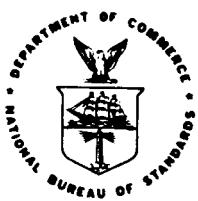


Tables of Bias Functions, B_1 and B_2 , for Variances Based on Finite Samples of Processes with Power Law Spectral Densities

J.A. Barnes

**Time and Frequency Division
Institute for Basic Standards
National Bureau of Standards
Boulder, Colorado 80302**

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TABLES OF BIAS FUNCTIONS, B_1 AND B_2 , FOR VARIANCES
BASED ON FINITE SAMPLES OF PROCESSES WITH POWER
LAW SPECTRAL DENSITIES

J. A. Barnes

ABSTRACT

D. W. Allan showed that if $y(t)$ is a sample function of a random noise process with a power law spectral density (i.e., $S_y(f) = h|f|^\alpha$), then there is generally bias to the estimated variance of y , defined as

$$\sigma_y^2(N, T, \tau) = \frac{1}{N-1} \sum_{n=1}^N (\bar{y}_n - \langle \bar{y} \rangle)^2,$$

where N is the number of samples, \bar{y}_n is the average value of $y(t)$ over the n -th interval of duration τ , T is the time between the beginnings of any two successive sample intervals, and

$$\langle \bar{y} \rangle = \frac{1}{N} \sum_{n=1}^N \bar{y}_n.$$

Allan also showed that, under these conditions, the expectation value of the estimated variance is proportional to τ^μ where μ is a constant related to α , the exponent in the spectral density; i.e.,

$$E[\sigma_y^2(N, T, \tau)] \propto \tau^\mu.$$

Based on this work one may define the two bias functions

$$B_1(N, r, \mu) \equiv \frac{E[\sigma_y^2(N, T, \tau)]}{E[\sigma_y^2(2, T, \tau)]}$$

and

$$B_2(r, \mu) \equiv \frac{E[\sigma_y^2(2, T, \tau)]}{E[\sigma_y^2(2, \tau, \tau)]}$$

where $r \equiv T/\tau$ and the B 's are functions of μ through their dependence on $y(t)$.

If one has a sample variance, $\sigma_y^2(N_1, T_1, \tau_1)$, the bias functions allow one to give an unbiased estimate for $\sigma_y^2(N_2, T_2, \tau_2)$ provided the spectral type is known (i.e., μ is known).

The tables give values of $B_1(N, r, \mu)$ and $B_2(r, \mu)$ accurate to four significant figures for the following values of N, r, μ :

$\mu = -2.0$ to 2.0 in steps of 0.2 ;

$N = 4, 8, 16, 32, 64, 128, 256, 512, 1024, \infty$;

$r = 0.001, 0.003, 0.01, 0.03, 0.1, 0.2, 0.4, 0.8, 1, 1.01, 1.1,$

$2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, \infty$.

Key Words: Statistics, variance, spectral density, unbiased estimate.

TABLES OF BIAS FUNCTIONS, B_1 AND B_2 , FOR VARIANCES
BASED ON FINITE SAMPLES OF PROCESSES WITH POWER
LAW SPECTRAL DENSITIES

Consider a random variable y with mean m . The variance of y is defined as the expectation value of $(y - m)^2$, that is

$$\text{Var } y \equiv E[(y - m)^2].$$

This is defined as an average over the entire ensemble but, for an ergodic process, $y(t)$, it can alternatively be defined as an average over all time, t .

Typically, the variance of y is estimated from a finite set of experimental values according to the relation

$$(\text{Var } y)_{\text{est.}} = \frac{1}{N-1} \sum_{i=1}^N (y_i - \langle y \rangle)^2 \quad (1)$$

where $\langle y \rangle \equiv \frac{1}{N} \sum_{i=1}^N y_i$ is the mean of the y_i . The factor $\frac{1}{N-1}$ is used in order that the estimate have no bias for non-correlated y : that is, one typically wants to obtain the true variance which would be obtained as $N \rightarrow \infty$. For finite N , the estimated variance has some expected value,

$$E \left[\frac{1}{N-1} \sum_{n=1}^N (y_n - \langle y \rangle)^2 \right].$$

If the y_i are independent (actually, non-correlated is sufficient) random variables, then the expected value of this estimated variance for finite

N is exactly equal to the true (infinite N) variance. If the y_i are correlated, however, the estimate based on (1) may indeed be biased. This fact has been recognized and discussed in some detail for the case of power law spectral densities by Allan [1].

