ACCURATE MEASUREMENTS OF SPECTRAL DENSITY OF PHASE NOISE IN DEVICES

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ABSTRACT

Systematic errors larger than 10 dB can occur in the measurement of the spectral density of phase unless considerable caution is exercised. Some potential problems due to the shape of the analyzer passband and the Fourier frequency dependence of mixers are discussed. Three measurement systems are analyzed to determine the conditions under which they may be used to make spectral density measurements with an accuracy of 0.2 dB.

INTRODUCTION

The systematic errors in the measurement of the spectral density of phase which we discuss concern the shape of the analyzer passband and the Fourier frequency dependence of double-balanced mixers. Both of these problems are well known. However, careful calculations and measurements indicate that in certain regions of the Fourier frequency spectrum the potential errors are substantially larger than had been expected. With respect to the wave analyzer passband, we find that errors larger than 10 dB can occur when making measurements at a frequency of three times the noise bandwidth of the analyzer when the spectral density diverges as rapidly as f^{-5}. We therefore present three figures which show the potential error due to the width and shape of the analyzer filter as a function of Fourier frequency. Curves are given for 2,4,6,8 and 16 pole filters and for spectral densities which diverge as f^{-1}, f^{-3}, and f^{-5} so that the region of significant error may be defined for a wide variety of measurement situations.

Measurements have been made to characterize the performance of double-balanced mixers in detail because it has been found that the phase to voltage conversion sensitivity of these devices can vary 15 dB over a Fourier frequency range smaller than 0.1% of the nominal bandwidth. Several figures are included which demonstrate the dependence of such mixers on the input drive levels, the input impedances, and the output termination. Based on this data, we analyze specific techniques for obtaining high accuracy measurements. This discussion stresses wide applicability, calibration accuracy and convenience, noise floor, and immunity from spurious effects. One technique stands out from the remainder because it is equal to or superior to the others in all respects. By using the methods described, we conclude that convenient measurements of spectral density of phase may be made to an accuracy of 0.2 dB.

FILTER SHAPES

In order to actually measure the spectral density of phase one would need a delta function filter. In this section, we present three graphs which quantify the errors which one would make in approximating the delta function by a real filter having a bandpass shape which is equivalent to n/2 high pass poles and an equal number of low pass poles. The specific function which has been assumed for the purpose of these calculations is:

$$\text{relative power} = \frac{1}{[1 + (\Delta f)^2]^{n/2}}$$

where Δf is the frequency difference from the center of the filter. When defined in this simple manner, the noise bandwidth of the filter is a function of the number of poles. The noise bandwidths for n = 2, 4, 6, 8 and 16 are listed in table I.

TABLE I

Number of poles, n	Noise bandwidth, Hz
2	3.14
4	1.57
6	1.18
8	0.983
16	0.631

The most common technique for measuring the spectral density of phase involves applying a signal voltage and a reference voltage to a double-balanced mixer and analyzing the mixer output. The noise voltage measured at the IF port is simply related to the phase fluctuations between the two inputs provided they are very near phase quadrature and the total phase fluctuations are small compared to 1 radian. If two oscillators are being compared, a loose phase-lock loop is often used to maintain the quadrature condition. However, the transfer function of such a loop will have the effect of high pass filtering the phase fluctuations [1]. Even if a phase-lock loop is not employed, it is often necessary to high pass filter the noise in order to obtain sufficient amplification to measure low noise levels. Consequently we have assumed, for the purpose of this calculation, that the spectral density being measured varies as:

$$S_\phi(f) = [1 + (f/0.25\ \mathrm{Hz})^2]^{-p/2},$$

$$\text{for } p = 1,\ 3 \text{ and } 5.$$

Figures 1, 2 and 3 show the relative error in estimating such a spectral density with an n pole filter, where n = 2, 4, 6, 8 and 16. The relative error is defined as:

$$\text{relative error} = \left| \frac{\text{measured value} - S_\phi(f)}{S_\phi(f)} \right|$$

The potential errors increase very rapidly as the slope of $S_\phi(f)$ increases and as f approaches the analyzer noise bandwidth. For example, a situation which could occur would be the measurement of a high level quartz oscillator at 1 Hz. In this case the spectral density may vary as f^{-5}. If one were to make this measurement with a commercial analyzer having a noise bandwidth of 1 Hz and an 8 pole bandpass filter, then the expected error is 15 dB at f = 2 Hz. Since the error depends on Fourier frequency, not only is the absolute value of the measured spectral density in error, but its measured dependence on frequency is also incorrect. It is therefore possible to draw incorrect concusions about the fundamental noise processes which are involved. By scaling figures 1, 2 and 3 according to the noise bandwidth one may use them to estimate the minimum frequency at which $S_\phi(f)$ may be measured with less than a selected error limit. The accuracy of this estimate will depend on the deviations from the assumptions which have been described above.

