DESIGN STUDIES FOR A LASER-COOLED SPACE CLOCK

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Abstract

We present a theoretical comparison between a TE_{01n} cavity and a traditional Ramsey cavity when used with a laser-cooled atom source in a microgravity clock.

Introduction

As part of our program to put a laser-cooled Cs atomic clock in space, we have studied system designs with regard to the accuracy and stability that can be achieved. These designs look very much like a traditional atomic beam clock with atoms traversing a microwave cavity. A cylindrical TE_{01n} cavity[1], n > 1, appears to be advantageous for several reasons: simple high tolerance fabrication, much reduced end-to-end phase shift, large apertures giving high atom flux. Because of these potential advantages, we explored other possible limitations of this cavity design relative to its use with a very high accuracy frequency standard. One of the important differences between the TE_{01n} cavity and the traditional Ramsey cavity is that the microwave field is always driving atoms in transit through the cavity. The resulting lineshape depends on microwave power and shows significant power broadening and saturation. The limitations discussed below relate to these effects on the lineshape and stability. The stability for a TE_{01n} cavity is compared with that for a Ramsey cavity, which does not show power broadening effects. The resulting stability is better for the Ramsey cavity.

Discussion

In Figure 1 we show a concept drawing for our space clock. Although details may change, this drawing gives a basis for modeling. A ball of atoms is collected, cooled, launched, and transverse cooled before entering the microwave cavity. Cavity designs other than the standard cavity pictured are considered. Laser beams are shown as broad arrows; those on the left are for atom collecting, cooling, and launching. Those on the far right are for detection. Lasers are turned on or off in concert with shutter closings or openings so as to avoid frequency shifts caused by scattered light. Three shutters shown as short, bold vertical bars are used to prevent light from entering the cavity region while atoms are being prepared or detected. Velocity selection is also done at the second shutter giving $v_{z0}-v_{zcut} \leq v_z \leq v_{z0}+v_{zcut}$ where v_z is the longitudinal velocity and v_{z0} is the most probable longitudinal velocity of the launched ball of atoms. The resulting width of the velocity distribution entering the cavity is 2 x v_{zcut}.

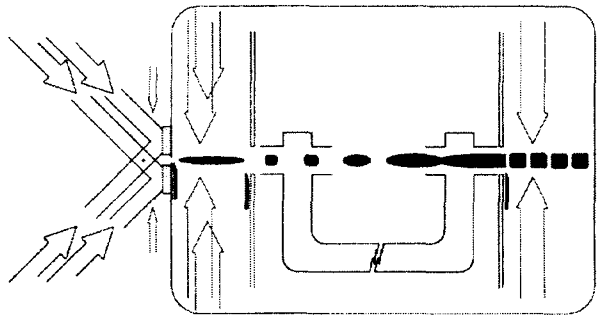

Figure 1. Concept drawing for spaceclock.

In this Figure the following parameters used below can be indentified: distance between the atom collection region at far left and the second shutter is L_1; distance between the second and third shutters is the end-to-end cavity length plus two beyond-cutoff waveguide sections, L+2 x L_{cutoff}.

TE₀₁ₙ Cavity

We report modeling studies on the TE_{01n} cavity including comparison with a traditional Ramsey cavity geometry. Specific issues considered are: (a) the locations of frequency detuning on the central fringe giving the minimum change in signal with microwave power; (b) the effect on frequency stability produced by choosing such an operating point; (c) the dependence of frequency stability on cavity length, aperture size, and characteristics of the cold-atom source.

In our initial model, we assume the cavity mode is ideal, with no corrections for atom trajectory through the cavity field, or for field corrections due to using waveguides beyond cutoff at the entrance and exit apertures. To calculate velocity-averaged lineshapes, an analytic expression is used for the atom flux from the cold-atom source through the microwave cavity. For simplicity it is assumed that the source is a delta function in space at z=0 and has transverse and longitudinal temperatures, T_{trans} and T_z. This model approximates a magnetooptic trap (MOT) followed in time by launching at velocity v_{z0}, with additional transverse cooling accompanied by longitudinal heating.

The lineshape for the assumed two-level atom moving with velocity v_z is calculated by numerical integration of the first-order coupled differential equations for the time-dependent coefficients of wavefunctions.[2] The velocity-averaged lineshape is found by weighting the v_z-lineshapes with the velocity distribution in the detection region. The fringe pattern shows essentially 100% contrast with typical parameter choices. In comparison with the Ramsey two-cavity geometry, the microwave field is always driving the atoms and the TE_{01n} cavity lineshape shows saturation as power is increased. To illustrate such effects, we assume for now a mono-velocity atom ensemble launched through a $TE_{01,13}$ cavity.