The Bias Functions, B_1 and B_2

Following Allan [1], consider a random process $y(t)$ with continuous sample functions. We assume that $y(t)$ has a spectral density, $S_y(f)$, which obeys the law

$$S_y(f) = h|f|^{\alpha}, \quad f_l < |f| < f_u \quad (2)$$

where h is a constant, the limit frequencies f_l and f_u satisfy the relations

$$0 \leq f_l < < f_u < \infty ,$$

and any intervals of time, Δt , of any significance satisfy the relations

$$\frac{1}{f_u} << \Delta t << \frac{1}{f_l} .$$

In short, $y(t)$ has a power law spectral density over the entire range of significance.

Consider a measurement process which determines an average value of $y(t)$ over the interval t to $t + \tau$. That is,

$$\bar{y}(t) = \frac{1}{\tau} \int_t^{t+\tau} y(t') dt' . \quad (3)$$

One, now, may determine an estimated variance from a group of N such measurements spaced every T units of time; that is,

$$\sigma_y^2(N, T, \tau) = \frac{1}{N-1} \sum_{n=1}^N \left\{ \bar{y}(t+nT) - \frac{1}{N} \sum_{k=1}^N \bar{y}(t+kT) \right\}^2, \quad (4)$$

which is called the "Allan variance" [1].

Allan [1] has shown that under these conditions,

$$E[\sigma_y^2(N, T, \tau)] \propto \tau^\mu, \quad N \text{ and } T/\tau \text{ constant,}$$

where μ is related* to α according to the mapping shown in Figure 1 (see references 1 and 2). The relation between μ and α may be given as

$$\mu = \begin{cases} -2 & \text{if } \alpha \geq 1 \\ -\alpha - 1 & \text{if } -3 < \alpha \leq 1 \\ \text{not defined otherwise.} & \end{cases}$$

This mapping involves a simple extension of Allan's work [1] to the range $0 < \mu < 2$. This extension was also mentioned in [3].

Allan [1] considered in some detail the case where $T = \tau$. This is the case of exactly adjacent sample averages--no "dead time" between measurements. Allan defined a function, $\chi(N, \mu)$, as follows

$$\chi(N, \mu) \equiv \frac{E[\sigma_y^2(N, \tau, \tau)]}{E[\sigma_y^2(2, \tau, \tau)]}, \quad (5)$$

where it is again assumed that

*It should be noted that in reference 1 the exponent, α , corresponds to the spectrum of phase fluctuations while variances are taken over average frequency fluctuations. In the present paper, α is equal to the exponent, α , in [1] plus two. Thus, in this paper, all considerations are confined to one variable, $y(t)$ (analogous to frequency in [1]) and the spectral density of y , $S_y(f)$. This paper does not consider the spectrum of the integral of $y(t)$.