One can draw several conclusions about how to make more accurate measurements at very low Fourier frequencies. In the first place, one should use the narrowest filters with the steepest skirt functions. Fast Fourier transform analyzers are now available which have bandwidths of a few millihertz and correspondingly steep skirts. In the second place, it is possible to lessen the severity of this problem by prewhitening the noise. One straightforward method for doing this is to use a tight phase-lock loop instead of a loose lock loop. The noise voltage which one measures, inside the loop bandwidth, is proportional to $S_y(f)$ and diverges a factor of f^2 more slowly than $S_\phi(f)$. The disadvantage of this technique is that the voltage to frequency coefficient of the oscillator tuning element must be measured. This coefficient may depend on Fourier frequency [2].

DOUBLE-BALANCED MIXER PHASE SENSITIVITY

In order to relate the voltage measured at the output of a mixer to the spectrum of the phase noise, it is necessary to know the phase sensitivity of the mixer (V/rad). Two significant problems are often encountered in its measurement. If two oscillators are being compared, then the mixer can be calibrated by allowing the oscillators to free run. However, very stable oscillators are often tunable over only a few Hz. One must answer the question: How can the calibration at low frequencies be extrapolated to ten or one-hundred kHz? If two components are being compared using a phase bridge, then the possibility of obtaining a beat signal without significantly altering the measurement conditions does not exist. The calibration is often performed by inserting a signal from a second oscillator in one side of the bridge, duplicating as far as possible power level and impedance conditions. We must answer the question: How sensitive is the calibration to changes in drive level and driving impedance?

Careful measurements reveal that the answers to these questions are rather complex. Only under certain termination conditions do the mixers appear broadband, to 100 kHz, and these same conditions seriously degrade the noise floor of the measurement system. On the other hand, when the noise floor is optimized by reactively terminating the mixer, the bandwidth degrades severely and it is necessary to calibrate the system at every frequency for which phase noise measurements are desired. The results of a variety of measurements are presented in this section to demonstrate the dependence of double-balanced mixers on drive level, input impedance, and output impedance. In the following section several measurement systems are discussed which bring these variables under control and permit fast, convenient and highly accurate phase noise measurements to be made with double balanced mixers.

The measurement system which was used to characterize the double-balanced mixer is shown in Figure 4. The drive conditions are maintained nearly identical at the R and L ports of the mixer by the two isolation amplifiers which have output impedance Z_d. The mixer is terminated by an impedance Z_L and isolated from the other loads by the LC filter. The synthesizer, in the lower signal path, makes possible beat frequency measurements of the phase sensitivity at any offset frequency. The calibrated electronic phase shifter, in the upper signal path, allows direct measurement of the phase sensitivity at any Fourier frequency by the insertion of a coherent phase modulation spectrum. It is described in more detail in the next section. For these measurements, the delay line phase-shifter is used to maintain the two input signals to the mixer in the quadrature condition. When using the same voltmeter for all measurements, the two methods agree to within 0.2 dB, so the electronic phase shifter was used for the mixer calibrations since it is significantly easier to use.

Figure 5 shows three sets of phase sensitivity calibrations. Between the two curves of each pair, the only change is the drive level which is defined as the amount of power delivered when the mixer is replaced by a 50Ω load. Both the driving impedance and the load impedance vary between the sets. These measurements reconfirm the well-known result that, independent of other conditions, the phase sensitivity depends strongly on how much current the diodes in the mixer conduct. The dependence on drive power makes it impossible to make any substitution of devices on the input of the mixer without introducing about 3 dB uncertainty in the calibration unless some method, such as the one discussed later, is used to significantly reduce this dependence.

If the mixer is terminated at its X-port by a 50Ω load, then the beat frequency output is nearly sinusoidal. When a high impendance termination, for example 1 kΩ, is used, the beat waveform becomes triangular and when the appropriate capacitive termination is used the waveform is more nearly square. This means that the slope through zero, i.e., the phase sensitivity, is increased [3]. Quantitative results are shown in Figure 6. It is seen that the available increase in sensitivity is more than 10 dB. In addition to decreasing the phase sensitivity, a 50Ω termination increases the white noise level. As a result, noise floor considerations often require the use of the capacitive termination.

The trade-off is an increased variation of the calibration with Fourier frequency. The full extent of this dependence is illustrated in Figure 7. When the driving impedance is low, 10Ω, and the load capacitance is 0.01 μF, the peak-to-peak variation in phase sensitivity is only 0.4 dB, but in the case of the constant current drive and a 0.1 μF load the variation in phase sensitivity is 15 dB over the 1 to 100 kHz range. Neither the phase sensitivity nor its frequency dependence remain constant for even the smallest changes in a measurement system. For this reason, the figures presented here should be used only as a guide. A careful calibration must be made after every change in a measurement system. Some ways for making such calibrations more convenient are discussed in the next section.

