USES AND POSSIBILITIES OF PIEZOELECTRIC OSCILLATORS*

By

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Certain crystals [1] which lack symmetry exhibit electrical charges in particular regions when subjected to stress or when heated or cooled. When the electrical charges are due to stress the effect is called piezoelectric, and, when due to heat, pyroelectric. All piezoelectric substances are pyroelectric, and it is doubtful if any pyroelectric effect would be obtained if stresses were eliminated. The effect of stress was discovered by P. and J. Curie.[2]

Though Rochelle salt has the greatest piezoelectric effect, and quartz a comparatively small one, the latter substance, on account of its mechanical properties, is more suitable for the uses and applications here described. Pioneer work in the practical applications of the piezoelectric effect has been described[3] by several experimenters. Prof. Cady and Prof. Pierce have been responsible for most of the applications of value in radio communication.

Quartz has a crystalline structure, as shown in Figure 1, OX being the optic axis. A plate cut out with its surface parallel to the OX axis and either of the other two principal axes, OY or OZ, shows piezoelectric effects which are very pronounced.

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[1] Rochelle salt, tourmaline, silicate of zinc, cane sugar, quartz, boracite, etc.

[2] Comptes Rendus, vol. 91, pp. 294 and 383, 1880. Voigt, "Lehrbuch der Kristallphysik," Leipsig, 1910; Graetz, "Handbuch der Elektrizitat und des Magnetismus," vol. 1, Leipzig, 1914. For similar observations consult Gehlers Physikalische Woerterb. Bd. 3, p. 255, 827. W. C. Roentgen, Weid. Ann. Bd. 18, p. 534, Bd. 19, p. 523, 1883. M. G. Lippman, Ann. d. Chemie (5), T. 24, p. 45, 1881. W. Thomson, Phil. Mag., vol. 36, p. 331, 1893.

[3] A. M. Nicolson, Proc. A. I. E. E., vol. 38, p. 1315, 1919; W. G. Cady, Proceedings of The Institute of Radio Engineers, vol. 10, p. 83, 1922; and G. W. Pierce, Proc. American Academy of Arts and Sciences, vol. 59, No. 4, p. 81, 1923.

Axes AB, CD, and EF are known as the piezoelectric axes. Piezoelectric effects are observed when the plate is placed between two metal sheets. The planes of the sheets are then perpendicular to one of the piezoelectric axes and parallel to the optic axis of the crystal. If the quartz plate is compressed, opposite charges will be induced in the two conducting sheets. Elongation reverses the polarity. Hence if the circuit is closed by a conductor connecting the two metal sheets, and the plate is subjected to alternating mechanical impulses, it generates corresponding alternating electric currents. An alternating e.m.f. impressed across the piezoelectric plate will also cause a similar mechanical vibration in it. This is true even if the metal sheets are not quite touching the respective faces of the plate.

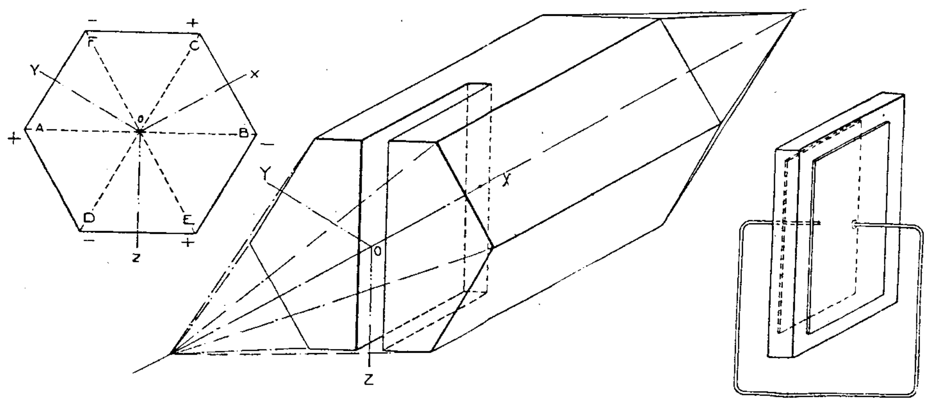

FIGURE 1—Quartz Crystal and Piezoelectric Pla.e

Piezoelectric Oscillator—From the foregoing it is evident that a plate of quartz can be used for converting mechanical vibrations into electrical oscillations and vice versa. The effects are greatest when the electrical oscillation is adjusted to resonance with a possible natural mechanical vibration of the piezoelectric plate. Using an electron tube circuit it is possible to have the piezoelectric plate control the oscillations which are set up. Such an arrangement is shown in Figure 2 where a quartz plate can be inserted either between the grid and the filament or between the grid and the plate.

The action of the circuit is as follows: Upon closing the circuit or moving certain portions of it, a transient current is started in it whose decay assumes a frequency which is due to a possible mechanical vibration of the piezoelectric plate. Normally such an oscillation would die out before being noticed. If, however, an inductance, L, of proper magnitude is inserted in the

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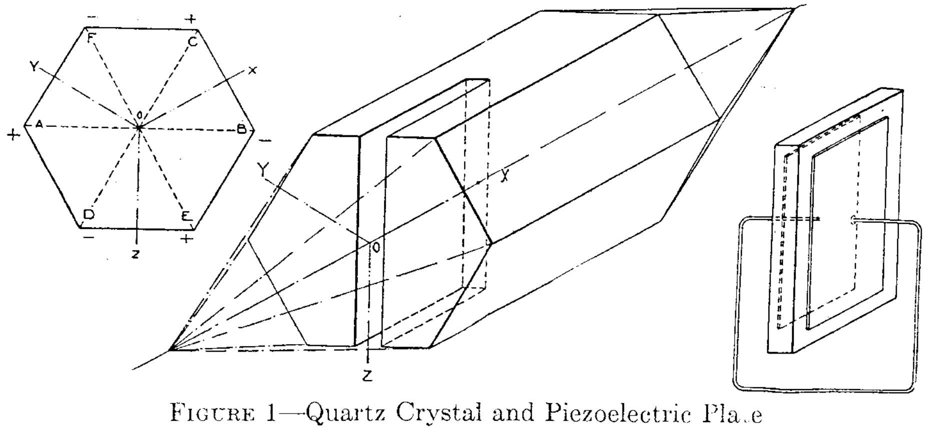

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The action of the circuit is as follows: Upon closing the circuit or moving certain portions of it, a transient current is started in it whose decay assumes a frequency which is due to a possible mechanical vibration of the piezoelectric plate. Normally such an oscillation would die out before being noticed. If, however, an inductance, L, of proper magnitude is inserted in the plate circuit, it will, by means of the feedback[4] through the tube itself, render the circuit regenerative, that is, produce the equivalent of a negative resistance between the grid and the filament, and sustain the oscillations due to the piezoelectric element. Whenever this happens, the plate current measured by a d. c. milliameter drops to a minimum value. The output can be increased by using a variable condenser, C, in parallel with the inductance, L, of the plate circuit. With the quartz plate connected between the filament and the grid, the condenser is set at its minimum position and its capacity is gradually increased. For a certain setting, the oscillations begin to build up and while increasing C, the plate current will decrease until the oscillations stop altogether when close to the resonance setting of the C-L circuit. For the quartz plate connected to the anode and the