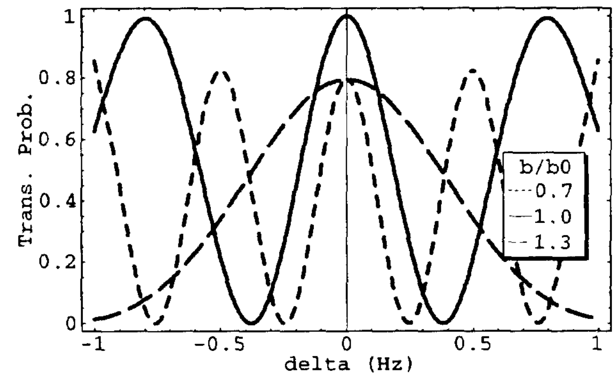

Figure 2. Fringes for a $TE_{01,13}$ cavity at three microwave field strengths vs. detuning, delta. The field for unit transition probability is b_0 for velocity v_{z0}. Parameters are given in the text.

Figure 2 shows several fringes around the central fringe at three field amplitudes, b/b_0= 0.7, 1, 1.3, where b_0 is the field value giving unit transition probability for atoms with velocity v_{z0}. The observed linewidth is greater than the Ramsey linewidth for the same end-to-end cavity length L. Parameters used are L=28.3 cm, launch velocity v_{z0}=10 cm/s, and cavity diameter of 6 cm.

When the side of the central fringe is interrogated at a given detuning from resonance, the dependence of signal on microwave power fluctuations can be interpreted as a frequency error by the frequency servo system. We search for detuning locations for a given power at which a small variation in power causes little or no change in transition probability. For the TE_{01n} cavity such locations correspond to a crossing of the lineshapes as power is changed.

Figure 3 illustrates such a crossing for a mono-velocity ensemble in an n=13 cavity. For the microwave field range illustrated, the detuning for this crossing offers the minimum frequency sensitivity to variations in power. Note that this detuning is not that of maximum slope, which would otherwise be optimum for use as a frequency discriminator.

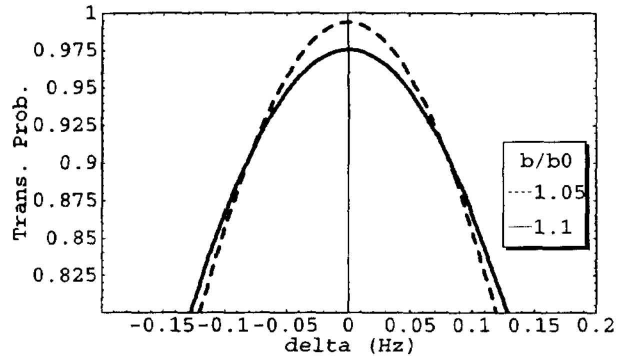

Figure 3. Fringe crossing at two microwave field values illustrating the approximate location of power insensitive detuning point of frequency for a $TE_{01,13}$ cavity.

By operating away from a position of maximum slope, the clock stability is penalized because the slope is reduced; however, the atom shot noise, proportional to $\sqrt{p(1-p)}$ where p is the transition probability, is actually reduced by operating at a transition probability $p \neq 1/2$. For the traditional Ramsey lineshape, the linewidth is not affected by microwave power. At powers near that which gives unit transition probability at resonance, the corresponding fringes overlap (rather than cross) so that any detuning offers minimal frequency sensitivity to power. Thus we can choose p_0 = 1, p = 1/2, and δ_0=FWHM/2, corresponding to the detuning for optimum slope. For high contrast, sinusoidal fringes, the fractional frequency stability, $\sigma_y(\tau)$, for a mono-velocity atom ensemble having a transition probability p is

$$\sigma_y(\tau) = \frac{\Delta\nu}{\alpha\nu_0}\sqrt{\frac{2\,p(1-p)}{m N_B}}\sqrt{\frac{T_c}{\tau}} \qquad (1)$$

where

α = lineshape slope factor $=\pi\, p_0 \sin(\pi\, \delta_0/\Delta\nu)$,
with p_0 = transition probability at resonance,
δ_0 = nominal detuning from resonance,
$\Delta\nu$= FWHM linewidth,
ν_0= resonant frequency,
p = transition probability for detuning δ_0,
N_B= number of detected atoms/ball,
m = number of balls launched/lineside,
T_c= time for one complete measurement cycle.