HIGH PRECISION MEASUREMENT SYSTEMS

The following measurement systems can be calibrated to an accuracy of 0.2 dB. Differences in noise performance, convenience and means of calibration are discussed. Figure 8 illustrates a simple test set for measuring $S_\phi(f)$ of an oscillator. The mixer noise is relatively large and the phase sensitivity minimum. However, the phase sensitivity is flat vs. Fourier frequency. This system can be used for a wide variety of carrier frequencies with a $S_\phi(f)$ noise floor approximately -160 dB relative to 1 rad^2/Hz. By simply measuring K_d, the phase slope at the zero crossing, using a strip chart recording or a scope trace, one can calibrate the system from dc to 100 kHz with an accuracy of about 0.2 dB. Specifically:

$$S_\phi(f) = \left[\frac{V_n(f)}{K_d(\text{volts/rad})\ G_1G_2}\right]^2 \frac{1}{f_h}$$

where $V_n(f)$ is the RMS noise measured with the spectrum analyzer at Fourier frequency f in a noise bandwidth f_h, and G_1G_2 is the measurement system gain. It is imperative that the measurements be made with exactly the same cables and mixer drive levels as was used during the calibration. Simply replacing a cable can change the calibration factor, i.e., phase sensitivity, by 3 dB or more. Because of this, this system is not very suited for measuring noise in amplifiers, etc.

The errors associated with the spectrum analyzer bandwidth and skirt selectivity can be estimated from Figures 1, 2 and 3, if the asymptotic form of the analyzer filter roll-off is known. However, to assure an accuracy of better than 10 dB for Fourier frequencies near the filter noise bandwidth, it is necessary to map the analyzer transfer function carefully and deconvolute the measured output. A word of caution - spectrum analyzer noise bandwidths may be as much as 3 dB different than advertised. Another common trouble is due to finite dynamic range; often it is necessary to precede the spectrum analyzer with a filter to remove out of band noise e.g., 60 Hz and its harmonics.

Another common trouble, which causes a change in mixer sensitivity, is phase drift between the oscillators due to finite gain in the phase-lock loop. This is easily avoided by using a second order phase lock loop which normally holds the phase error to less than 0.1 rad [3]. The phase lock loop attack time, τ_v, is adjusted to be equal or greater than $2/\pi f_{min}$ where f_{min} is the minimum Fourier frequency of interest. For Fourier frequencies below 0.1 Hz it is often necessary to measure $S_y(f)$ instead of $S_\phi(f)$. This can be accomplished by using a tight phase lock loop with τ_v less than $1/10\pi f_{max}$, where f_{max} is the highest Fourier frequency of interest. In this case the mixer output is proportional to $S_y(f)$.

$$S_y(f) = \frac{K_y^2}{\nu^2} \left[\frac{V_n}{G_1 G_2}\right]^2 \frac{1}{f_h} \quad \text{and}$$

$$S_\phi(f) = \frac{\nu^2}{f^2} S_y(f),$$

where K_y is the voltage to frequency sensitivity in Hz/volt of the oscillator tuning element, and ν is the carrier frequency. If $S_\phi(f)$ is varying rapidly, the tight phase lock loop method can be used to prewhiten the noise, enabling more accurate measurements.

Figure 9 illustrates a more general measurement system which is capable of measuring $S_\phi(f)$ of signal processing equipment and passive components, as well as oscillators. The calibration procedure is very simple and simuataneously calibrates mixer sensitivity, amplifier gain and spectrum analyzer gain. This system is the most convenient one of the three for measuring $S_y(f)$ with a tight phase-lock loop.

The calibration procedure consists of driving the precision phase shifter with a reference signal of known amplitude from 100 Hz to 100 kHz. This reference mixer output is then compared to the measured noise level. The calibrated phase shifter and its calibration are discussed below. This system allows the use of a mixer termination which maximizes its phase sensitivity and minimizes the noise. For a loose phase-lock system:

$$S_\phi(f) = \left[\frac{V_n(f)}{V_R(f)} K\right]^2 \frac{1}{f_h}$$

where K is the RMS phase modulation impressed by the phase shifter in radians, and $V_R(f)$ is the output reference voltage as measured on the spectrum analyzer as a function of f. Results obtained with this method and the beat frequency method agree to within 0.2 dB. The phase shifter can be calibrated to within 0.2 dB at dc using a time interval measurement, e.g., the dual mixer time difference system [2] or it can be calibrated with the beat frequency method to the same accuracy. At high Fourier frequencies, a storage scope can be used to check the flatness of the sensitivity. The present design, shown in Figure 10, uses a series tuned LCR circuit sandwiched between two high quality isolation amplifiers [3]. The isolation amplifiers guarantee that the phase shift is independent of input and/or output loading conditions. The LCR circuit has a Q factor of about 2 and therefore is very wideband. The residual amplitude modulation is typically -60 dB relative to the phase modulation of 1×10^{-3} rad. Measurements of $S_y(f)$ using the tight phase lock loop are calibrated using the calibrated phase shifter via:

$$S_y = \left[\frac{V_n(f)}{V_R(f)} \frac{f\,K}{\nu}\right]^2 \frac{1}{f_h},$$

where f, K, $V_n(f)$ and $V_R(f)$ have the previous definitions.