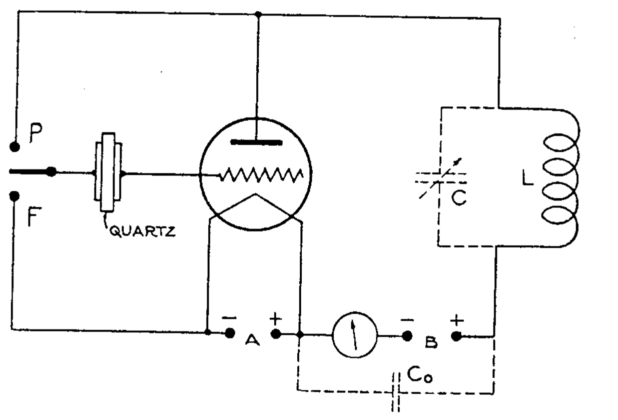

FIGURE 2—Circuit Connections for a Piezoelectric Oscillator

grid, the capacity, C, has to be gradually decreased from the maximum setting in order to start the oscillations. The building up of the oscillations takes place normally in a fraction of a second, but in some cases it may require several seconds, and it is therefore necessary to change C slowly. The slow building-up of the oscillations may be due to either a poor piece of quartz, incorrect value of the inductance L, or the lack of necessary freedom of the quartz plate. When using the quartz plate as an oscillator it is necessary for it to be free to move, while for use as a resonator the conductive layers can

[4] If the piezoelectric element is inserted between the filament and the grid the plate to grid capacity acts as the agency for the feedback of the plate actions into the grid branch. The filament to grid capacity provides the feedback, if the quartz plate is connected between the grid and the anode.

even be pasted on the piezoelectric plate or clamped against it without disturbing the operation.

Cutting of Quartz Plates Suitable for Oscillator Work.—Figure 3 shows a large piece of quartz which rests on a surface perpendicular to the optic axis along which no "direct" electrical effect is possible. Figure 4 shows another crystal of quartz likewise resting on a plane perpendicular to the optic axis. The lines indicate where a plate is to be cut out. Cuts of this type give a maximum piezoelectric effect with a small temperature coefficient which is either positive or negative. Although there are

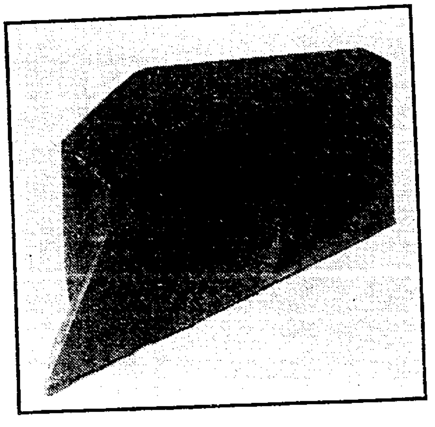

FIGURE 3—Piece of Quartz Resting on a Plane Perpendicular to the Optic Axis and Cut Along an Axis Such as OX, OY or OZ.

FIGURE 4—A Suitable Cut for Obtaining a Plate for Piezo Oscillators

cuts for which the temperature coefficient is zero it has been found more practical to cut as closely as possible along OX, OY or OZ directions (Figure 1) since then three well-defined natural vibrations occur. Figure 5 shows a quartz plate removed from the original crystal while Figure 6 shows several plates cut out from the same block. From such slices either rectangular plates are cut out as indicated on one of the slices or circular plates are cut out by means of a revolving brass tube. The latter shape can be secured more quickly since there are only two faces which need to be made parallel. There are several methods of cutting out slices from the natural crystal. One method is to use a plate of galvanized iron or copper which revolves against the crystal. To this is fed No. 150 carborundum powder mixed with about the same amount (by volume) of water. The work can be done somewhat faster by using a plate of copper whose circumference is finely ribbed and charged with diamond dust, using a steady flow of kerosene against the cutting edge. The faces are ground parallel first by means of No. 60 carborundum and water and then with No. 150 carborundum and water. The finishing is done by means of No. 140 emery powder mixed with water, then No. 302 emery and No. 303 emery in water solution. Most

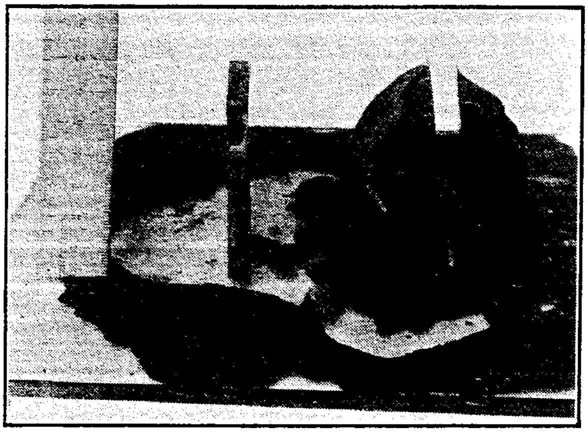

FIGURE 5—Quartz Plate Cut from the Natural Crystal

of the piezoelectric plates at the Bureau of Standards are polished to transparency by means of rouge and the edges somewhat bevelled. A polished plate sometimes has the tendency to chip

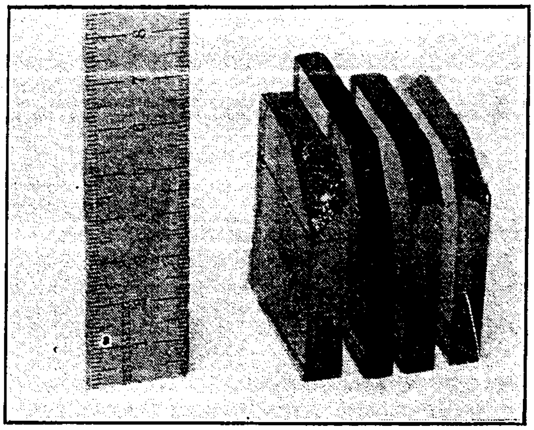

FIGURE 6—Several Piezoelectric Plates Cut from a Quartz Crystal

off near the edges. However, when the two main faces (parallel to the respective conducting layers) are polished, oscillations start more easily and such plates are usually more desirable. The breaking off of the small pieces may be due either to excessive mechanical vibrations in the plate or more probably to strains left near the polished surface, as in the case of Prince Rupert drops.

Important Natural Mechanical Vibrations of Quartz Plates.—When quartz plates are cut out as described above and indicated in Figures 1, 4 and 5, it will be found that in general three fundamental modes of natural oscillations are possible. Several of the many samples tested are described in Table I, from which it is seen that two of the natural frequencies are close together with respect to the third. The reason for this is that the thickness of the plates is small in comparison with the other two dimensions.

When rectangular plates are cut not along planes parallel with either OX, OY, or OZ (Figure 1) but along planes of zero temperature coefficient of frequency or planes closer to directions OA, OD, or OE, either three or more natural mechanical vibrations occur and with frequencies which are not always in agreement with those expected when the normal cuts are employed. For example, a rectangular plate $24.94 \times 19.29 \times 3.03$ mm. gave $f_1 = 105$; $f_2 = 612$, and $f_3 = 924$ kc., that is, two higher frequencies, f_2 and f_3 respectively, in comparison to the third frequency f_1. A normal cut would have given two low frequencies and a high one. In another case four natural frequencies were obtained, the plate having the dimensions $24.775 \times 19.92 \times 3.04$ mm., that is, almost the identical shape as in the first case. These frequencies were $f_1 = 105.6$; $f_2 = 606.5$; $f_3 = 819.5$ and $f_4 = 932.5$ kc.

Using normal cuts as shown in Figure 1, it can be said that three fundamental natural frequencies are possible, irrespective of whether the cut is bounded by a rectangle, square, ellipse or circle, as long as the thickness is small, but not too small, in comparison with the other dimensions. If a thickness is used which is comparable to the other dimensions such as in a cube, it would take a strong electric field to start the oscillations and there would be little gain since the frequencies would be close together. In the same way when the shortest dimension becomes small, less than about 2 mm., special measures have to be employed to start the highest frequency.