This expression assumes that only ideal atomic shot noise is present. Electronic or other noise sources will make an additional contribution to the total noise that will affect $\sigma_y(\tau)$.

The microwave power is assumed to have unwanted fluctuations. In a worst-case analysis, the power is taken as constant for one side of the line, then changed to another constant value for the second side of the line. For the $TE_{01,13}$ cavity, a power-insensitive parameter choice for the velocity-averaged lineshape gives FWHM~0.46 Hz, b/b_0 ~ 1.05, α ~ 0.88, and δ_0 ~ 0.09 Hz. Following this type of analysis, it should be possible to operate a cold atom Cs frequency standard using a $TE_{01,13}$ cavity at a microwave power such that the worst-case microwave power-induced frequency shift $(\delta\nu/\nu_0)_{\delta P}$ ~ 1×10^{-16} within a fractional microwave power range $\delta P/P$~0.003. For comparison, $\delta P/P$~0.01 for the Ramsey geometry. For a $TE_{01,12}$ cavity, giving a "dark" central fringe having a minimum at resonance, a parameter choice of b/b_0~0.64, FWHM~0.27 Hz, δ_0~0.11 Hz yields a range $\delta P/P$~0.006 with stability comparable to the n=13 case.

A comparison of stabilities is made for the $TE_{01,13}$ cavity with a Ramsey cavity geometry where both geometries have the same end-to-end lengths and aperture diameters. This keeps the atom flux the same for both geometries. Although the temperatures characterizing the cold atoms strongly affect the stability through the number of atoms reaching the detection region, a comparison using the ratio of stabilities having the same temperatures is independent of the temperature. The following parameters are used: 10 cm/s launch velocity; and 28.3 cm $TE_{01,13}$ cavity length. For b/b_0=0.7, the $TE_{01,13}$ geometry has a FWHM of ~0.25 Hz. The FWHM linewidth for the Ramsey geometry is ~0.19 Hz. If we operate at the detuning for maximum slope, or half the FWHM value, $\sigma_{TE01,13}(\tau)/\sigma_{Ram}(\tau)$=1.6. For this operating point with the $TE_{01,13}$ cavity, a fractional microwave power change of 10^{-3} causes a worst-case amplitude change equivalent to a fractional frequency change ~ 1.2×10^{-14}.

If we move to a power-insensitive detuning for the $TE_{01,13}$ cavity, with detuning ~ 0.09 Hz, and b/b_0 ~ 1.05, but keeping the optimum Ramsey-geometry parameters (detuning~0.1 Hz, b/b_0~1), we find $\sigma_{TE01,13}(\tau)/\sigma_{Ram}(\tau)$=2.6. Thus the Ramsey cavity provides an improvement in stability by a factor of ~2.6.

Ramsey Cavity Stability Considerations

In Figure 4, we show the number of atoms at launch in the m_F=0 state required for a stability of 10^{-14} in 1 s. This is done for the Ramsey geometry as a function of end-to-end distance L between the two cavities, using p=1/2, and p_0=1. Comparison with the $TE_{01,13}$ geometry can be made by using the σ-ratios given above for launch velocity of 10 cm/s.

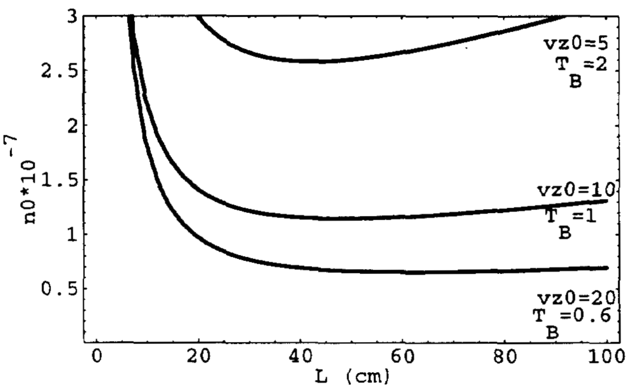

Figure 4. Required number of atoms at launch in m_F = 0 state for 10^{-14} stability in 1 s for a Ramsey cavity. L is the end-to-end cavity length. Three launch velocities are shown with corresponding times between ball launches T_B.