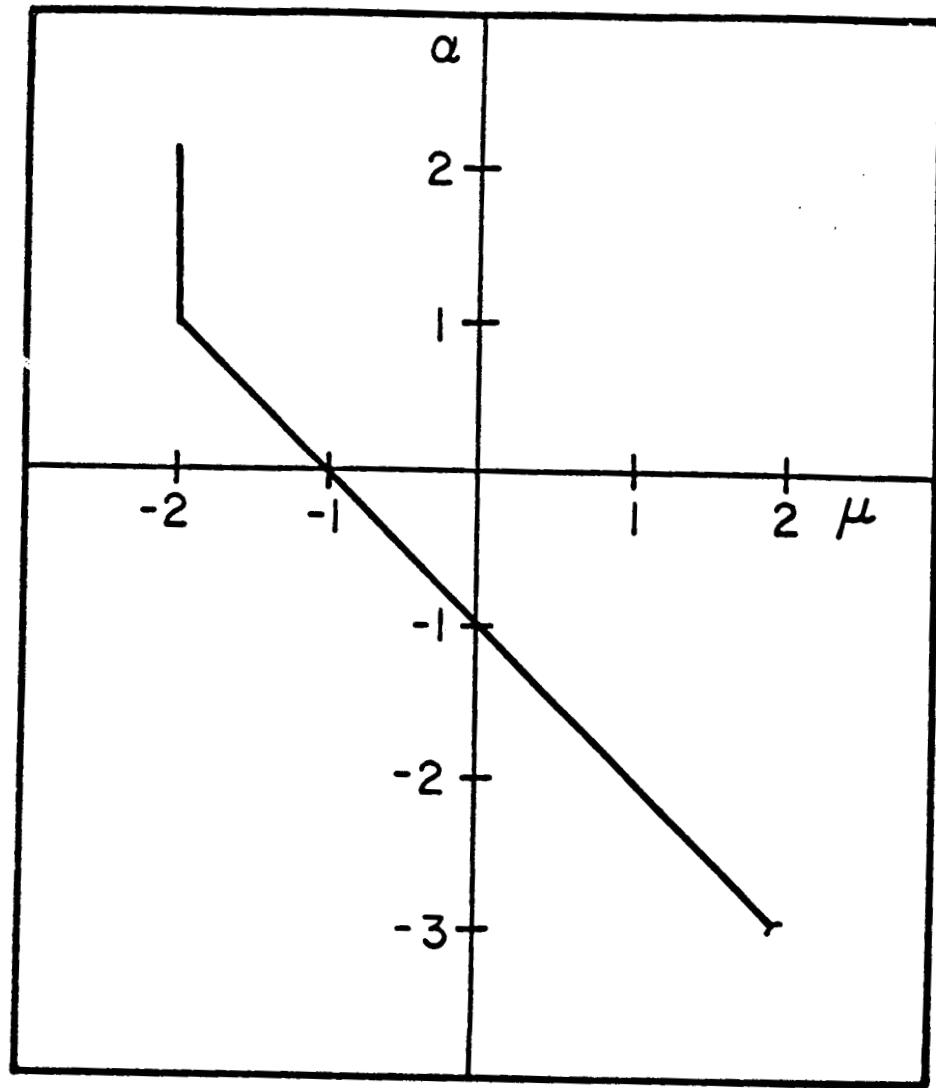


FIG. 1 μ - α MAPPING

$$E\left[\sigma_y^2(N, \tau, \tau)\right] \propto \tau^\mu, \quad N \text{ constant}.$$

Allan shows [1] that experimental evaluations of $\chi(N, \mu)$ may be used to infer μ and hence the spectral type by use of the mapping of Figure 1.

Since many experiments actually have dead time present, it is of value to make two different extensions of this function, $\chi(N, \mu)$. First, define $B_1(N, r, \mu)$ by the relations

$$B_1(N, r, \mu) = \frac{E\left[\sigma_y^2(N, T, \tau)\right]}{E\left[\sigma_y^2(2, T, \tau)\right]} \quad (6)$$

where $r \equiv T/\tau$ and

$$E\left[\sigma_y^2(N, T, \tau)\right] \propto \tau^\mu, \quad N \text{ and } r \text{ constant}.$$

The second function, $B_2(r, \mu)$, is defined according to the relation

$$B_2(r, \mu) = \frac{E\left[\sigma_y^2(2, T, \tau)\right]}{E\left[\sigma_y^2(2, \tau, \tau)\right]} \quad (7)$$

where $r \equiv T/\tau$. In words, B_1 is the ratio of the expected variance for N samples to the expected variance for 2 samples (everything else fixed); while B_2 is the ratio of the expected variance with dead time to that of no dead time (with $N = 2$ and τ held constant). The B 's, then, reflect bias relative to $N = 2$ rather than $N = \infty$. It is apparent that $B_1(N, r=1, \mu) \equiv \chi(N, \mu)$.

For the conditions given above and with reference to Allan [1], one may write expressions for both B_1 and B_2 , as follows:

$$B_1(N, r, \mu) = \frac{1 + \sum_{n=1}^{N-1} \frac{N-n}{N(N-1)} \left[2|nr|^{\mu+2} - |nr+1|^{\mu+2} - |nr-1|^{\mu+2} \right]}{1 + \frac{1}{2} \left[2|r|^{\mu+2} - |r+1|^{\mu+2} - |r-1|^{\mu+2} \right]}; \quad (8)$$

in particular for $r = 1$,

$$B_1(N, 1, \mu) = \frac{N(1-N^\mu)}{2(N-1)(1-2^\mu)}; \quad (9)$$

and

$$B_2(r, \mu) = \frac{1 + \frac{1}{2} \left[2|r|^{\mu+2} - |r+1|^{\mu+2} - |r-1|^{\mu+2} \right]}{2(1-2^\mu)}, \quad (10)$$

except that by definition, $B_2(1, \mu) \equiv 1$. The magnitude bars are essential on the $r-1$ term when $r < 1$, and, indeed, proper. Since Allan [1] was involved with $r \geq 1$ the magnitude bars were dropped in reference 1.