In all cases, irrespective of shape, the fundamental vibration due to the thickness can be estimated with a good degree of approximation from the expression

$$f_3 = \frac{2870}{t} \tag{1}$$

when f is expressed in kilocycles per second (kc.) and the thickness t measured in mm. It is therefore seen that the product of the frequency and the thickness is a constant

$$K_3 = f_3 \cdot t = 2870 \tag{2}$$

This is shown by the data of Table I and II.

Data on circular plates used as piezo oscillators, f_1 is the lowest f_2 the medium, and f_3 the highest natural frequency of the quartz plate.

Since for oscillator work the surfaces of the piezoelectric plate are free, it can be assumed that a fundamental vibration has its node at the center of the plate and consequently the thickness is equal to one-half wave length. If v_3 denotes the velocity of propagation along the thickness t, that is, along the electrostatic lines across the plate and λ_3, the wave length, we have

$$v_3 = \lambda_3 f_3 = 2\,t f_3 = 2\,K_3 \tag{3}$$

Hence twice the value of the constant K_3 gives the velocity of propagation, 574×10^3 cm./sec. This result agrees within about 5% with a value obtained from a theoretical formula.[5] This agreement would probably be much better if the appropriate modulus of elasticity were used. The other frequencies which are lower and due to larger dimensions can not be calculated by formula (1). However, for circular plates

$$f_1 = \frac{2715}{d} \tag{4}$$

$$f_2 = \frac{3830}{d} \tag{5}$$

where d is the diameter of the plate in millimeters and the respective frequencies are again expressed in kilocycles. The characteristic constants for these two oscillations are then

$$K_1 = f_1\,d = 2715 \tag{6}$$

and

$$K_2 = f_2\,d = 3830 \tag{7}$$

and the respective velocities of propagation along the diameters are:

$$v_1 = 2\,K_1 = 543 \times 10^3 \text{ cm./sec.} \tag{8}$$

$$v_2 = 2\,K_2 = 766 \times 10^3 \text{ cm./sec.} \tag{9}$$

using similar assumptions as for v_1.

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[5] $$v_3 = \sqrt{\frac{\textit{modulus of elasticity}}{\textit{density}}} = \sqrt{\frac{785 \times 10^{11}\,dynes/cm.^2}{2.654\,gn/cc}}$$
$$= 540 \times 10^3\, cm/cc$$

TABLE I

DATA ON SEVERAL QUARTZ PLATES USED IN PIEZO OSCILLATORS
CIRCULAR PLATES.

Quartz Plate	Shape	Dimensions in mm.				Natural Fundamental Frequencies in kilo: cycles per secomd			Correspomding Wave lengths in meters							
		t	d	a	b	f_1	f_2	f_3	λ_1	λ_2	λ_3	f_3t	f_1a	f_2b	f_1d	f_2d
A	Circular plate	6.307	36.15			75.05	105.91	454.2	3995	2831	659.8	2871			2715	3815
B	Circular plate	4.782	39.17			69.36	97.625	600	4354	3070	499.5	2874			2700	3827
C	Circular plate	6.0	57.97			46.80	66.27	475.25	6405	4523	631	2854			2715	3841
D	Circular plate	5.78	49.3			55.115	77.82	492.8	5445	3860	608.5	2852			2717	3831
E	Circular plate	4.87	49.27			55.23	77.76	583.6	5425	3855	505	2893			2727	3831
F	Circular plate	6.075	49.3			55.01	77.715	470	5445	3860	638	2857			2717	3831
G	Circular plate	3.4325	49.58			54.88	77.375	843.7	5461	3880	355.2	2899			3725	3831
H	Circular plate	3.4975	58.38			46.5	65.675	822	6445	4560	364.7	2879			2717	3836
I	Circular plate	4.277	39.37			68.5	97	670	4376	3090	447.5	2868			2700	3822
J	Circular plate	8.805	21.84			124.8	175.5	326.75	2401	1708	917.5	2882			2727	3836
K	Circular plate	8.28	29.2			93.4	131.8	346.25	3210	2273	866	2876			2727	3856
							Rectangular Plates									
L	Rectangular plate	3.12		31.855	25.155	79.925	105.41	924.57	3752	2843	324.2	2887	2549	2655		
M	Rectangular plate	4.909		39.005	30.295	70.69	102.42	587	4240	2928	510.5	2886	2759	3099		
N	Rectangular plate	7.177		42.08	39.62	66.05	85.43	404	4538	3508	742	2901	2786	3390		
O	Rectangular plate	3.264		36.82	25.2	80.485	106.67	873.5	3725	2810	343.4	2854	2967	2691		
P	Rectangular plate	7.59		44	35	64.6	83.4	382.8	4640	3595	783.5	2907	2846	2921		
Q	Rectangular plate	6.83		48.5	37.5	59	78.8	417.4	5080	3805	718	2855	2866	2956		
R	Rectangular plate	5.04		39	30	70.7	102.2	569.5	4240	2932	527	2871	2762	3067		

f_1 is the lowest, f_2 the medium, and f_3 the highest natural frequency of the piezo oscillators. t is the thickness of the rectangular plate, d the diameter of the circular plates, a and b are the length and breadth of the rectangular plates.

The optic axis is along a line which is parallel to the circular planes of the plate and so one diameter is parallel with the optic axis. Along a diameter perpendicular to the optic axis the modulus of elasticity is, as given in footnote 5, 7.85×10^{11} dynes/cm.^{2} and with the density value given in that formula confirms exactly the velocity, v_1, of equation (8). The value, v_2, is high and even if it is assumed that the vibration occurs along the direction of the optic axis for which the modulus would be 10.3×10^{11} dynes/cm.^{2} the theoretical velocity is only 623×10^3 cm./sec., which is about 22 percent low. The mechanism for this oscillation is

TABLE II

DATA ON CIRCULAR PLATES USED AS PIEZO OSCILLATOR

Quartz Plate	Natural Fundamental Frequencies in Kilocycles per second			Corresponding Wave lengths in meters			$f_1.d$	$f_2.d$	$f_3.t$
	f_1	f_2	f_3	λ_1	λ_2	λ_3			
A_1	74.95	105.5	452.5	4000	2842	663	2710	3814	2856
A_2	75.05	105.91	454.2	3995	2831	659.8	2713	3829	2865
A_3	74.9	105.15	454.25	4003	2853	660	2707	3800	2868
A_4	75.4	106.0	457.0	3976	2828	656	2726	3830	2873
A_5	74.75	105.35	454.5	4011	2847	659.5	2700	3810	2869
A_6	75.0	105.75	454.5	3997	2835	659.5	2710	3828	2869
A_7	75.1	106.25	453.75	3991	2822	661	2713	3840	2861
A_8	74.8	105.5	457.25	4007	2842	655.5	2703	3814	2887
A_9	75.25	106.5	453.25	3984	2815	661.5	2720	3850	2860
A_{10}	74.75	106.0	454.25	4011	2828	660	2700	3830	2868
A_{11}	74.8	105.95	453.25	4007	2829	661.5	2703	3830	2861
A_{12}	74.95	105.55	454.25	4000	2840	660	2710	3819	2868
A_{13}	74.7	105.55	454.5	4013	2840	659.5	2700	3819	2869
A_{14}	74.75	106.1	454.5	4011	2827	659.5	2700	3835	2869
A_{15}	75.15	106.0	456.0	3989	2828	657	2716	3830	2869
A_{16}	75.05	105.75	454.5	3995	2835	659.5	2713	3825	2860
A_{17}	74.65	105.15	453.75	4016	2853	661	2699	3800	2860
A_{18}	75.1	106.1	453.25	3991	2827	661.5	2714	3836	2860
A_{19}	74.9	105.35	452.5	4003	2847	663	2707	3810	2856

(Circular Quartz plates all of the same size, diameter = 36.15 mm. and thickness = 6.31 mm. cut from different pieces of quartz as indicated in Figure 1.)

probably more complicated than the other two. Nevertheless this in no way gives any practical difficulties since equation (5) can be used to a fair degree of approximation even though a better explanation cannot be given at the present time.

