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IMPROVING THE SHORT-TERM STABILITY OF LASER PUMPED RB CLOCKS BY REDUCING THE EFFECTS OF THE INTERROGATION OSCILLATOR[1]

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Abstract

This paper studies the limitations due to the local oscillator on the short-term stability of a laser pumped rubidium frequency standard. Two effects have been considered: the increase of current noise on the resonance signal and the "aliasing" effect originated by the microwave phase noise. After reducing these two effects, the standard has demonstrated a short-term stability of $\sigma_y(\tau) = 3\cdot10^{-13}\,\tau^{-1/2}$.

We have measured the clock stability as a function of local oscillator phase modulation (PM) noise at harmonics of the sine wave modulation frequency. At present we are not aware of a complete theory for PM noise at Fourier frequencies that are very large compared to the line width. In some cases the PM noise at 100 times the nominal line width dominated the short term frequency stability. The details presented here should prove useful for testing a complete theory of this effect and give some insight into the design requirements of local oscillators for passive standards.

1. Introduction

The paper is organized in the following manner. This section introduces the problem of evaluating the effects of microwave phase modulation (PM) noise on the stability of a clock. The second section outlines the experimental setup. The third section shows our experimental results and compares them to theoretical predictions. A summary concludes the paper.

Previous works have shown that passive, laser pumped, rubidium frequency standards can potentially attain a short-term frequency stability that is even better than that available from an active room temperature hydrogen maser [1, 2]. However, there is a large discrepancy between these predictions and the results that have been reported so far. This communication continues the analysis of the limiting factors which affect the stability of laser pumped gas-cell standards. Our previous studies [3, 4] focused on the effects related to the laser. We reported a significant improvement of the short-term stability ($\sigma_y(\tau) < 1\cdot10^{-12}\,\tau^{-1/2}$), obtained after reducing the instabilities due to the laser AM and FM noise.

Another serious problem in achieving good short-term frequency stability with passive standards is the PM noise in the interrogating signal [5-10]. In [3, 4], we mentioned two limitations due to our microwave synthesizer: (1) additional photocurrent noise on the resonance signal and (2) the intermodulation effects related to the microwave PM noise. After reporting the improvement obtained with the reduction of the first effect (§ 3.1), the paper focuses on the potential further improvements by decreasing the second effect (§ 3.2).

We also analyze one of our previous experimental works, regarding the effectiveness of notch filters in the microwave synthesizer at $\nu_0 \pm 2f_m$ (f_m and ν_0 are the modulation and carrier frequencies) [11]. In fact, our results seemed to demonstrate that the stability could be improved by at least a factor of 20 with such notch filters. On the other hand, theoretical work by the LHA group at the University of Paris [12] indicated that the sensitivity to the other even harmonics should be so large that the notch filter would provide less than a factor-of-2 improvement. We now have experimental data that provide an explanation for the large discrepancy between the two approaches.

2. Experimental setup

The three main components of our clock (physics package, laser, and microwave synthesizer) have been described in [2]. A DBR laser is used to optically pump a rubidium vapor in a buffer gas cell. The laser is stabilized to a saturated absorption line of a separate Rb^{87} evacuated cell. Sideband locking at 12 MHz is used [13]. Since our goal is short-term stability, we do not fine-tune the laser to the zero light shift frequency ($\nu_{LS=0}$). The laser is locked to the $5S_{1/2}$, F=2 → $5P_{3/2}$, F'=3 and F'=2 cross-over transition at 780 nm. Thus, the laser frequency is about 20 MHz higher than $\nu_{LS=0}$. The total shift due to the light shift effect is $1\cdot10^{-9}$ and $(\Delta\nu_{clock}/(\nu_{clock}\cdot\Delta\nu_{laser}\cdot\Delta I_{laser}) \approx 2\cdot10^{-11}/(\text{MHz}\cdot\mu\text{A}))$.

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3.2 "Aliasing" effect of the local oscillator PM noise.

G. Kramer [5] first pointed out the "aliasing" problem in which, the PM noise in the interrogating source, at the even harmonics of the sinewave modulation frequency, degrades the performance of passive standards. Various analyses have been published for standards operating either in pulsed [8] or continuous [9, 10] mode. None of these analysis treat the case investigated here, namely the sensitivity to PM noise at Fourier frequencies that are large compared to the nominal linewidth. References [9] and [10] investigate the "quasi-static" case where the modulation frequency is very small compared to the line width. This approach is expected to break down when the modulation frequency approaches the line width.