For $\mu = 0$, equations (8), (9), and (10) are indeterminate of form $0/0$ and must be evaluated by L'Hospital's rule. Special attention must also be given when expressions of the form 0^0 arise.

One may obtain the following results:

$$\begin{aligned} B_1(2, r, \mu) &\equiv 1 \\ B_1(N, r, 2) &= \frac{N(N+1)}{6} \\ B_1(N, 1, 1) &= \frac{N}{2} \\ B_1(N, r, -1) &= 1 \text{ if } r \geq 1 \\ B_1(N, r, -2) &= 1 \text{ if } r \neq 1 \text{ or } 0 \\ B_2(0, \mu) &\equiv 0 \\ B_2(1, \mu) &\equiv 1 \end{aligned}$$

$$B_2(r, 2) = r^2$$

$$B_2(r, 1) = \frac{1}{2}(3r - 1) \text{ if } r \geq 1$$

$$B_2(r, -1) = \begin{cases} r & \text{if } 0 \leq r \leq 1 \\ 1 & \text{if } r \geq 1 \end{cases}$$

$$B_2(r, -2) = \begin{cases} 0 & \text{if } r = 0 \\ 1 & \text{if } r = 1 \\ 2/3 & \text{otherwise} \end{cases}$$

Values of the functions $B_1(N, r, \mu)$ and $B_2(r, \mu)$ are tabulated on the following pages for values of N, r, μ as shown below:

$\mu = -2.0$ to 2.0 in steps of 0.2 ;

$N = 4, 8, 16, 32, 64, 128, 256, 512, 1024, \infty$;

$r = 0.001, 0.003, 0.01, 0.03, 0.1, 0.2, 0.4, 0.8,$

$1, 1.01, 1.1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024,$

$2048, \infty$.

Figure 2 is a graphical representation of $B_2(r, \mu)$ for $0 \leq r \leq 2$ and $-2 \leq \mu \leq 2$.

Examples of the use of the bias functions

The spectral type, that is, the value of μ , may be inferred by varying τ , the sample time [1, 2]. Another useful way, however, of determining the value of μ is by using $B_1(N, r, \mu)$ as follows: calculate an estimate of $E[\sigma_y^2(N, T, \tau)]$ and of $E[\sigma_y^2(2, T, \tau)]$ and hence $B_1(N, r, \mu)$; then by use of the tables the value of μ may be inferred.

Suppose one has an experimental value of $\sigma_y^2(N_1, T_1, \tau_1)$ and its spectral type is known--that is, μ is known. Suppose also that one wishes to know the variance at some other set of measurement parameters, N_2, T_2, τ_2 . An unbiased estimate of $\sigma_y^2(N_2, T_2, \tau_2)$ may be calculated by the equation:

$$E\left[\sigma_y^2(N_2, T_2, \tau_2)\right] = \left(\frac{\tau_2}{\tau_1}\right)^\mu \left[\frac{B_1(N_2, r_2, \mu)B_2(r_2, \mu)}{B_1(N_1, r_1, \mu)B_2(r_1, \mu)} \right] E\left[\sigma_y^2(N_1, T_1, \tau_1)\right]$$

where $r_1 = T_1/\tau_1$ and $r_2 = T_2/\tau_2$.

Obviously one might be interested in $N_2 = \infty$. In this case if $\mu \geq 0$, the expected value of $\sigma_y^2(\infty, T_2, \tau_2)$ is also infinite. This is true because,

$$\lim_{N_2 \rightarrow \infty} B_1(N_2, r_2, \mu) = \infty,$$

for $\mu \geq 0$.

Also, it should be noted that, for $\mu = 2$, $E\left[\sigma_y^2(N, T, \tau)\right]$ is a function of f_ℓ for any $N \geq 2$, T , τ , even though $B_1(N, r, 2)$ and $B_2(r, 2)$ as determined from (8), (9), and (10) are finite and well behaved [3]. In this region, $\mu \sim 2$, the low frequency behavior is critically important.