According to Table I and many other data, it is more difficult to give good approximation formulas for the two low frequencies, f_1 and f_2, respectively, of rectangular plates since some values deviate too much from the average value. The average value of the lowest frequency gives a characteristic constant of about $K_1 = 2785$ and for the medium frequency $K_2 = 2945$. The corresponding velocity values, $v_1 = 2\ K_1$, and $v_2 = 2\ K_2$, are reasonable. It is interesting to note that their average value gives about the value for the constant K_3. From these observations it looks as though the three characteristic vibrations occur along the three main dimensions. This appears to contradict the theory of piezoelectricity, at least at first sight, since one dimension is parallel to the optic axis along which no piezoelectric effect is possible. But since a contraction across a set of small faces perpendicular to the optic axis produces an expansion along this axis and vice versa, there will be a disarrangement of the molecules. The effect of this probably produces the third vibration.

Another explanation would be that the piezoelectric plate acts like a coupled circuit which produces two frequencies, one which is somewhat lower and another which is somewhat higher than the expected frequency. The degree of coupling is dependent on the relative magnitude of the three main axis.

A third explanation makes use of the fact that the vibration parallel to the two conducting layers may be due to both a transverse and a longitudinal wave motion which have different velocities of propagation and so produce two waves of different frequencies.

Experience indicates that it is best to use circular plates. It is essential that the faces be exactly parallel since otherwise the frequency spectrum of the highest frequency may give several values or not appear at all. It happens sometimes that a plate will not oscillate at all, although the faces appear to be parallel. A little grinding which would hardly be noticed with a micrometer brings in the oscillation. This is probably due to the fact that a certain stiffness exists so as to annul the effect. Sometimes after grinding, the oscillation disappears again and further grinding starts another higher oscillation and so on, which somewhat confirms the last explanation. Some plates work exceedingly well because one fundamental oscillation is a multiple of another fundamental oscillation.

Besides the three fundamental oscillations a piezo oscillator (Figure 2) will give a series of harmonics. They are due mostly to the distortion produced by the tube circuit. They should be distinguished from the higher modes of oscillation when the piezoelectric plate is used as a resonator and for which no harmonic relations necessarily exist.

Calibration of Frequency Meters by Means of a Piezo Oscillator

The method, which is due to Prof. G. W. Pierce (see footnote No. 3), is illustrated in Figure 7. Use is made of the three fundamental frequencies due to the quartz plate, the harmonics of the corresponding electrical oscillations as well as of the harmonics of the auxiliary generator.

The procedure of measurement is as follows:

(1) By means of the inductance L_1 and the variable air condenser C_1 the piezo oscillator is set to one of the three fundamental vibrations of the quartz plate, say for instance, to 80 kc. This is accompanied by using an inductance L_1 which tunes for a certain setting of C_1 to this frequency.

With the piezoelectric plate connected between the filament and the grid, the capacity C_1 is gradually increased until a decrease is noted in the milliammeter in the plate circuit. The capacity of C_1 is further increased until[6] the oscillations stop.

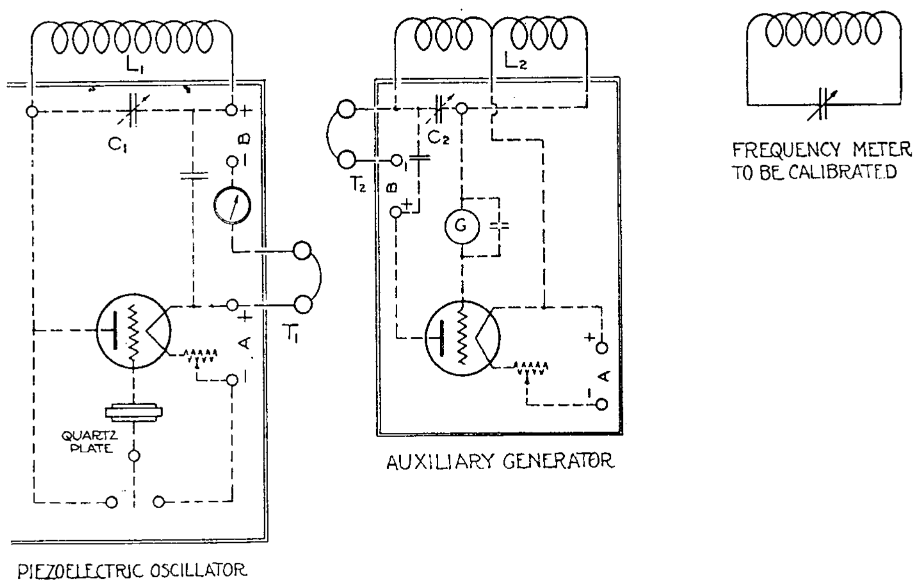

FIGURE 7—Arrangement for Calibrating a Frequency Meter

This happens when the C_1-L_1 branch becomes a capacity reactance and produces a positive resistance between the grid and the filament. The circuit is then no longer regenerative and cannot sustain the oscillations set up by the piezoelectric plate.

The oscillations are started once more by setting the condenser at a point just below the resonance setting. Such a condition will, as a rule, produce a current which is rich in harmonics with a fundamental frequency of remarkable steadiness.[7]

(2) The loosely coupled auxiliary generator is set to the same fundamental frequency by noting the beat note either with the

[6] A milliammeter giving 5 to 10 ma. for maximum deflection is suitable for this work if an ordinary receiving tube is being used with about 80 volts on the plate.

[7] A weight of about 50 grams when resting on a plate vibrating at about 10^5 cycles per second does not change the frequency more than about 10 cycles, and one degree Centigrade change in temperature produces a frequency change of about 20 parts in one million.

telephone receiver T_1 or T_2. Resonance occurs where the critical silence condition (zero beat) within the range of the audible note is secured.[8]

(3) The frequency meter to be calibrated is loosely coupled to the auxiliary generator and its setting varied until the grid milliammeter G of the generator shows a decrease. When this is a minimum the frequency meter is tuned to resonance with the auxiliary generator, *i. e.*, to the fundamental[9] frequency of the piezo oscillator which, for example, is 80 kc.

(4) The frequency of the auxiliary generator is then increased until the next best beat note is heard and the critical silence position adjusted. The grid current decrease produced by the resonance of the frequency meter then gives the calibration for the second harmonic which is for the example 2×80 kc. Without the aid of an amplifier it is possible to go up to about the 20th harmonic. Using one or two stages of audio-frequency amplification for the beat note, it is possible to hear beat notes up to about the 200th harmonic. For ondinary work, the amplifiers can be dispensed with since the twenty harmonics of each of the three fundamental vibrations of the quartz plate give sufficient points. For beat notes due to harmonics of the piezo osillator with the fundamental of the auxiliary generator, it is best to use the receivers T_1 because the beat notes are then louder.

(5) By adjusting the fundamantal frequency of the auxiliary generator to half of the frequency of the piezo oscillator, that is, in the above case to 40 kc., the second harmonic (2×40 kc.) of the auxiliary generator will beat with the fundamental current of the piezo oscillator and the grid current decrease will give the calibration for $f/2 = 40$ kc., if f denotes the fundamental frequency of the piezo oscillator. In a similar way we get calibrations for $f/3$, $f/4$, $f/5$, etc., and this can be readily carried on at least to $f/20$ without the use of an amplifier. For practical frequencies the telephone receiver T_2 should be used since the harmonics of the auxiliary generator are being utilized. Since there are normally three fundamental frequencies, f_1, f_2 and f_3 of the quartz plate, it is evident that one quartz plate is sufficient to

[8] The beat note passes from a high pitch through a low note, narrow silence region, and again up to a higher note, while the condenser C of the auxiliary generator is being varied. The critical silence position can be set within about 15 cycles per second and even somewhat less by means of telephone receivers. If a still greater accuracy is required, a milliammeter can be used instead of phones for watching any of the slower beats between zero and 15 cycles.

