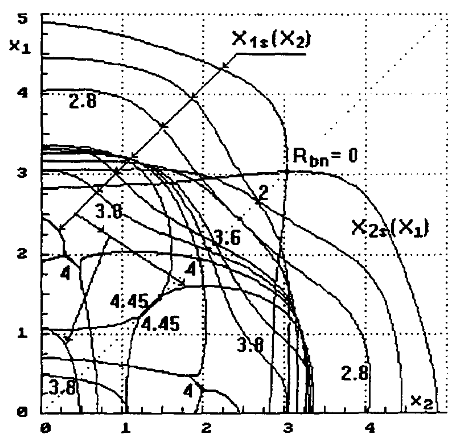

Figure 4. Symmetric steady-state regimes in the case of $FR_1 = FR_2 = 3$, $X_{00} = 0$, and different bias resistors.

3. NOISE CHARACTERISTICS

The noise components $U_f(t)$ and $\phi_f(t)$ arise in the output signal as the oscillator reaction to primary noise sources (Fig. 1). For simplicity consider the case of fundamental δ-correlated noise. Taking each particular variable X_k as the sum of a steady value X_{sk}, and a noise variation X_{fk} using the SSE method, we obtain a system of linear equations for the first approximation to the disturbances

$$\begin{pmatrix} a_{11} - pT_1 & a_{12} & a_{13} \\ a_{21} & a_{22} - pT_1 & a_{23} \\ a_{31} & a_{32} & a_{33} - pT_b \end{pmatrix} \times \begin{pmatrix} X_{f1} \\ X_{f2} \\ X_{f0} \end{pmatrix} = \begin{pmatrix} -I_{||1} \\ -I_{||2} \\ -X_{bn} \end{pmatrix}. \qquad (10)$$

The elements a_{ij} of the matrix (**A**) are first-order partial derivatives of right hand side regular parts of equations

(6) with respect to the variables X_1, X_2, X_0 evaluated at the stationary point. Thus, $a_{33} = -(1 + R_{bn}\frac{\partial I_0}{\partial X_0})$.

Oscillator noise depends on the specific form of VAC. The graphs in Fig. 5-10, obtained for a monofrequency oscillator with a constant bias, allow us to check our «arctg»-type VAC in this sense.

From (10) it follows that the power spectral density of AM noise takes a form

$$S_a(f) = \frac{S_{\parallel}(f)}{(G_1 - \sigma_1)^2 + (2\pi T G_1 f)^2},$$

where $\sigma_1(U) = \partial I_1/\partial U$ is the local first harmonic AD transconductance unlike the averaged one $G_1 = I_1/U$. Random amplitude perturbations decay with a time constant $T_a = T(1 - \sigma/G_1)$ which is regime dependent (Fig. 5). For regime stability $T_a > 0$, that is, $G_1 > \sigma_1$. Under this condition, σ_1 can have any sign and value.

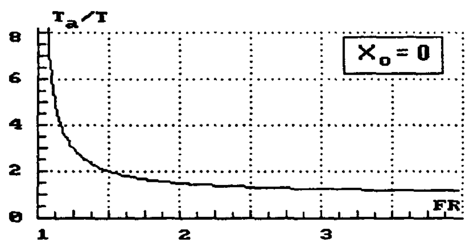

Figure 5. Dependence of the time constant for amplitude noise on a regeneration factor.

The plots of AM noise mean square value $\overline{U_f^2}(U) = \frac{\pi}{2TG_1(G_1 - \sigma)}S_{\parallel}$ and the fractional AM noise $\overline{U_f^2} / U_{ss}^2$ in Fig. 6 demonstrate quite different behavior with respect to FR.

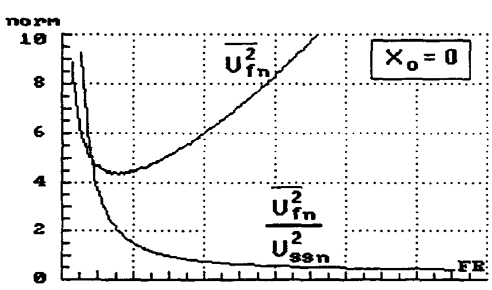

Figure 6. Mean squared AM noise and fractional AM noise vs. regeneration factor.

PM noise is described by $\pounds(f) = S_{\perp}(f) / 2G_1^2 U_{ss}^2 \xi^2$. Here $S_{\perp}(f)$ is the power spectral density of the quadratic noise current component $I_{\perp}$, $\xi = \Omega T = 2\Omega Q/\omega_o = 2fQ/\nu_o$ is the extended tank circuit detuning at the Fourier frequency f. This formula gives the same results as Leeson's formula for the amplifier input [13].