3.2.1 The quasi-static approach.

The limit to the frequency stability due to PM noise at even harmonics of the square wave modulation frequency has been calculated in the quasi-static approximation (f_m « resonance width) and sinewave demodulation. The results are given by [9]

$$\sigma_y(\tau)_{PM\,noise} = \sqrt{\sum_{n=1}^{\infty} C_{2n}{}^2 S_\Phi(2nf_m)} \cdot \tau^{-1/2} \qquad (1)$$

For square wave modulation, the general formula and first coefficients are given by Eq. (2).

$$\begin{aligned}
&\textit{General} && C_{2n} = \frac{2n}{(2n-1)\cdot(2n+1)} \cdot \frac{f_m}{\nu_0} \\
&2^{nd}\ \textit{harm.} && C_2 = \frac{2}{3} \cdot \frac{f_m}{\nu_0} \\
&4^{th}\ \textit{harm.} && C_4 = \frac{4}{15} \cdot \frac{f_m}{\nu_0} = 0.4 \cdot C_2 \qquad (2) \\
&6^{th}\ \textit{harm.} && C_6 = \frac{6}{35} \cdot \frac{f_m}{\nu_0} \approx 0.26 \cdot C_2 \\
&8^{th}\ \textit{harm.} && C_8 = \frac{8}{63} \cdot \frac{f_m}{\nu_0} \approx 0.19 \cdot C_2
\end{aligned}$$

Eqs. (1) and (2) are useful for estimating the effect of the synthesizer PM noise, but cannot provide a precise evaluation for our device. In fact, we use sine wave modulation and f_m (~300 Hz) is one third of the resonance linewidth (~900 Hz).

3.2.2 Experimental results : C_2, C_4, C_6, and C_8.

To evaluate the effect of the microwave PM noise on the stability, we have measured these coefficients. We used the experimental setup described by Fig. 1, with the switch on position a. At a given modulation frequency, we added narrow band PM noise centered on the 2nd, 4th, 6th and 8th harmonic. We then measured the clock stability as a function of the PM noise. f_m was varied from 21.5 to 607 Hz.

The results are given in Figs. 4a, b, c and d. For low modulation frequencies, there is good qualitative and even quantitative agreement between the quasi-static predictions and the experimental results. The coefficients increase linearly with the modulation frequency, with a slope slightly different than predicted, probably because the theory was established for square wave modulation while we used sine wave modulation. When $2 \cdot n \cdot f_m$ becomes large compared to the resonance half-width, the coefficients stop increasing linearly. They appear to become constant.

With this technique, we have measured the effect of the PM noise centered on the odd harmonics of f_m. The noise added on the 1st harmonic produced an instability 40 times lower than for the 2nd. The noise added on the 3rd has a coefficient 5 times lower than C4, and actually smaller than the coefficient of the 350th. Thus, we conclude that, as predicted by the "quasi-static" model, the effect of the PM noise at the odd harmonics of f_m is negligible as compared to the effect of the PM noise at the even harmonics.

3.2.3 Experimental results : C_{10}, to C_{304}.

A second type of measurement was performed to evaluate the effect of higher harmonics. The modulation frequency was fixed at 287 Hz. The pass band filter after the noise source was adjusted so that, instead of one harmonic, the noise spectrum (typically 25 kHz wide) included a large number of harmonics. An estimate of the average coefficients could then be obtained by measuring the clock stability as a function of the noise power. The results are given in Fig. 5.

On Figs. 4 and 5, we observe that the magnitude of the coefficients decreases by one order of magnitude when passing from C_2 to C_4, but then decreases very slowly. In fact, it hardly changes by 50% from C_8 to C_{304}. If we use the "quasi-static" formula (2), we obtain 97% change between C_8 and C_{304}. However, the "quasi-static" approach is not valid for high harmonics, since $2 \cdot n \cdot f_m \gg \Delta\nu$. The consequence of the slow decrease of high order coefficients is that broadband PM noise in the synthesizer signal can significantly affect the clock stability (because of the high number of terms which contribute to the instability, cf. Eq. (1)).