Figure 11 shows a measurement system for $S_\phi(f)$ which holds its calibration factor over a 15 dB change in input power, has very low noise, and can be made very flat from dc to 100 kHz Fourier frequency offset from the carrier. This system is based on the use of low noise current coupled pairs as odd-order multipliers [4]. This has many advantages:

(1) The output level and drive impedance is virtually independent of input drive over a large range (See Figure 12) with the consequence that the mixer sensitivity is constant. Therefore, only one calibration is necessary.

(2) The multiplication raises the input noise by n^2 so that the mixer noise and the following audio amplifer noise is less important. Therefore the mixer can be resistively terminated in order to keep it flat vs. Fourier frequency without sacrificing noise performance. Both oscillators and passive components can be measured.

(3) One can freely substitute components and/or devices on the input and still maintain the same system calibration factor to within 0.1 dB, allowing precise comparisons of the added noise of various system subassemblies.

(4) Noise floors below $S_\phi(f) = -180$ dB relative to 1 rad^2/Hz have been achieved with a fundamental carrier frequency of 5 MHz and a x9 commercially available multiplier.

CONCLUSION

It has been shown that a failure to take into account the effect of the analyzer window shape and/or the frequency dependence of the mixer can lead to errors in the measurement of $S_\phi(f)$ as large as 15 dB. To solve this problem, several measurement systems are presented along with calibration procedures which allow fast and convenient spectral density measurements with 0.2 dB accuracy. Although the basic concepts of these systems are already well known, their applicability to high accuracy measurements has not been demonstrated previously. To obtain an accuracy of 0.2 dB, careful calibration of the entire measurement system is an absolute requirement; with few exceptions, the most trivial change requires recalibration. The ease of measurement, calibration, and wide applicibility has been stressed in the measurement systems described. One measurement system presented retains its calibration factor over a 15 dB change in input power level and possesses an inherent noise floor which is below virtually all other present signal processing equipment and oscillators.

ACKNOWLEDGEMENTS

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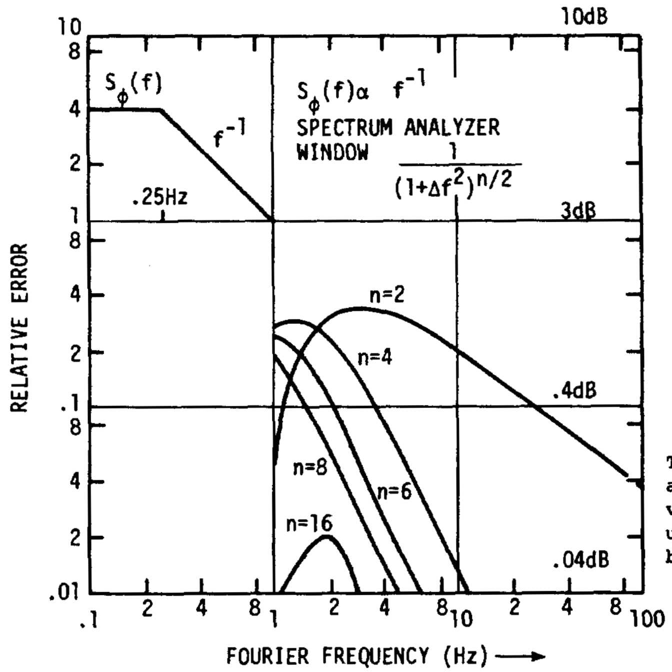

Fig. 1

The relative error of a spectral density of phase measurement as a function of Fourier frequency. The spectral density varies like f^{-1} for frequencies above 0.25 Hz as shown in the upper left hand corner. The different curves are parametrized by n, the number of poles in the bandpass filter.

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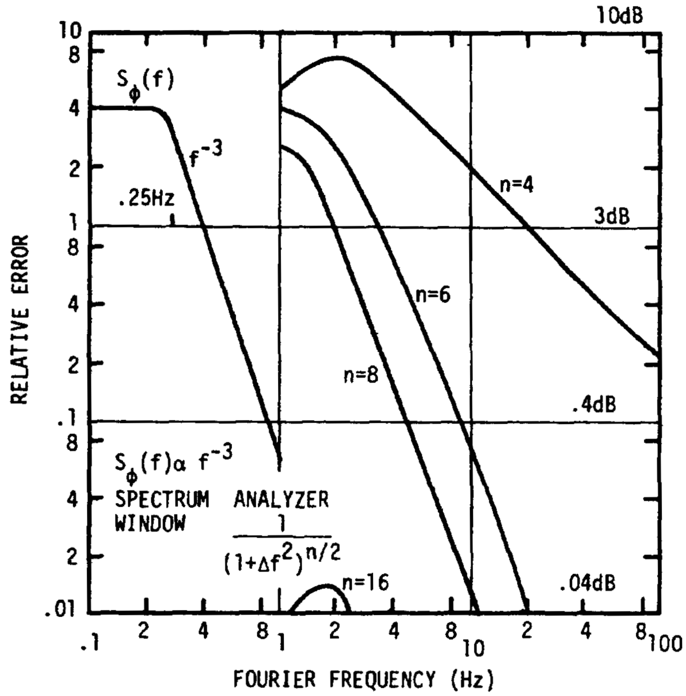

Fig. 2

The relative error of a spectral density of phase measurement as a function of Fourier frequency. The spectral density varies like f^{-3} for frequencies above 0.25 Hz as shown in the upper left hand corner. The different curves are parametrized by n, the number of poles in the bandpass filter.