[9] By means of the grid milliammeter, the frequency meter can be coupled very loosely to the auxiliary generator with a negligible effect on the frequency.

check the entire range of frequencies used in radio communication. Figures 8a and 8b show characteristic frequency spectra for a circular quartz plate and a rectangular plate, respectively. The exact values are given in Tables III and IV and are computed from only three primary calibrations, namely from the three fundamental frequencies f_1, f_2 and f_3 of the piezoelectric plate.

Example—Suppose a certain frequency meter is to be calibrated for a range between 10 and 50 degrees of the condenser

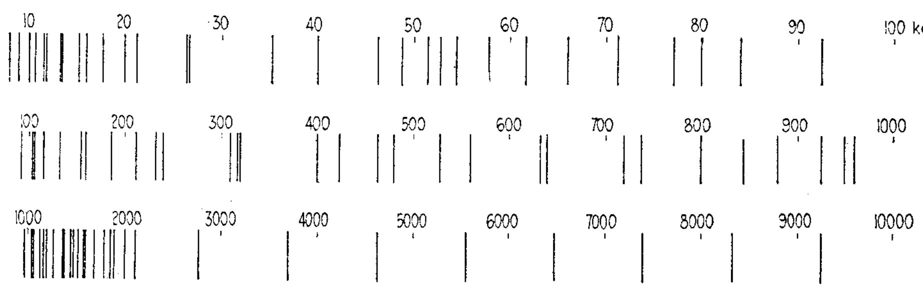

FIGURE 8a—Frequencies Produced by a Piezo Oscillator Using a Circular Quartz Plate 6.307 mm. Thick, and a Diameter of 36.15 mm. The Three Fundamental Frequencies are 75.05, 105.91 and 454.2 kc.

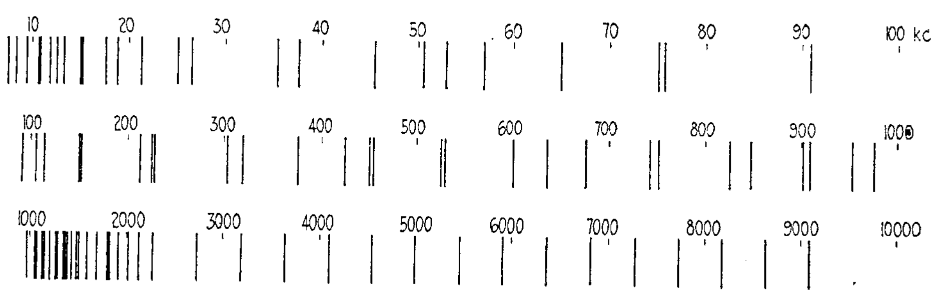

FIGURE 8b—Frequencies Produced by a Piezo Oscillator Using a Rectangular Quartz Plate 3.12 x 25.155 x 31.855 mm. The Three Fundamental Frequencies are 79.925, 105.41 and 924.57 kc.

setting. By means of the auxiliary generator whose settings are approximately known, a few points can be readily found to an accuracy of about one percent. Otherwise the old calibration of the frequency meter can be used or the calibration be estimated from the capacity and the approximate value of the inductance. Suppose that with one of the above procedures it has been found that at 20 degrees the frequency is roughly 226 kc.; at 30 degrees it is 190 kc., and at 40 degrees it is 168 kc. Using the piezo oscillator with the circular plate we find from Table III that 225.15 kc. is closest to 226 kc. and is due to the third harmonic ($3\,f_1$) of the lowest frequency of the piezoelectric plate, while 227.1 kc. corresponds to a setting of the auxiliary generator of half of the frequency ($f_3/2$) of the highest natural frequency of the plate.

TABLE III

FREQUENCIES AVAILABLE FROM PIEZO OSCILLATOR A (Circular Plate)

$f_1 = 75.05$ kc.	$f_2 = 105.91$ kc.	$f_3 = 454.2$ kc.
7.505 $= f_1/10$	211.82 $= 2 f_2$	1350.9 $= 18 f_1$
8.3389 $= f_1/9$	225.15 $= 3 f_1$	1362.6 $= 3 f_3$
9.38125 $= f_1/8$	227.1 $= f_3/2$	1376.83 $= 13 f_2$
10.591 $= f_2/10$	300.2 $= 4 f_1$	1425.95 $= 19 f_1$
10.7214 $= f_1/7$	317.73 $= 3 f_2$	1482.74 $= 14 f_2$
11.7678 $= f_2/9$	375.25 $= 5 f_1$	1501.0 $= 20 f_1$
12.5083 $= f_1/6$	423.64 $= 4 f_2$	1588.65 $= 15 f_2$
13.2388 $= f_2/8$	450.3 $= 6 f_1$	1694.56 $= 16 f_2$
15.01 $= f_1/5$	454.2 $= f_3$	1800.47 $= 17 f_2$
15.13 $= f_2/7$	525.35 $= 7 f_1$	1816.6 $= 4 f_3$
17.6517 $= f_2/6$	529.55 $= 5 f_2$	1906.38 $= 18 f_2$
18.7625 $= f_1/4$	600.4 $= 8 f_1$	2012.29 $= 19 f_2$
21.182 $= f_2/5$	635.46 $= 6 f_2$	2118.2 $= 20 f_2$
25.0167 $= f_1/3$	675.45 $= 9 f_1$	2271.0 $= 5 f_3$
26.4775 $= f_2/4$	741.37 $= 7 f_2$	2725.2 $= 6 f_3$
35.3033 $= f_2/3$	750.5 $= 10 f_1$	3179.4 $= 7 f_3$
37.525 $= f_1/2$	825.55 $= 11 f_1$	3633.6 $= 8 f_3$
45.42 $= f_3/10$	847.28 $= 8 f_2$	4087.8 $= 9 f_3$
50.4667 $= f_3/9$	900.6 $= 12 f_1$	4542.0 $= 10 f_3$
52.955 $= f_2/2$	908.4 $= 2 f_3$	4996.2 $= 11 f_3$
56.775 $= f_3/8$	953.19 $= 9 f_2$	5450.4 $= 12 f_3$
64.8857 $= f_3/7$	975.65 $= 13 f_1$	5904.6 $= 13 f_3$
75.05 $= f_1$	1050.7 $= 14 f_1$	6358.8 $= 14 f_3$
75.7 $= f_3/6$	1059.1 $= 10 f_2$	6813.0 $= 15 f_3$
90.84 $= f_3/5$	1125.75 $= 15 f_1$	7267.2 $= 16 f_3$
105.91 $= f_2$	1165.01 $= 11 f_2$	7721.4 $= 17 f_3$
113.55 $= f_3/4$	1200.8 $= 16 f_1$	8175.6 $= 18 f_3$
150.1 $= 2 f_1$	1270.92 $= 12 f_2$	8629.8 $= 19 f_3$
151.4 $= f_3/3$	1275.85 $= 17 f_1$	9084.0 $= 20 f_3$

TABLE IV

FREQUENCIES AVAILABLE FROM PIEZO OSCILLATOR B (Rectangular Plate)