3.2.4 Limit due to microwave synthesizer PM noise.

We measured the PM noise of our synthesizer at 100 MHz by using the "three-cornered-hat cross-correlation technique" [14], the results are in Eq. (3).

$$\begin{aligned}
&S_\Phi(f) = 1.2\cdot10^{-14}\ \text{rad}^2/\text{Hz} && @\ 600\ \text{Hz} \\
&S_\Phi(f) = 1\cdot10^{-14}\ \text{rad}^2/\text{Hz} && 1\ \text{kHz} < f < 1\ \text{MHz} \qquad (3) \\
&S_\Phi(f) < 5\cdot10^{-15}\ \text{rad}^2/\text{Hz} && f > 1\ \text{MHz}
\end{aligned}$$

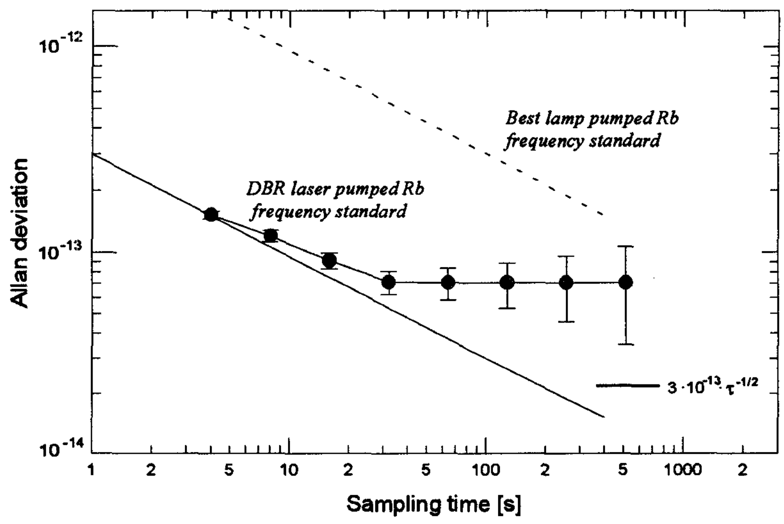


Figure 2. Short-term stability of the Rb clock of Fig. 1.

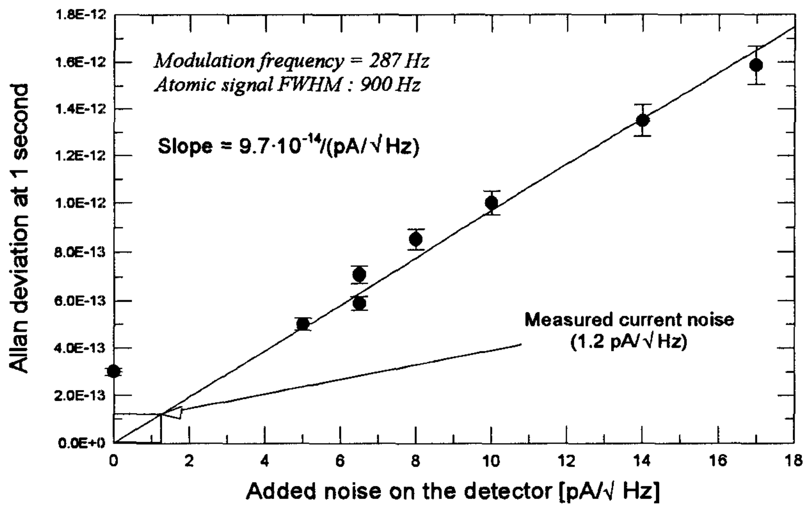
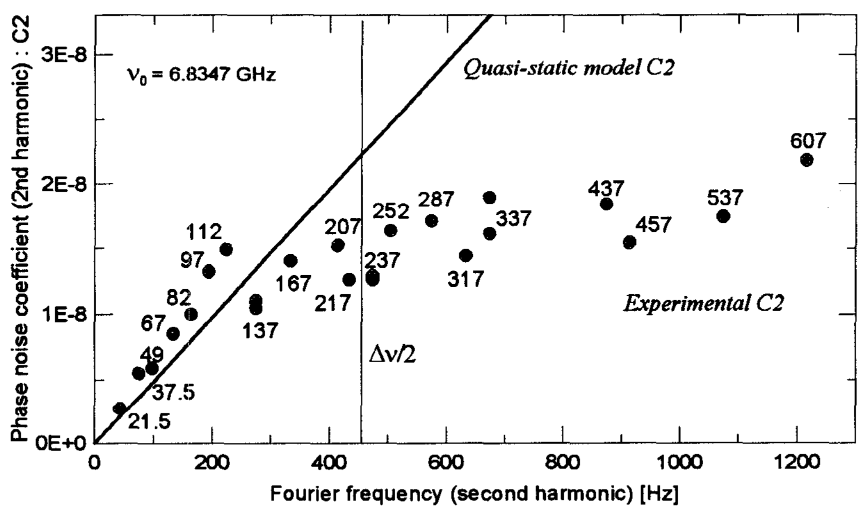
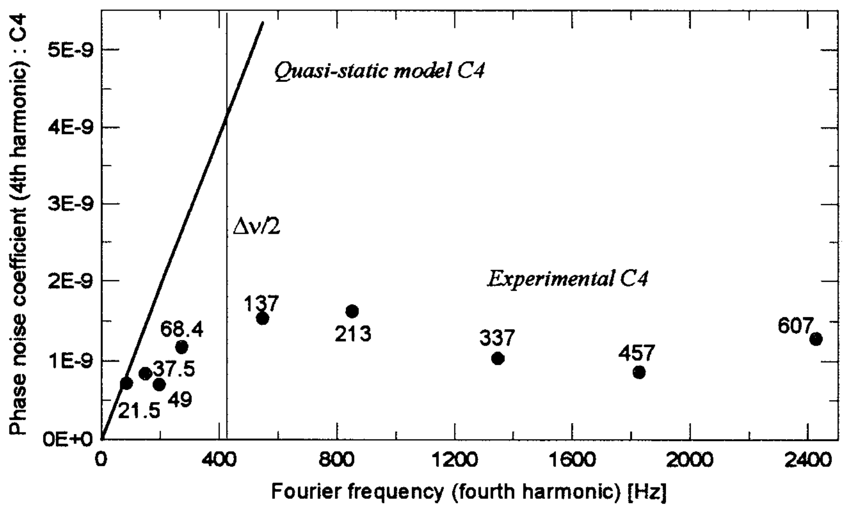