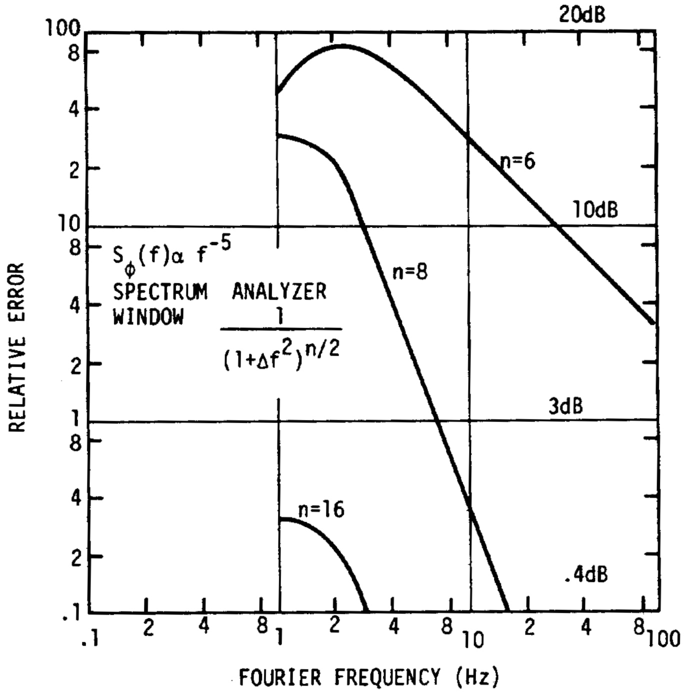

Fig. 3

The relative error of a spectral density of phase measurement as a function of Fourier frequency. The spectral density varies like f^{-5} for frequencies above 0.25 Hz. The different curves are parametrized by n, the number of poles in the bandpass filter.

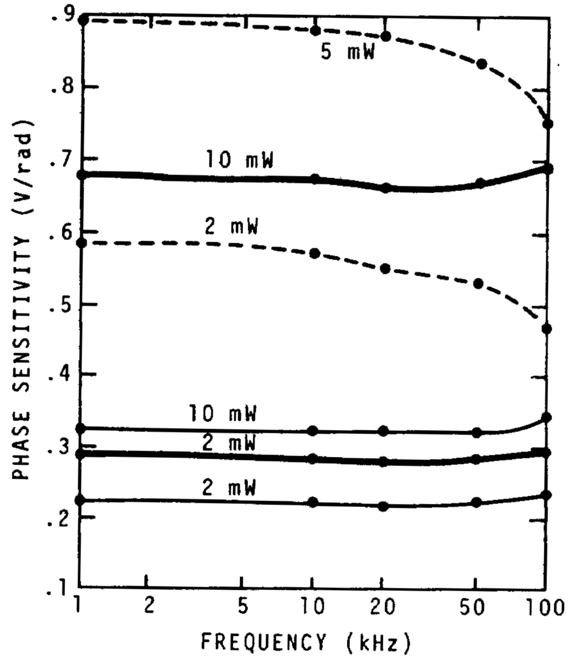

Fig. 5

The double-balanced mixer phase sensitivity as a function of Fourier frequency for different drive levels. Each pair of curves represents identical conditions except for the drive level while input/output conditions vary widely between the pairs.

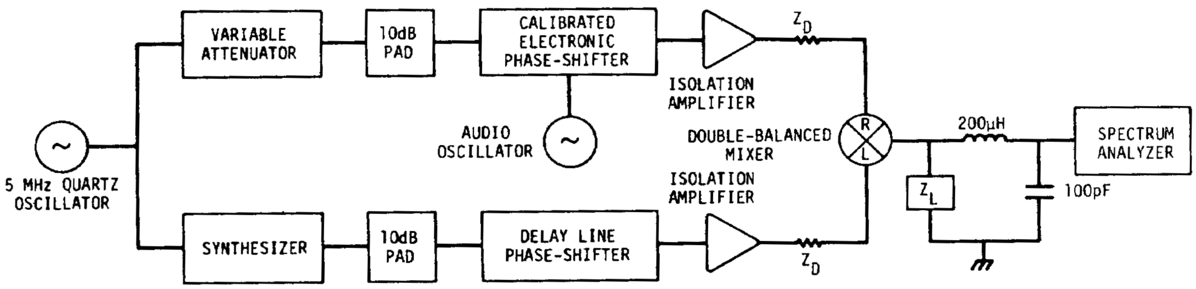

Fig. 4 Schematic of the measurement system for evaluating the dependence of the mixer phase sensitivity on drive level, drive impedance, and output impedance.