ACKNOWLEDGMENT

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- 3 J. A. Barnes and D. W. Allan, "An Approach to the Prediction of Coordinated Universal Time," Frequency, Vol. 5, No. 6, November/December 1967, pp. 15-20.

TABLES OF B_1 AND B_2

The tables are photographic reproductions of the computer output. Each entry for the value of the functions, B_1 and B_2 consists of a decimal number followed by an integer which is the exponent of 10. Ten raised to this power should multiply the decimal number. Thus the table entry $2.752 + 003$ could be written 2.752×10^3 or, simply 2752. Similarly, " $9.869 - 001$ " = 9.869×10^{-1} = 0.9869.

B1 (N=30, M1) FOR R= 0.001

2

B1(N,P,MI) FOR R= 0.003

R1 (W,D,MU) FOR R= 0..n10

R1(N•R•MU) FOR R= 0.030

N									
MU	4	8	16	32	64	128	256	512	1024
2.00	3.333•000	1.200•001	4.533•001	1.760•002	6.933•002	2.752•003	1.097•004	4.378•004	1.749•005
1.80	3.331•000	1.196•001	4.487•001	1.704•002	6.374•002	2.304•003	8.142•003	2.849•004	9.936•004
1.60	3.327•000	1.191•001	4.431•001	1.650•002	5.851•002	1.930•003	6.068•003	1.866•004	5.684•004
1.40	3.320•000	1.184•001	4.360•001	1.589•002	5.352•002	1.615•003	4.535•003	1.228•004	3.276•004
1.20	3.309•000	1.172•001	4.267•001	1.519•002	4.867•002	1.348•003	3.392•003	8.128•003	1.903•004
1.00	3.290•000	1.155•001	4.141•001	1.437•002	4.320•002	1.117•003	2.533•003	5.345•003	1.114•004
0.80	3.257•000	1.128•001	3.966•001	1.330•002	3.887•002	9.152•002	1.879•003	3.580•003	6.554•003
0.60	3.203•000	1.087•001	3.728•001	1.220•002	3.372•002	7.357•002	1.376•003	2.364•003	3.868•003
0.40	3.119•000	1.027•001	3.412•001	1.077•002	2.845•002	5.750•002	9.868•002	1.541•003	2.277•003
0.20	2.992•000	9.460•000	3.013•001	9.127•001	2.946•002	4.324•002	6.956•002	9.834•002	1.328•003
-0.00	2.819•000	8.430•000	2.546•001	7.348•001	1.748•002	3.096•002	4.668•002	6.082•002	7.611•002
-0.20	2.604•000	7.242•000	2.047•001	5.581•001	1.259•002	2.094•002	2.997•002	3.619•002	4.256•002
-0.40	2.362•000	6.005•000	1.566•001	3.993•001	8.524•001	1.335•002	1.744•002	2.066•002	2.314•002
-0.60	2.114•000	4.838•000	1.147•001	2.704•001	5.441•001	8.049•001	9.997•001	1.134•002	1.225•002
-0.80	1.878•000	3.125•000	1.141•000	1.753•001	3.304•001	4.626•001	5.499•001	6.029•001	6.341•001
-1.00	1.657•000	3.000•000	5.667•000	1.100•001	1.928•001	2.556•001	2.929•001	3.127•001	3.229•001
-1.20	1.484•000	2.355•000	3.918•000	6.768•000	1.043•001	1.378•001	1.525•001	1.595•001	1.626•001
-1.40	1.329•000	1.863•000	2.719•000	4.133•000	6.080•000	7.268•000	7.815•000	8.044•000	8.135•000
-1.60	1.199•000	1.491•000	1.910•000	2.532•000	3.341•000	3.782•000	3.963•000	4.030•000	4.053•000
-1.80	1.091•000	1.211•000	1.346•000	1.573•000	1.877•000	1.950•000	1.995•000	2.010•000	2.015•000
-2.00	1.000•000	1.000•000	1.000•000	1.000•000	1.000•000	1.000•000	1.000•000	1.000•000	1.000•000