Figure 3. Short-term stability as a function of the photocurrent noise density.

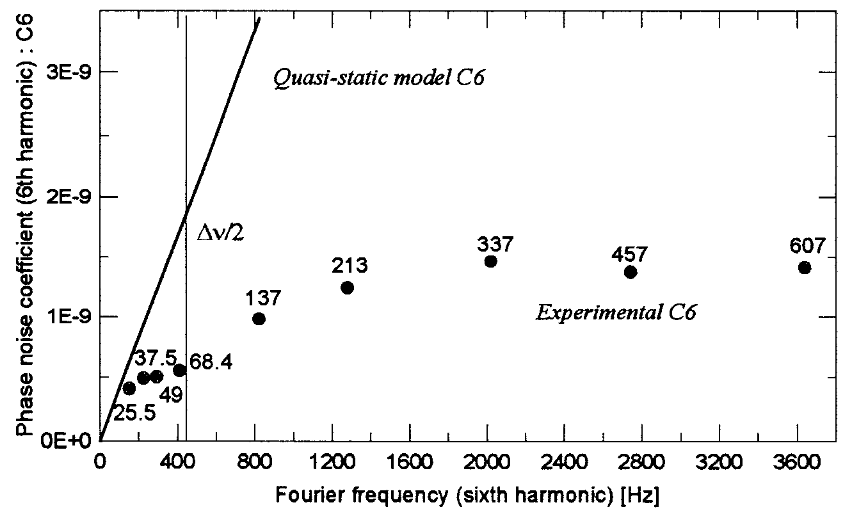
4a)



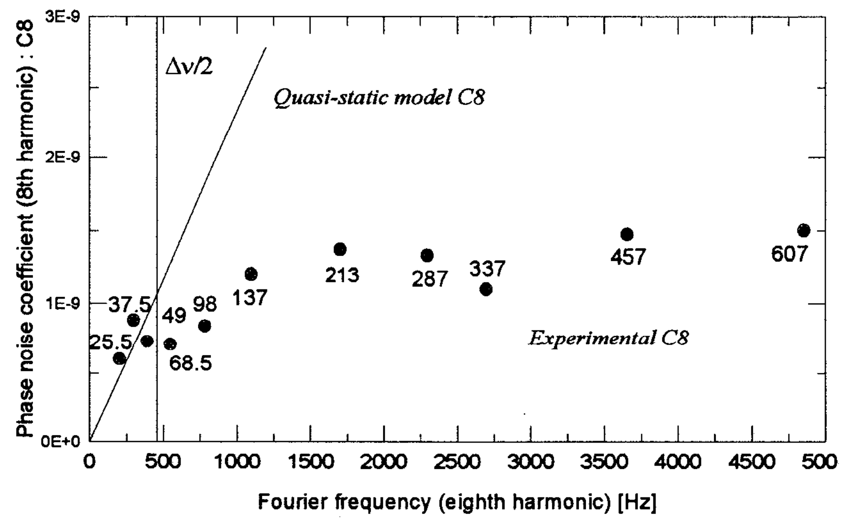
4b)



4c)



4d)



Figures 4a, b, c, d. PM noise coefficient : C_2, C_4, C_6, and C_8 at ν_0=6.8347 GHz. The modulation frequency is indicated. $\Delta\nu$ (=900 Hz) is the atomic resonance width.