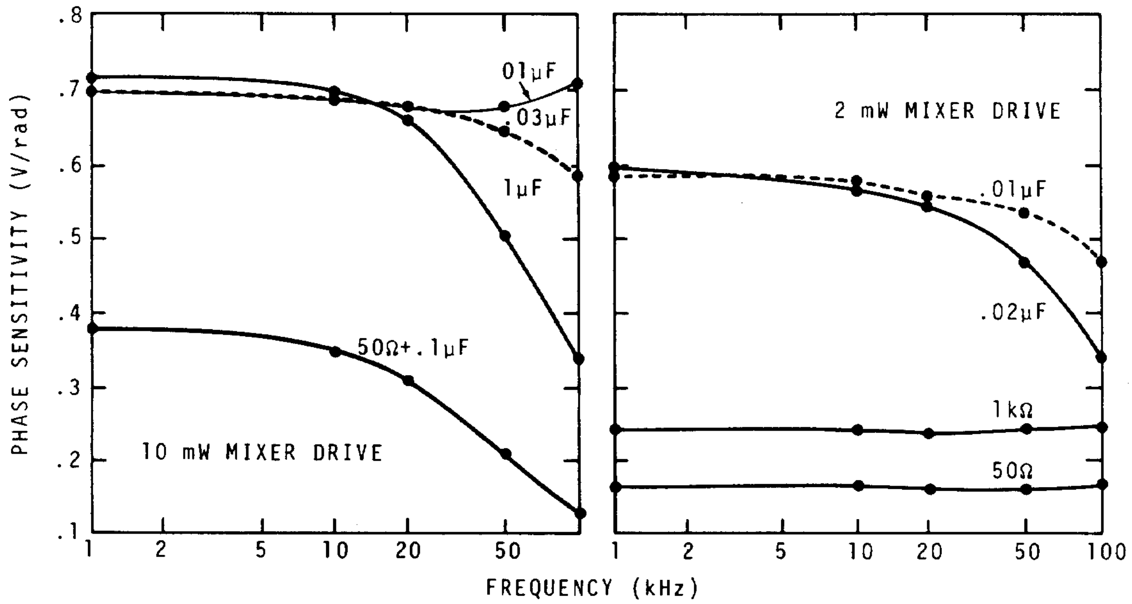

Fig. 6 Double-balanced mixer phase sensitivity as a function of frequency for various output terminations. The curves on the left were obtained with 10 mW drive while those on the right were obtained with 2 mW drive. The data demonstrate a clear choice between constant, but low sensitivity or much higher, but frequency dependent sensitivity.

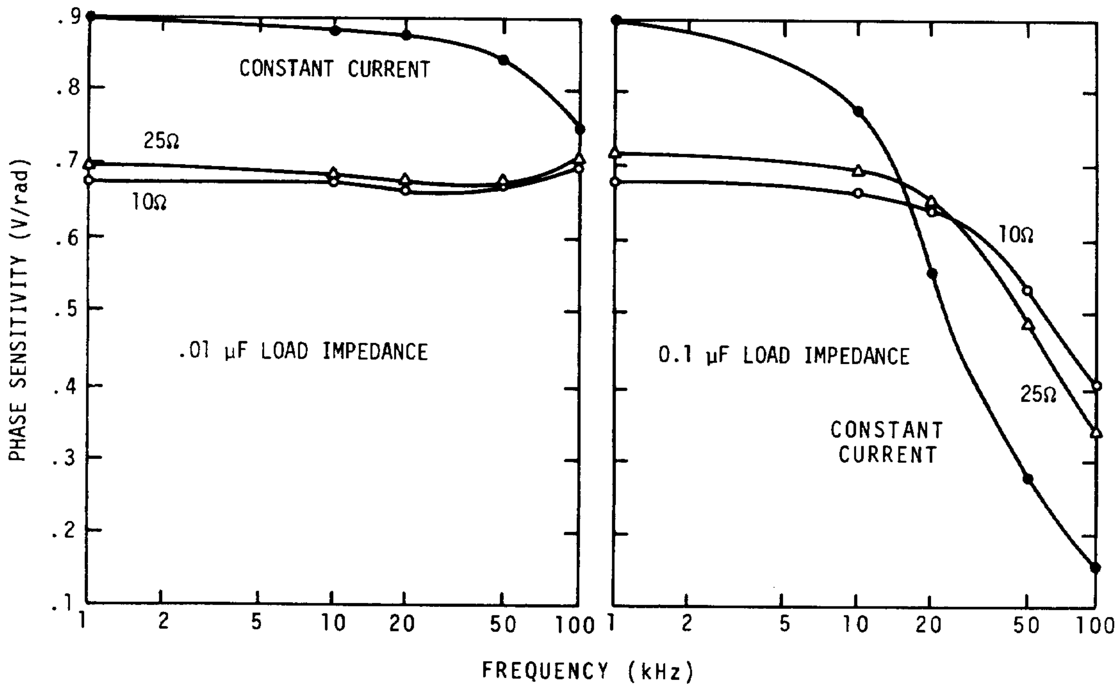

Fig. 7 Double-balanced mixer phase sensitivity as a function of Fourier frequency for different driving impedances. The curves on the left and right differ in the value of the mixer output termination. The frequency dependence of the phase sensitivity depends strongly on the value of the termination capacitor.