$f_1 = 79.925$ kc.	$f_2 = 105.41$ kc.	$f_3 = 924.54$ kc.
7.9925 $= f_1/10$	115.567 $= f_3/8$	1264.92 $= 12 f_2$
8.8805 $= f_1/9$	132.077 $= f_3/7$	1278.8 $= 16 f_1$
9.9906 $= f_1/8$	154.09 $= f_3/6$	1358.725 $= 17 f_1$
10.541 $= f_2/10$	159.85 $= 2 f_1$	1370.33 $= 13 f_2$
11.4178 $= f_1/7$	184.908 $= f_3/5$	1438.65 $= 18 f_1$
11.712 $= f_2/9$	210.82 $= 2 f_2$	1475.74 $= 14 f_2$
13.176 $= f_2/8$	231.135 $= f_3/4$	1518.575 $= 19 f$
13.321 $= f_1/6$	239.775 $= 3 f_1$	1581.15 $= 15 f_2$
15.058 $= f_2/7$	308.18 $= f_3/3$	1598.5 $= 20 f_1$
15.985 $= f_1/5$	316.23 $= 3 f_2$	1686.56 $= 16 f_2$
17.568 $= f_2/6$	319.7 $= 4 f_1$	1791.97 $= 17 f_2$
19.981 $= f_1/4$	399.625 $= 5 f_1$	1849.08 $= 2 f_3$
21.082 $= f_2/5$	421.64 $= 4 f_2$	1897.38 $= 18 f_2$
26.352 $= f_2/4$	462.27 $= f_3/2$	2002.79 $= 19 f_2$
26.642 $= f_1/3$	479.55 $= 6 f_1$	2108.2 $= 20 f_2$
35.137 $= f_2/3$	527.05 $= 5 f_2$	2773.62 $= 3 f_3$
39.9625 $= f_1/2$	559.475 $= 7 f_1$	3698.16 $= 4 f_3$
46.23 $= f_3/20$	632.46 $= 6 f_2$	4622.7 $= 5 f_3$
48.66 $= f_3/19$	639.4 $= 8 f_1$	5547.24 $= 6 f_3$
51.36 $= f_3/18$	719325 $= 9 f_1$	6471.78 $= 7 f_3$
52.705 $= f_2/2$	737.87 $= 7 f_2$	7396.32 $= 8 f_3$
54.38 $= f_3/17$	799.25 $= 10 f_1$	8320.86 $= 9 f_3$
57.78 $= f_3/16$	843.28 $= 8 f_2$	9245.4 $= 10 f_3$
61.64 $= f_3/15$	879.175 $= 11 f_1$	10169.94 $= 11 f_3$
66.04 $= f_3/14$	924.54 $= f_3$	11094.48 $= 12 f_3$
71.12 $= f_3/13$	948.69 $= 9 f_2$	12019.02 $= 13 f_3$
77.04 $= f_3/12$	959.1 $= 12 f_1$	12943.56 $= 14 f_3$
79.925 $= f_1$	1039.025 $= 13 f_1$	13868.1 $= 15 f_3$
84.05 $= f_3/11$	1054.1 $= 10 f_2$	14792.64 $= 16 f_3$
92.454 $= f_3/10$	1118.95 $= 14 f_1$	15717.18 $= 17 f_3$
102.727 $= f_3/9$	1159.51 $= 11 f_2$	16641.72 $= 18 f_3$
105.41 $= f_2$	1198.875 $= 15 f_1$	17566.26 $= 14 f_3$
		18490.8 $= 20 f_3$

Either one can be used since the points are rather close together. In a similar way 151.4 kc. $=f_3/3$ requires that the auxiliary generator is set to one-third of the frequency of the highest frequency of the piezoelectric plate corresponding roughly to about 50 degrees on the scale of the auxiliary generator. It is convenient to have a rough calibration for the auxiliary generator used. The accurate calibration is then as follows:

(a) The piezo oscillator is excited to the higher frequency which in this particular case is $f_3=454.2$ kc.

(b) The auxiliary generator is set to a frequency of about 151 kc. according to the rough calibration given on the curve of the coil and the condenser C_2 varied slightly until a beat note is heard and then adjusted to the point of zero beat.

(c) The dip [12] in the grid current produced by the resonance of the frequency meter then gives the calibration for 151.4 kc.

(d) By searching by means of C_2 and for the same vibration ($f_3=454.2$ kc.) of the plate in the neighborhood of 225 kc. for a beat note and setting again to the critical silence point, calibration for 227.1 kc. is secured.

(e) Next, the piezo oscillator is adjusted so that the plate vibrates at $f_1=75.05$ kc., and a beat note found near 225 kc. After securing the critical silence point and obtaining the grid current dip, the calibration for 225.15 kc. is obtained.

Beats Between Harmonics and Their Application

When the coupling between the auxiliary generator and the piezo oscillator is somewhat closer but still loose enough to avoid any objectionable interaction between the respective circuits, it is possible to hear weak beat notes, which correspond to 1.25, 1.33, 1.5, etc., times a fundamental frequency f of the quartz plates of zero beat settings for $f/1.25$, $f/1.33$, $f/1.5$, etc. Such beats are caused by the interference of harmonic currents of the piezo oscillator with harmonic currents of the auxiliary generator. This is evident when we express $1.25f$ by $5f/4$ and note that for a fundamental frequency f of the auxiliary generator and its adjustment to zero beat within the region of such an interference the relation

$$5f\frac{=F}{4}$$

[12] For certain couplings (which are not extremely loose) as the frequency meter gets gradually in resonance with the auxiliary generator a low beat note appears again and disappears as resonance occurs. This method can be used for checking the grid dip.

or
$$5f=4F$$
holds. Hence the fifth harmonic of the piezo oscillator produces a zero beat with the fourth harmonic of the auxiliary generator. In a similar way the case of $1.33f=\frac{4f}{3}$ shows that the fourth harmonic of the piezo oscillator is beating with the third harmonic of the auxiliary generator and that for $1.5f$ the third harmonic of the piezo oscillator beats with the second harmonic of the auxiliary generator.

Such beats can also be explained by means of beats of beat currents. Suppose the piezo oscillator is excited with a fundamental frequency $f=80$ kc., and that the fundamental frequency of the auxiliary generator is set to $F=125$ kc., then an interference takes place between the fundamental currents of the respective high frequency sources. The amplitude variation of the resultant current occurs at the rate of $F-f=45$ kc., which corresponds to a high-frequency variation which is not audible. Another amplitude variation which is possible is due to the interference between the second harmonic of the piezo oscillator and the fundamental current of the auxiliary generator which produce again a high-frequency variation but of frequency $2f-F=35$ kc. But these two high-frequency currents can beat again and with each other producing an audible current of frequency $45-35=10$ kc. If the auxiliary genenator is, therefore, varied until $F=120$ kc., then
$$F-f=2f-F=40 \text{ kc.}$$
and a zero beat condition is attained which confirms the case of $1.5f=120$ kc. According to this explanation the so-called "spurious" beat notes are due to beats between beat currents, which accounts for the fact that they are, as a rule, weaker than the beat notes giving the settings as expected directly from the theorm of Fourier. The first explanation by means of the interference between the harmonics of each circuit confirms the law
$$a.f=b.F \qquad (10)$$
where a and b are whole numbers and is perhaps the most direct way of explaining the phenomenon.

For ordinary work, it seems best to utilize only the main harmonics as shown in Tables III and IV, but if more points are required, for example, two more points between 90.84 and 105.91 kc. (Table III), it is possible to secure them by means of the fundamental plate vibration $f_1=75.05$ kc. by using $\frac{5f_1}{4}=93.81$

and $\frac{4f_1}{3} = 99.82$ kc.