The original multiplication chain of the synthesizer is the following. A low phase noise 5 MHz quartz oscillator is sinusoidally phase-modulated and doubled. The 10 MHz frequency is multiplied by 50. A digital synthesizer at 11.8 MHz is subtracted from the 500 MHz. After being filtered, the 488 MHz power is amplified and used to drive a step recovery diode. The 14th harmonic coincides with the rubidium "clock" transition resonance frequency and is injected into the microwave cavity. Figure 1 describes our experimental setup. The function of the broadband noise source and of the adjustable pass band filter are described below.

3. Experimental results

3.1 Reducing the noise on the photocurrent.

In the presence of optical pumping, the microwave field at resonance induces a reduction of the photocurrent. The total absorption of light due to the interrogation constitutes the double resonance signal. Unfortunately, it can introduce noise on the detector.

In our previous setup, the noise increased from 1.5 to 6 pA/√Hz when the microwave radiation was present. This noise increased strongly on the side of the resonance, indicating that it was more likely due to PM than AM noise. The theoretical limit of the clock stability was then $6\cdot10^{-13}\ \tau^{-1/2}$, and we typically measured $8\cdot10^{-13}\ \tau^{-1/2}$.

We first tried to improve the stability without changing the principle of the synthesizer multiplication chain. The spectrum of the 6.8 GHz signal contained spurious sidebands which could be removed by filtering the adjustable synthesizer with a phase-locked, low-noise quartz oscillator. After this modification, the photocurrent noise decreased to 1.2 pA/√Hz when the microwave radiation was present.

Figure 2 shows the frequency stability after these improvements. The short-term stability has been notably enhanced; $\sigma_y(\tau)$ reaches 10^{-13} after less than 10 s., which corresponds to a white frequency component of $3\cdot10^{-13}\cdot\tau^{-1/2}$. A flicker floor of $7\cdot10^{-14}$ is then reached, and the stability does not decrease further.

To determine whether the short-term stability was still limited by the photocurrent noise, we performed the following measurement. Noise centered around the modulation frequency of 287 Hz was added to the photocurrent signal (Fig. 1, switch on b). We measured the clock stability as a function of the corresponding added photocurrent noise (Fig. 3). The slope agrees with our previous estimate (Eqs. (1) and (2) in [4]): $\sigma_y(\tau) \cong 1\cdot10^{-13}\cdot$ Noise [pA/√Hz]$\cdot\tau^{-1/2}$. With a noise of 1.2 pA/√Hz, the limit for the stability due to this parameter is $1.2\cdot10^{-13}\cdot\tau^{-1/2}$. Since the measured stability is $3\cdot10^{-13}\cdot\tau^{-1/2}$, the photocurrent noise is no longer the limiting factor for the clock short-term stability.