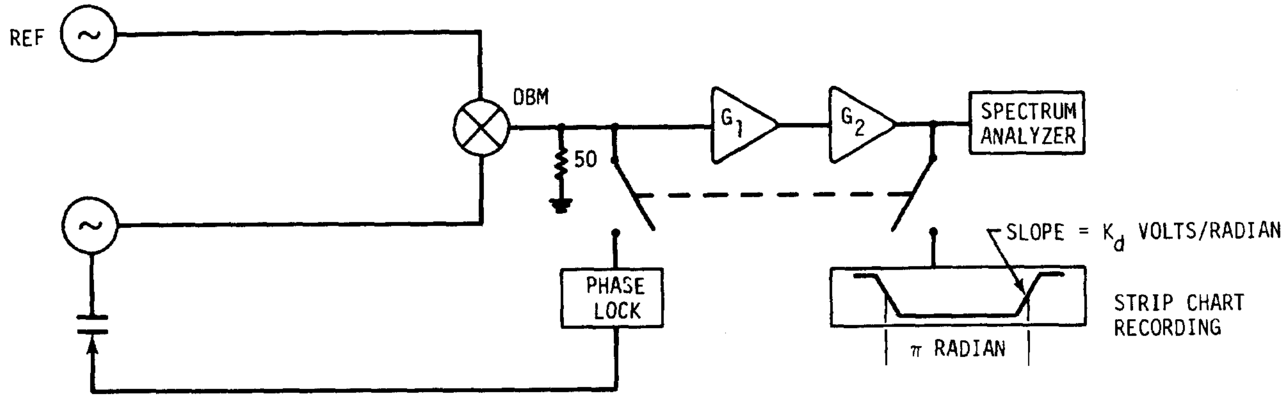

Fig. 8 Precision phase measurement system. Calibration requires an external strip chart or storage scope to measure the slope at the zero crossing. The accuracy is 0.2 dB from dc to 100 kHz Fourier frequency offset from the carrier. This system is suitable only for oscillators. A wide range of carrier frequencies can be accommodated with the same measurement system.

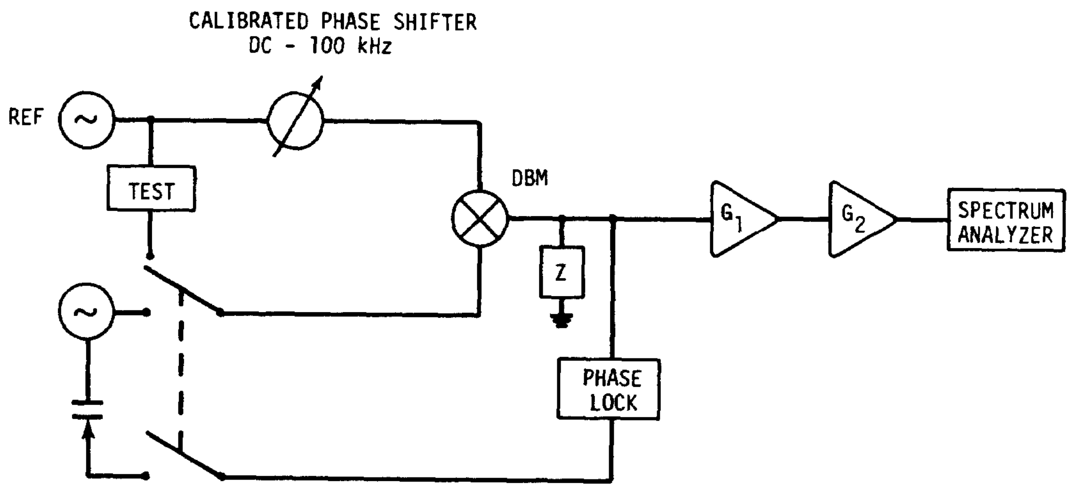

Fig. 9 Precision phase measurement system featuring self calibration to 0.2 dB accuracy from dc to 100 kHz Fourier frequency offset from carrier. This system is suitable for measuring signal handling equipment and passive components, as well as oscillators. Different carrier frequencies require a different phase shifter.