R1 (H, P, MU) FOR R= 0.100

MU	4	8	16	32	64	128	256	512	1024	∞
2.00	3.3330000	1.2000001	4.5330001	1.7600002	6.9730002	2.7520003	1.097004	4.378004	1.749005	∞
1.80	3.3150000	1.1740001	4.2520001	1.5170002	5.3100002	1.8580003	6.467003	2.250004	7.830004	∞
1.60	3.2930000	1.1470001	3.9320001	1.3090002	4.1130002	1.2830003	3.843003	1.166004	3.535004	∞
1.40	3.2650000	1.1190001	3.7210001	1.1300002	3.1830002	8.6360002	2.3040003	6.103003	1.613004	∞
1.20	3.2300000	1.0940001	3.4630001	9.7280001	2.4700002	5.9430002	1.3940003	3.2320003	7.453003	∞
1.00	3.1840100	1.06450001	3.2040001	9.3470001	1.9180002	4.1140002	8.5230002	1.7350003	3.5000003	∞
0.80	3.1230000	9.9890000	2.9380001	7.1190001	1.4870002	2.8620002	5.2690002	9.4660002	1.678003	∞
0.60	3.0430000	9.4450000	2.6630001	6.0140001	1.1490002	1.9900002	3.2950002	5.2650002	8.2540002	∞
0.40	2.9400000	8.8040000	2.3760001	5.0150001	8.8160001	1.3970002	2.8840002	2.9930002	4.1940002	∞
0.20	2.8100000	8.0650000	2.0800001	4.1110001	6.6900001	9.7500001	1.3310002	1.7410002	2.2140002	∞
-0.00	2.6530000	7.2360000	1.7790001	3.3000001	5.0030001	6.7710001	8.5660001	1.0370002	1.2190002	∞
-0.20	2.4720000	6.3440000	1.4310001	2.5850001	3.6730001	4.6640001	5.5420001	6.3140001	6.9890001	1.1560002
-0.40	2.2710000	5.4300000	1.1990001	1.9710001	2.6420001	3.1770001	3.5920001	3.9100001	4.1530001	4.9220001
-0.60	2.0650000	4.5400000	9.4230000	1.4610001	1.8590001	2.1370001	2.3260001	2.4520001	2.5370001	2.7020001
-0.80	1.8690000	3.7200000	7.2040000	1.0560001	1.2810001	1.4190001	1.5020001	1.5500001	1.5770001	1.6160001
-1.00	1.6670000	3.0000000	5.3750000	7.4290000	8.6530000	9.3120000	9.4520000	9.8250000	9.9120000	1.0000001
-1.20	1.4910000	2.3960000	3.9310000	5.1250000	5.7510000	6.0460000	6.1770000	6.2350000	6.2600000	6.2780000
-1.40	1.3370000	1.9070000	2.8320000	3.4770000	3.7720000	3.8920000	3.9380000	3.9540000	3.9600000	3.9620000
-1.60	1.2050000	1.5220000	2.0210000	2.3200000	2.4490000	2.4900000	2.5020000	2.5050000	2.5050000	2.5040000
-1.80	1.0930000	1.2260000	1.4380000	1.5460000	1.5790000	1.5860000	1.5850000	1.5840000	1.5830000	1.5830000
-2.00	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000