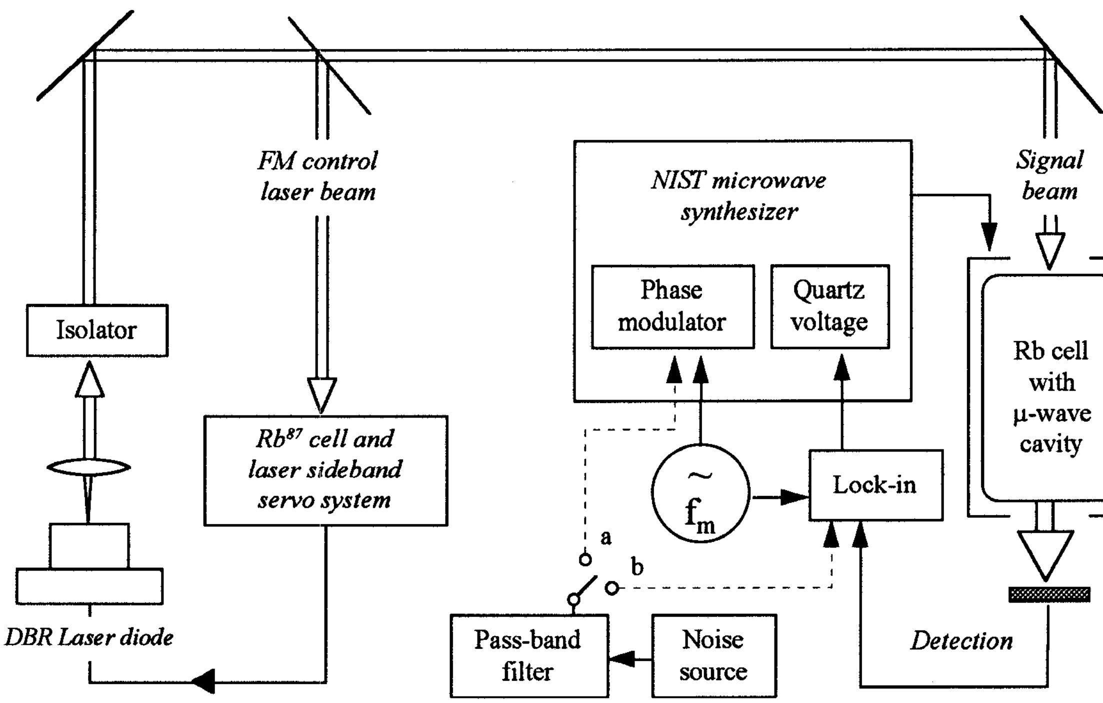


Figure 1. Block diagram of the Rb clock. The role of the noise source is described in the text (§ 3.1 & 3.2).

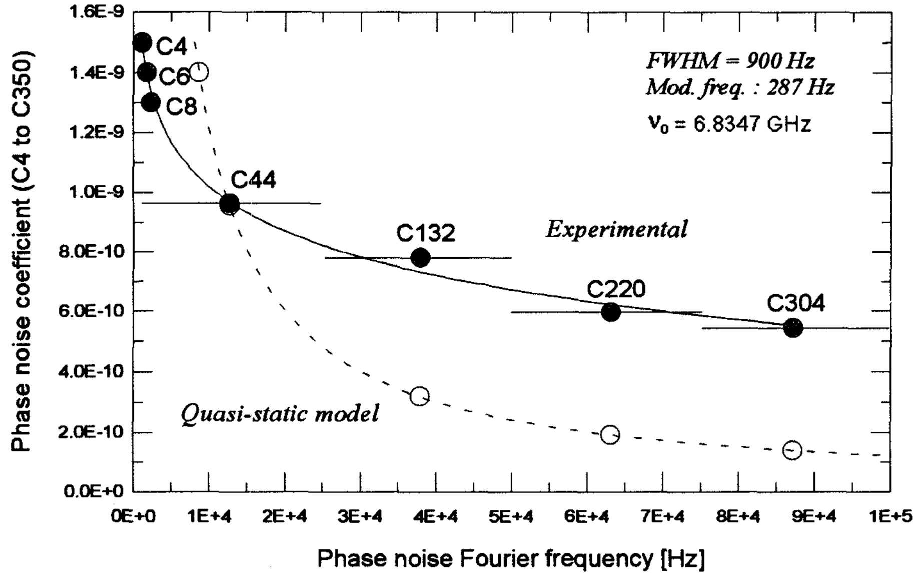


Figure 5. High order PM noise coefficients.

The effect of the microwave PM noise on the clock instability can be calculated from Eq. (3). Table 1 summarizes the results obtained with the coefficients corresponding to the quasi-static model, and with the coefficients measured in the "dynamic" case. Even though the two total limits are very close, we can observe a significant difference regarding the origin of these two limits. In the quasi-static case the contribution from the low order harmonics are dominant, and the effect of the broadband noise is negligible. In the "dynamic" case, the effect of the low order harmonics are much lower, but the broadband noise contribution is not negligible and is actually even larger than the effect of the 2nd harmonic.

Table 1 *Limit to the Rb clock short-term stability (Allan deviation at 1 s.) due to the microwave PM noise. From Eq. (1), the PM noise of Eq. (3) and the coefficients of Eq. (2) and Figs. 4 and 5 (f_m = 287 Hz).*

Harmonic contribution in eq. (1)	2nd 574 Hz	4th 1148 Hz	4th to 350th 1-100 kHz	350th to 3500th 0.1-1 MHz	Total limit
quasi-static (theory)	$2.1\cdot10^{-13}$	$8\cdot10^{-14}$	$1.2\cdot10^{-13}$	$<1\cdot10^{-14}$	$\mathbf{2.4\cdot10^{-13}}$
dynamic (exp.)	$1.3\cdot10^{-13}$	$1\cdot10^{-14}$	$7\cdot10^{-14}$	$1.4\cdot10^{-13}$	$\mathbf{2\cdot10^{-13}}$

The total limit on the short-term stability due to the synthesizer PM noise ($2\cdot10^{-13}\ \tau^{-1/2}$) is lower than the measured stability of the clock ($3\cdot10^{-13}\ \tau^{-1/2}$). However, this limit is based on the measurements of the PM noise at 100 MHz, while the interaction with the atoms occurs at 6.8 GHz. Since we can reasonably expect a slight degradation of the signal in the multiplication chain, this limit is probably optimistic. In addition, if we also include the effect of the photocurrent noise ($1.2\cdot10^{-13}\ \tau^{-1/2}$), and the light shift ($2\cdot10^{-13}\ \tau^{-1/2}$ [3, 4, 15]) we obtain a limit for $\sigma_y(\tau)$ ($3\cdot10^{-13}\ \tau^{-1/2}$) which is equal to the measured Allan deviation.