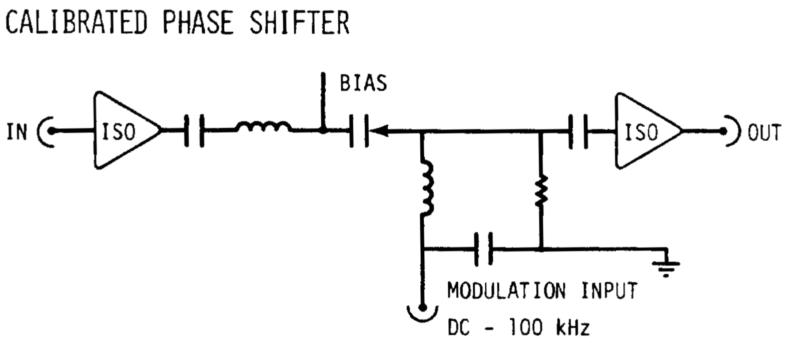

Fig. 10 Isolated phase modulator is insensitive to input or output loading. The LCR series tuned circuit has a Q of about 2. The phase modulation sensitivity is constant to 0.1 dB from dc to 100 kHz.

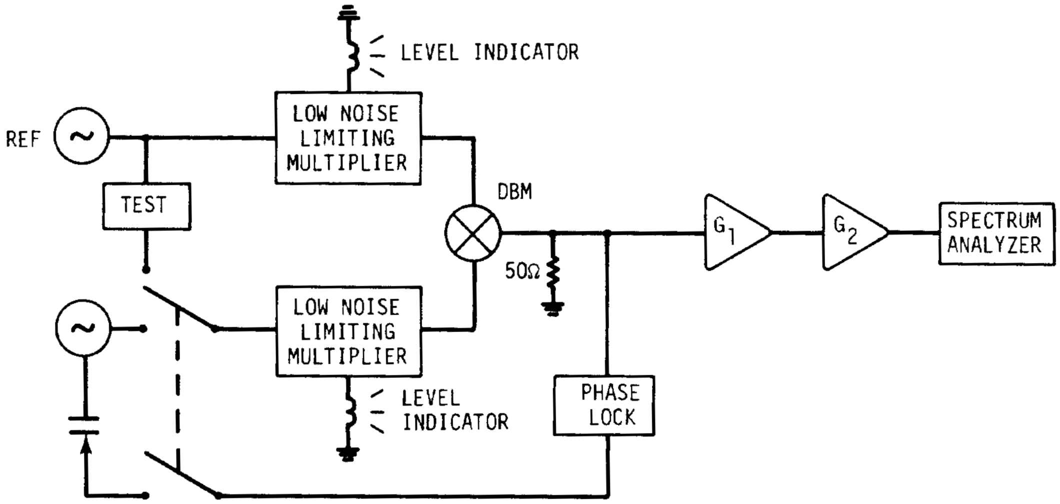

Fig. 11 Precision phase measurement system featuring constant calibration factor for a wide range of input signal levels and Fourier frequency. This system also features the lowest noise floor, about -180 dB relative to 1 rad^2/Hz and is the most suitable for measuring phase noise of signal handling devices, passive components or oscillators. Different carrier frequencies require a different pair of multipliers.

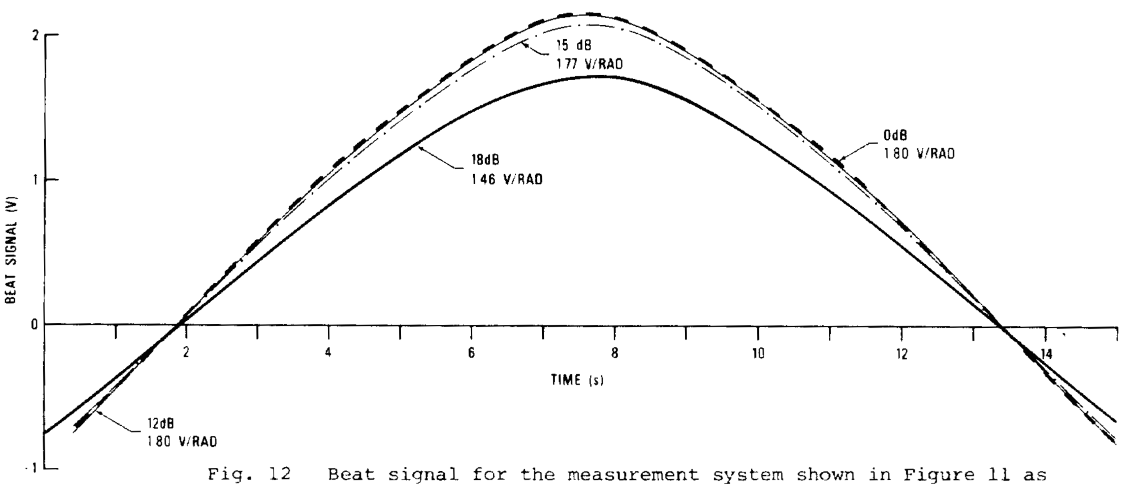

Fig. 12 Beat signal for the measurement system shown in Figure 11 as a function of attenuation of the input drive level.