To the first approximation, $S_{\perp} = S_{\parallel} \propto I_0$. The dependence of PM noise on the regime calculated on this basis is shown in Fig. 7. The larger FR the lower the PM noise.

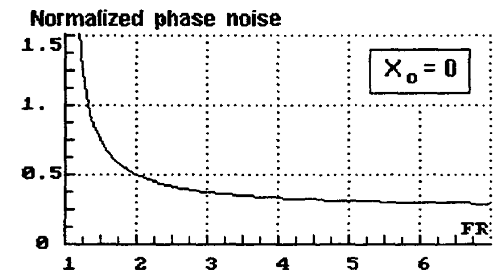

Figure 7. Influence of the regeneration factor on oscillator output PM noise spectral density.

So far our oscillator noise model is based on the assumption of a linear time-invariant system. In reality, any oscillator is a periodically time-varying system and its time-varying nature must be taken into account to permit accurate modeling of noise [14,15]. Periodic nonstationarity of the noise process leads to unequal correlation functions, and so power spectral densities of the in-phase and orthogonal noise current components. In general, there is some mutual correlation. The values of possible AM and PM noise changes due to noise cyclostationarity are illustrated in Fig.8 where we defined [15]

$$\Delta S_a = (F_o + F_{2c}) / F_o, \quad \Delta\pounds = (F_o - F_{2c}) / F_o. \quad (11)$$

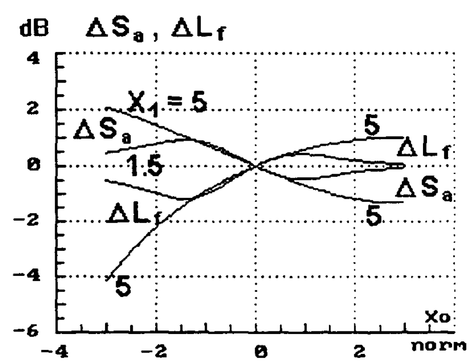

Figure 8. Influence of the regeneration factor on oscillator output phase noise spectral density.

The variations in Fig. 8 do not exceed ±2 to 4 dB. This means, noise cyclostationarity can be neglected when we calculate oscillator noise for the chosen VAC form. In the biharmonic regime the second oscillation influences the average current thus changing the initial noise. However, the effects stipulated by noise cyclostationarity remain the same order of magnitude as in a mono-frequency case.

Consider now PM noise in the symmetric two-frequency regime with $FR_1 = FR_2 = FR$. This leads to $X_{s1(2)} = X_{(s)}$, $I_{11(2)} = I_1$. The information on PM noise regime dependence is represented in Fig. 9 where we drew in logarithmic scale the direct to first harmonic current ratio vs. FR. This ratio reproduces PM noise behavior since $I_{\perp i} \propto I_0$. The solid line characterizes the two-frequency regime, and the marked one "-" the mono- frequency regime.

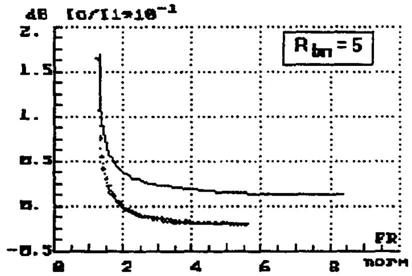

Figure 9. PM noise dependence on the regeneration factor for an oscillator with auto-biasing.

It follows that the symmetric two-frequency regime has 3.to.4 dB higher PM noise than the mono-frequency one. This is because the second oscillation suppresses the rf current more than the direct one.

AM noise in the two-frequency regime, as well in a mono-frequency one, is influenced strongly with bias circuitry inertiality described by the T_b/T_1, T_b/T_2 ratios. According to Eq. (10), AM noise at each frequency depends on the two partial in-phase rf noise components and the noise associated with the bias. Thus, AM noise at the two frequencies is partially correlated.

4. CONCLUSION

Our investigation confirmed that the chosen symmetric VAC allows us to reproduce all the basic effects known earlier for two-frequency asynchronous oscillations. Our results reveal the features of steady-state regimes and noise in such oscillators. Unfortunately, our analysis also predicts that the PM noise is higher in a two-frequency regime than in the mono-frequency regime when the oscillator uses a single AD operating in a symmetric mode.

5. REFERENCES

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