Since the clock instability is due to both the PM noise at the 2nd harmonic and the broadband noise, it is necessary to reduce the amplitude and the spectral width of the PM noise in order to significantly diminish this effect. A notch filter at the 2nd harmonic alone would not be sufficient in this case.

3.2.5 Improving the microwave synthesizer.

In order to reduce the effect of the microwave radiation, the synthesizer has been redesigned in the following manner. A low-noise 100 MHz quartz oscillator, phase-locked to the multiplied 5 MHz quartz, has been introduced. After being phase-modulated, the 100 MHz signal is multiplied up to 6.8347 GHz following the same scheme described previously (in section 2). Equation (4) displays the measured PM noise of the 100 MHz after the phase modulator. The broadband noise spectrum has been reduced for f > 300 Hz, and narrowed to 12 kHz. The resulting limit for the short-term stability of the clock is given in Table 2.

$S_\Phi(f) = 4\cdot10^{-13}$ rad²/Hz @ 10 Hz

$S_\Phi(f) = 2.6\cdot10^{-13}$ rad²/Hz @ 50 Hz

$S_\Phi(f) = 1.6\cdot10^{-13}$ rad²/Hz @ 100 Hz

$S_\Phi(f) = 3.2\cdot10^{-14}$ rad²/Hz @ 300 Hz (4)

$S_\Phi(f) = 6.4\cdot10^{-15}$ rad²/Hz @ 600 Hz

$S_\Phi(f) < 1.2\cdot10^{-15}$ rad²/Hz 1 kHz < f < 12 kHz

$S_\Phi(f) < 1\cdot10^{-16}$ rad²/Hz f > 12 kHz

Table 2 *Limit to the Rb clock stability (σ_y (1 s)) with the improved synthesizer. From Eq. (1), the PM noise in Eq. (4) and the Figs. 4 and 5 (f_m = 287 Hz).*

Harmonic contribution	2nd 574 Hz	4th 1148 Hz	4th to 42nd 1-12 kHz	Total limit	With notch on 2nd harm.
quasi-static (theory)	$1.6\cdot10^{-13}$	$3\cdot10^{-14}$	$4\cdot10^{-14}$	$\mathbf{1.6\cdot10^{-13}}$	$4\cdot10^{-14}$
dynamic (exp.)	$1\cdot10^{-13}$	$4\cdot10^{-15}$	$1\cdot10^{-14}$	$\mathbf{1\cdot10^{-13}}$	$1\cdot10^{-14}$

According to Tables 1 and 2, the effect of the PM noise at the 2nd harmonic is reduced by a factor √2, and the effect of the broadband PM noise is now negligible. According to this analysis, we should measure a clock stability of $2.5\cdot10^{-13}\ \tau^{-1/2}$, and be essentially limited by the photocurrent noise and the light shift. If so, the stability might be further optimized by changing the buffer-gas pressure and composition (since the atomic discriminator slope might be increased) or by reducing the laser noise. However, our results show that the ultimate limit for laser-pumped, rubidium frequency standards using a buffer-gas cell will be reached soon, at least for the short-term stability. After that, additional efforts will be needed to improve the medium and long-term stability (and reduce the flicker-floor level). As described in previous analysis [2-4], the next step could be to use wall-coated cells (§ 3.2.7).

3.2.6 Comparison with a previous experiment [11].

In a previous experiment [11], PM noise was added to the microwave synthesizer in order to determine the potential improvement with a notch filter at the second harmonic. The modulation frequency was 37.5 Hz. The phase noise for f < 300 Hz, at ν_0 = 10 MHz, after the phase modulator (noise on) is given by Eq. (5). It then decreases by 20 dB per decade. The limit on the clock stability can be calculated by using our measured coefficients with f_m = 37.5 Hz (Fig. 4). The results are given in Table 3.