The parameters chosen for Figure 4 are as follows:

T_{trans} ~ 0.2 μK = transverse atom temperature,
T_z ~ 15 μK = longitudinal atom temperature,
L = end-to-end distance between cavities,
L_{cutoff} = 2 cm,
L_I = 5 cm; distance from cold atom source to shutter,
v_{zcut} = 2 cm/s,
Diameter of cavity aperture = 1 cm,
Cavity diameter = 6 cm,
Number of balls/lineside = m = 10,
Launch velocity v_{z0} = 5, 10, 20 cm/s,
T_B= time between ball launches on given lineside,
t_C= time to collect atom ball = 0.33 s.

For short L, the rise in the required number of atoms n_0 at launch is due to the increase in linewidth caused by a short T_{Ram}. (T_{Ram}, the Ramsey time, is essentially the drift time between cavities.) Little or no loss of atoms is encountered in passing through the cavities in this case. At the other extreme, for large L we again find a rise in n_0: the number of atoms/ball N_B reaching the detection region is proportional to $1/T_{Ram}^2$ due to transverse expansion of the atom balls and the resulting atom loss due to a finite aperture diameter. The linewidth $\Delta\nu$ is proportional to $1/T_{Ram}$, so that in the stability formula, the effects of $\Delta\nu$ and N_B parameters cancel each other. In this limit of long L (long T_{Ram}), T_c is proportional to T_{Ram}, causing the rise in stability for large L. For intermediate values of L, the linewidth, atom loss, and T_c compensate each other resulting in a nearly constant n_0. A soft minimum occurs around the optimum L for a given parameter choice. Smaller values of the number of balls/lineside, m, *e.g.* m=1, give sharper minima. The advantage of using a shutter is that we can collect, cool, and launch atoms at a higher rate than without the shutter while keeping light out of the cavity and drift region.

The stability and hence the required number of atoms at launch depends on T_c. This in turn depends on the details of the launch cycle: number of balls/lineside, time between launches, choice of shutters, and shutter timing. One choice, omitting the shutter closest to the MOT, but still preventing light from entering the cavity gives $T_c = 2*[t_{L1}+t_{L2} - T_{L1} + (m-1)*(t_c + t_{L1})]$.

Here

t_{L1}=time for slowest atoms to move distance L_1,

T_{L1}=time for fastest atoms to move distance L_1,

t_{L2} = time for slow atoms to move a distance,

$2 \times L_{cutoff} + L$,

t_C =time to collect atom ball.

The slow and fast atom velocities are respectively $v_{0z}-v_{zcut}$ and $v_{0z}+v_{zcut}$. The expression for T_c includes the time for clearing atoms from the cavity between a frequency change in moving to the other side of the line.

An accuracy of 1 part in 10^{16} requires knowing, measuring, and correcting systematic effects so that a linewidth of, for example, 0.1 Hz can be reliably split to 1 part in 10^5. Our goal for a frequency stability of 1×10^{-14} in 1 s is made with special concern for other systematic errors (such as collisional shift and LO stability) which are not discussed here, but which are important for attaining an accuracy of 1 part in 10^{16}.

Summary

We have modeled the frequency sensitivity to changes in microwave power for the TE_{01n} cavity, n=12 and 13. A stability comparison with a Ramsey cavity geometry is made. The Ramsey geometry offers a fixed linewidth not subject to the saturation found in the TE_{01n} cavity. Operation at a power-insensitive parameter choice favors the Ramsey geometry; stability is ~2.6 times improved over the $TE_{01,13}$ cavity for our choice of parameters. A calculation of the number of cold atoms needed at launch *vs.* the end-to-end cavity length is presented for the Ramsey geometry, allowing a choice of optimum length. Other questions will need to be addressed in the future if the TE_{01n} cavity is used for high accuracy applications. These include sensitivity to nonuniform magnetic fields; careful evaluation of distributed cavity phase shift associated with the apertures and cut-off waveguides; lineshape effects due to spatial averaging of atomic trajectories through the cavity; excitation and effects of unwanted cavity modes; and evaluation of the end-to-end cavity phase shift. We have not addressed systematic effects for the Ramsey geometry, including the critical issue of determining the end-to-end cavity phase shift.

References

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2. A. DeMarchi, R.E. Drullinger, and J.H. Shirley (1990), "Interference Fringes from Single-Cavity Excitation of an Atomic Beam," *Proc. 44th Annual Symposium on Frequency Control*, pp. 34-38.