Method Used for Grinding Piezoelectric Plates Accuratey to the Required Frequency

It is possible to grind a quartz plate accurately within a small fraction of a desired frequency, even though the desired frequency is of the order of 10^6 cycles per second. An ordinary standard frequency meter does not have the resolving power to indicate such accurate settings, but the beat method, with a visual beat indicator, can be used. The principle of this method is as follows: An auxiliary generator is required, which can be set at the desired frequency and can maintain it constant for some time. The piezoelectric plate, after first being ground to the approximate frequency according to the frequency formulas given in this paper, is connected as indicated in Figure 2. The holder for the plate is shown in Figure 9b and provides an air

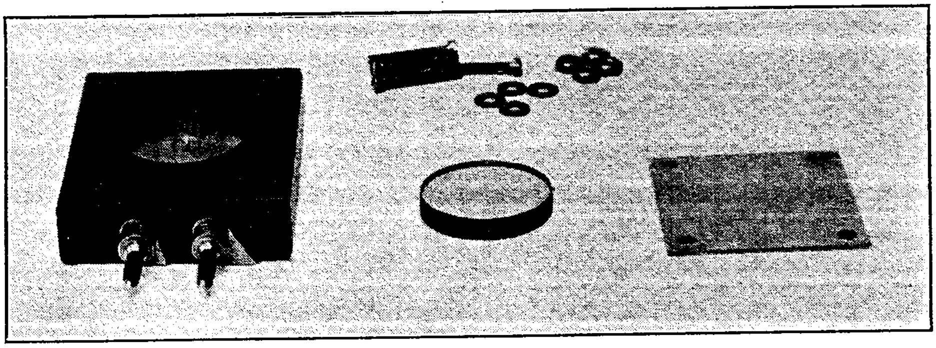

Figure 9a—Quartz Plate and Holder for Piezo Oscillator

gap of about one-third mm. between the upper face of the plate and the brass cover. Figure 9a shows the holder open. The plate rests on a polished brass plate. A note will be heard in the telephone receiver of the piezo oscillator, since the frequency of the plate is somewhat off. The plate is taken out of the holder and moved slightly over a grinding plate, using fine emery with water and tried again and again until the note becomes so low as to be difficult to hear. This indicates that the frequency difference is about 15 cycles, the exact frequency depending on the observer. By using a portab... alvanometer instead of the phones, the slower beats between 15 and zero cycles can be indicated. It is convenient to connect a portable galvanometer using about 1 to 2 ma. current for the maximum deflection in series with a crystal detector and a coil coupled loosely to both the piezo oscillator and the auxiliary generator. When the pointer swings to and fro twice in one second, it indicates that the frequency is off by two cycles per second; and if the pointer moves once in two minutes, the frequency is only off by 1/120th of a cycle per second. Accuracies of such a nature are seldom required, and the grinding, according to the formulas given here, is usually sufficiently accurate.

Rough Test for Suitable Piezoelectric Materials

For a rough test of material to determine the suitability for piezo oscillators, a plate or disk is cut from the material which

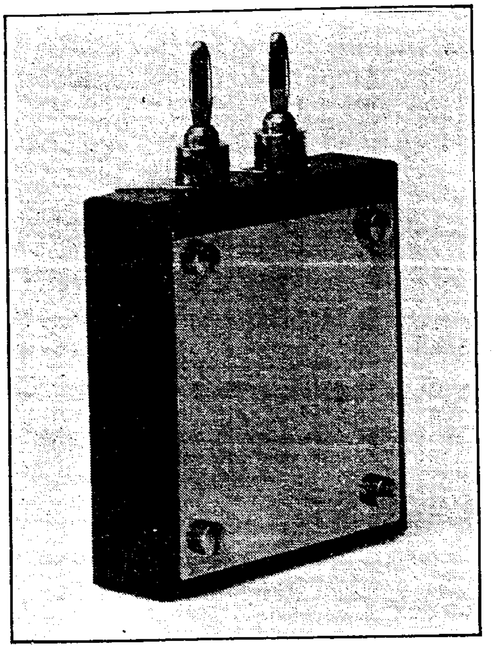

Figure 9b—Quartz Plate Holder for Piezo Oscillator Assembled

does not need to be exactly parallel), and this plate is inserted between the plate and the grid of an auxiliary generator which has a telephone receiver in the plate circuit. If the sample is suitable it will give several distinctive absorption noises while acting as a resonator. The frequency of the generator is gradually varied and a click will be heard for the different modes of resonator actions.

Method for Controlling Power by Means of Piezoelectric Plate

Since it is not possible to generate much power[13] in the circuit in which the quartz plate is connected, an amplifier which is free from self-oscillation must be used. An arrangement is shown in Figure 10. The piezo oscillator is set for maximum output so that its fundamental current is almost sinusoidal and pronounced. This is done by varying the capacity of C (circuit as in Figure 2), until the oscillation is about to stop. The amplifier tube (50-watt tube) as well as the power tube (250 watts) use negative grid voltages so that they deliver no plate current at times when the piezoelectric plate is not vibrating. These tubes are therefore only loaded at such times as they are required to deliver power. Experiment shows that there is no transfer of power back to the piezo oscillator, and that the arrangement does not generate but merely amplifies the current of the first circuit. This can be demonstrated by keying the switch K. With the

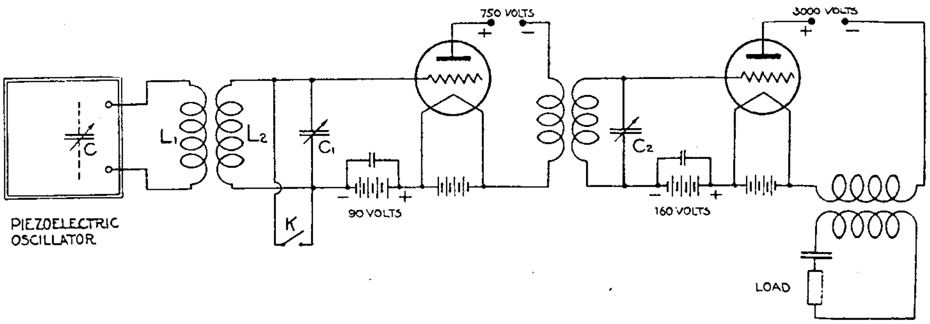

FIGURE 10—Control of Power by Means of Piezoelectric Oscillator

switch open, incandescent lamps consuming 250 watts in place of the load burn brightly; with the switch closed they are extinguished. The arrangement indicated works well. To control more power, one or more stages may be added. The control of power is of special interest for high-frequency work (2,000 kc. and above), where the constancy of the frequency is otherwise greatly impaired by body capacity, etc. Special means have to be used then to produce the very high-frequency oscillations. One is, for instance, by using a suitable auxiliary voltage in the grid circuit of the piezo oscillator. If an appropriate choke coil is used instead of the auxiliary voltage in the grid branch, care muct be taken that the tube does not produce oscillations which are due to the constants of the circuit.

AUDIO-FREQUENCY CURRENTS FROM PIEZO PLATES OF MODERATE SIZE

Since the frequency in kilocycles of a quartz plate is roughly 3,000 divided by the dimension in mm. along which the vibration occurs, it is evident that a very large plate has to be used for producing audio currents. Audio frequencies can, however, be obtained when the interference vibrations of two high frequencies are produced, giving beats of audio frequency. There are three methods for accomplishing this.

(1) Use of two piezo oscillators. Suppose one marked A has a fundamental frequency of 100 kc. and another marked B a frequency of 99 kc. Each plate is connected in a separate piezo oscillator as shown in Figure 2. The two circuits may be coupled to a third circuit with a detecting device and will give an audio frequency current of 1 kc. Either of the two circuits may be used also as a detector, and the audio frequency taken directly from one of them. For ordinary work this gives the audio-frequency currents readily. However, one circuit has the tendency to affect the other, that is, by adjusting the amplitude of one high-frequency component the frequency of the audio current varies somewhat, often as much as 10 to 20 cycles per second. This is a disadvantage when a high precision is required unless the circuit is calibrated and used only for certain amplitudes (condenser settings of the respective piezo oscillators).