$S_\Phi(f) = 2\cdot10^{-10}$ rad²/Hz @ 75 Hz

$S_\Phi(f) = 2\cdot10^{-11}$ rad²/Hz @ 150 Hz (5)

$S_\Phi(f) < 1\cdot10^{-11}$ rad²/Hz f > 300 Hz

Table 3 *Limit to the Rb clock short-term stability (σ_y(1 s)) in the case of reference [11] (f_m = 37.5 Hz).*

Harmonic contribution	2nd 75 Hz	4th 150 Hz	4th-27th 1-1 kHz	Total limit	With notch on 2nd h.
according to this work	$6.1\cdot10^{-11}$	$3\cdot10^{-12}$	$5\cdot10^{-12}$	$\mathbf{6\cdot10^{-11}}$	$\mathbf{5\cdot10^{-12}}$
measured in ref. [11]				$\mathbf{7\cdot10^{-11}}$	$\mathbf{5\cdot10^{-12}}$

In the experiment, the clock had a stability of $\sigma_y(\tau)=7\cdot10^{-11}\ \tau^{-1/2}$ without notch filter and $5\cdot10^{-12}\ \tau^{-1/2}$ with the notch filter. The stability measured with the notch filter corresponds exactly to the value in Table 4. The experimental value without notch filter is slightly higher than the theoretical prediction. The agreement between the two results is thus very good.

The main purpose of this experiment was to quantify the effectiveness of the notch filter at the second harmonic. Both the previous measurements and this analysis agreed on the fact that in that particular setup, the notch filter improved the stability by at least a factor of 10. However, it must be noted that this strong effect was also due to the fact that a low pass filter was used in the setup which added PM noise. Without this filter, the contribution of the higher order harmonics would have been more important and the effectiveness of the notch filter lower.

As Tables 1, 2, and 3 show, the improvement of passive clock stability using notch filters strongly depends on the broadband PM noise spectrum and on the modulation frequency. The narrower the noise (as compared to f_m), the more effective the notch filter.

3.2.7 Scaling for a lower atomic linewidth.

As demonstrated by previous studies [16], the resonance signal linewidth can be reduced to less than 100 Hz with the use of a wall-coated cell instead of the buffer-gas cell. In this case, the modulation frequency will be lower, around 30 Hz typically. In Table 4, we have summarized our estimation of the limit on the stability due to the microwave PM noise with the improved synthesizer. This estimation was obtained assuming that the PM noise coefficients correspond to the quasi-static model for modulation frequencies and harmonics below the half atomic linewidth and then saturate, similarly to the curves in Figs 4 and 5. The estimated limit is $7.6\cdot10^{-14}\ \tau^{-1/2}$ and could be reduced to $3\cdot10^{-14}\cdot\tau^{-1/2}$ with a notch filter on the second harmonic. This limit is higher than the expected "shot noise" limit with a wall-coated cell ($1\cdot10^{-14}\cdot\tau^{-1/2}$). Thus, with additional efforts to reduce the PM noise at low frequencies the expected "shot noise" ($1\cdot10^{-14}\cdot\tau^{-1/2}$) might be reached.

Table 4 *Extrapolated limit to the Rb clock stability (σ_y (1 s)) due to the microwave PM noise when wall-coated cells are used. ($\Delta\nu$ = 100 Hz, f_m = 37.5 Hz).*

Harmonic contribution in eq. (1)	2nd 75 Hz	4th 150 Hz	4th to 320th 1-12 kHz	Total limit	With notch on 2nd har.
quasi-static (theoretical)	$1.1\cdot10^{-13}$	$3.3\cdot10^{-14}$	$7\cdot10^{-14}$	$\mathbf{1.3\cdot10^{-13}}$	$7\cdot10^{-14}$
dynamic (extrapolated)	$7\cdot10^{-14}$	$4\cdot10^{-15}$	$3\cdot10^{-14}$	$\mathbf{7.6\cdot10^{-14}}$	$3\cdot10^{-14}$

4. Summary

We have built a diode laser pumped gas-cell rubidium frequency standard and demonstrated a frequency stability of $3\cdot10^{-13}\,\tau^{-1/2}$.

We have measured the effect on the frequency stability of our Rb clock due to PM noise, at even harmonics of the modulation, on the interrogating signal. Coefficients for the degradation in the white frequency noise have been experimentally determined for PM noise at Fourier frequencies up to the 350th harmonic or approximately 200 times the half bandwidth of the resonance. This data shows that the coefficients for the even harmonics larger than 10 times the resonance bandwidth fall very slowly with increasing harmonic number. This data enabled us to refine our microwave synthesis so that it now contributes approximately $1\cdot10^{-13}\,\tau^{-1/2}$ to the overall instability We can use the results of these experiments to roughly estimate how these effect would scale to another clock with a different resonance line width.

This data should also prove helpful in testing a complete theory of this effect.

Acknowledgments

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