(2) Use of two quartz plates in the same circuit. The two plates A and B may be connected in parallel but in separate plate holders and in the same circuit which gives directly the audio-frequency current. The audio-frequency oscillation is then a little harder to start since both high frequencies are produced by the same tube. It may happen that one of the two high-frequency vibrations builds up faster than the other and uses all of the available power and annuls the effect of the other oscillation. This is sometimes accompanied by a short whistle during which period both oscillations exist. Sometimes there is no whistle at all, in which case only one oscillation starts up.

By using frequencies in the neighborhood of 100 kc. and higher, it is easy to find a plate inductance which starts both frequencies and produces the desired audio-frequency current. If it is done properly the audio frequency can be produced for a range of condenser setting (about 10 to 20 degrees) and adjusted to a point for which maximum loudness exists. This is the point for which the oscillator should be calibrated and used.

(3) Use of a single quartz plate. A single piezoelectric element may be used for producing the audio-frequency current directly. To accomplish this a plate is ground first for producing the component vibration A and then a small step ground in it[14]

[13] Not more than about 6 watts for a plate of average rating.

[14] The height of the step is exceedingly small so that the quartz has still the shape of a plate to the eye and can be used in an ordinary plate holder with an air gap of about 1/3 mm.

as indicated in Figure 11 in order to superimpose on it the high-frequency vibration B. The plate is used in the ordinary way, (Figure 2) and works well. The oscillogram of Figure 12 shows the beat current which produces an audible note. Figure 13

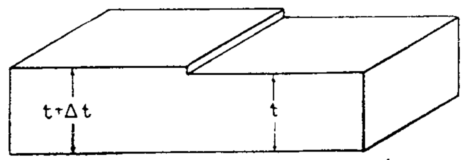

FIGURE 11—A Piezoelectric Plate with a Minute Step in it for Audio Frequency Currents

gives an arrangement using a plate as indicated in Figure 11, when more output is required. It is an arrangement which is self-starting, that is, a fixed condenser is used in the piezo circuit and the audio frequency current will start upon closing it. The

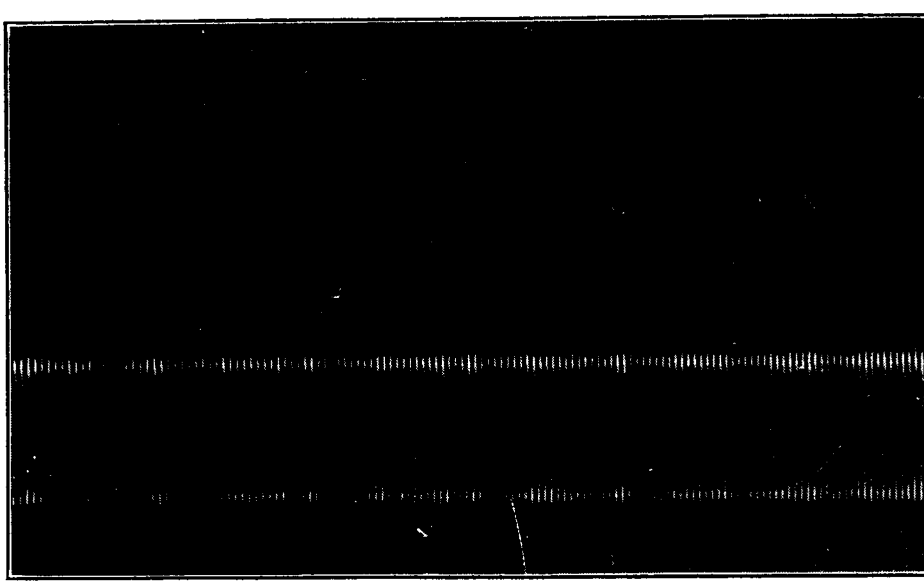

FIGURE 12—Beat Current Produced by a Single Small Piezoelectric Oscillator. (f . . 1912.1 Cycles per Second, Upper Oscillogram is the Timing Wave.)

arrangement of Figure 13 was used for producing the wave shown in Figure 12. For the oscillographic work and other applications it is also possible to use the piezoelectric plate circuit directly in the output circuit as long as not more than about 6 watts are required. It is to be noted that the output of an audio-frequency oscillator is somewhat smaller than when only one component current is flowing. It is not necessary always to use a load resistance across the output branch as indicated in Figure 13. Any amplifier circuit arrangement will be satisfactory.

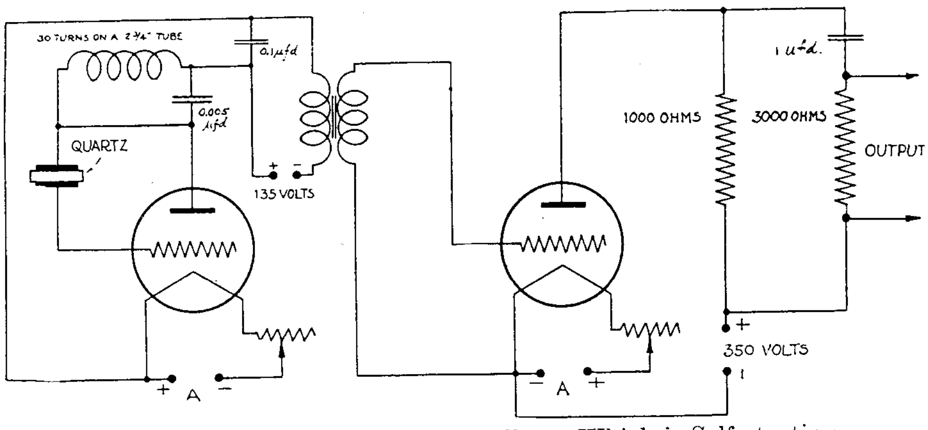

FIGURE 13—Audio-frequency Oscillator Which is Self-starting

MISCELLANEOUS APPLICATIONS

By using the method herein described, it is possible to grind a plate to any suitable frequency. It is, therefore, possible to design the equivalent of a second pendulum or any other timing device. The disadvantages described in connection with the first method for producing audio-frequency currents can be used to advantage for making a radio or audio-frequency generator whose frequency can be varied somewhat without regrinding the piezoelectric plates.

A change of a few cycles and less in a plate for radio frequency currents is due to a change in dimension which can not be noted with a micrometer. It can, therefore, be used for measuring very small variations in the thickness of piezoelectric materials.

Using a sphere of quartz the optical axis can be roughly determined electrically (no direct electrical effect along it) as well as the main piezoelectric axis by placing the sphere between two cup-shaped electrodes and noting the strength of any resonator effects when placed along different diameters of the sphere.

Working backwards, using the electrical data (frqeuency), and the dimensions, it gives a means for finding certain mechanical properties of the substance such as its elasticity from the velocity and the density.

Using a vibrating piezoelectric plate in front of a fine slot or another vibrating plate, a shutter can be designed which opens and closes at a very high rate. This may open up a new field in experimental optics for direct and reflected light rays, and give a means of determining the velocity of light.

Conclusions

(1) Experiments with quartz plates have shown that they can be used in an electron tube circuit for producing radio-frequency currents of fixed frequencies bearing a definite relation to the dimensions of the plate.

(2) The piezo oscillator can be used together with an auxiliary generator for standardizing a frequency meter.

(3) A single piezoelectric plate can be employed as a standard for the entire range of frequencies used in radio communication.

(4) By using special arrangements a small plate can be employed for producing audio-frequency currents.

(5) Methods are given for grinding a plate accurately to a given frequency.

(6) Formulas are given for designing plates to a desired frequency to a fair degree of accuracy.

(7) Other miscellaneous applications are described.