$A_1(n, p, \mu)$ FOR $R = 0.200$

R1 (N,P,MU) FOR R= 0.400

2

$R_1(N, P, \mu)$ FOR $R = 0.800$

R1(N,P,MU) FOR R= 1.00

		N								
MU	N	4	8	16	32	64	128	256	512	1024
2.00	3.333•000	1.200•001	4.533•001	1.760•002	6.933•002	2.752•003	1.097•004	4.378•004	1.749•005	6.69
1.80	2.988•000	9.490•000	3.138•001	1.063•002	3.646•002	1.260•003	4.372•003	1.519•004	5.286•004	6.69
1.60	2.688•000	7.555•000	2.191•001	6.479•001	1.938•002	5.833•002	1.762•003	5.331•003	1.615•004	6.69
1.40	2.426•000	6.059•000	1.546•001	3.999•001	1.044•002	2.738•002	7.202•002	1.897•003	5.003•003	6.69
1.20	2.198•000	4.900•000	1.104•001	2.504•001	5.717•001	1.304•002	2.999•002	6.881•002	1.580•003	6.69
1.00	2.000•000	4.000•000	8.000•000	1.600•001	3.200•001	6.400•001	1.280•002	2.560•002	5.120•002	6.69
0.80	1.827•000	3.299•000	5.894•000	1.045•001	1.841•001	3.230•001	5.652•001	9.872•001	1.722•002	6.69
0.60	1.677•000	2.750•000	4.424•000	7.004•000	1.096•001	1.698•001	2.614•001	4.005•001	6.114•001	6.69
0.40	1.546•000	2.320•000	3.391•000	4.846•000	6.801•000	9.407•000	1.287•001	1.744•001	2.350•001	6.69
0.20	1.432•000	1.982•000	2.658•000	3.471•000	4.432•000	5.555•000	6.858•000	8.363•000	1.010•001	6.69
-0.00	1.333•000	1.714•000	2.133•000	2.581•000	3.048•000	3.528•000	4.016•000	4.509•000	5.005•000	6.69
-0.20	1.247•000	1.502•000	1.754•000	1.994•000	2.216•000	2.418•000	2.499•000	2.759•001	2.900•000	3.863•000
-0.40	1.172•000	1.333•000	1.476•000	1.599•000	1.700•000	1.742•000	1.847•000	1.898•000	1.938•000	2.065•000
-0.60	1.107•000	1.197•000	1.271•000	1.327•000	1.370•000	1.401•000	1.422•000	1.438•000	1.448•000	1.470•000
-0.80	1.050•000	1.088•000	1.117•000	1.137•000	1.150•000	1.160•000	1.165•000	1.169•000	1.171•000	1.175•000
-1.00	1.000•000	1.000•000	1.000•000	1.000•000	1.000•000	1.000•000	1.000•000	1.000•000	1.000•000	1.000•000
-1.20	9.569•001	9.284•001	9.105•001	8.997•001	8.933•001	8.97•001	8.977•001	8.866•001	8.865•001	8.854•001
-1.40	9.193•001	8.700•001	8.410•001	8.245•001	8.154•001	8.105•001	8.079•001	8.065•001	8.058•001	8.051•001
-1.60	8.866•001	8.221•001	7.864•001	7.672•001	7.570•001	7.517•001	7.490•001	7.476•001	7.468•001	7.461•001
-1.80	8.581•001	7.827•001	7.431•001	7.226•001	7.122•001	7.068•001	7.042•001	7.028•001	7.021•001	7.018•001
-2.00	8.333•001	7.500•001	7.083•001	6.771•001	6.719•001	6.680•001	6.693•001	6.673•001	6.667•001	6.667•001

R1(N•P•MU) FOR Q = 1.01

MU	4	8	16	32	64	128	256	512	1024	88	N	
											1.000+000	1.000+000
2.00	3.333+000	1.200+001	4.533+001	1.760+002	6.933+002	2.752+003	1.097+004	4.378+004	1.749+005	88		
1.80	2.986+000	9.482+000	3.135+001	1.062+002	3.643+002	1.259+003	4.367+003	1.518+004	5.280+004	88		
1.60	2.685+000	7.542+000	2.187+001	6.466+001	1.934+002	5.822+002	1.758+003	5.321+003	1.611+004	88		
1.40	2.422+000	6.045+000	1.542+001	3.988+001	1.041+002	2.730+002	7.180+002	1.892+003	4.988+003	88		
1.20	2.194+000	4.885+000	1.100+001	2.497+001	5.695+001	1.303+002	2.987+002	6.854+002	1.573+003	88		
1.00	1.995+000	3.985+000	7.966+000	1.593+001	3.185+001	6.369+001	1.274+002	2.547+002	5.095+002	88		
0.80	1.822+000	3.285+000	5.864+000	1.039+001	1.810+001	3.212+001	5.619+001	9.814+001	1.712+002	88		
0.60	1.672+000	2.738+000	4.400+000	6.963+000	1.089+001	1.687+001	2.597+001	3.978+001	6.073+001	88		
0.40	1.541+000	2.309+000	3.371+000	4.814+000	6.744+000	9.340+000	1.277+001	1.731+001	2.332+001	88		
0.20	1.428+000	1.972+000	2.642+000	3.447+000	4.400+000	5.512+000	6.804+000	8.296+000	1.002+001	88		
-0.00	1.329+000	1.706+000	2.120+000	2.563+000	3.025+000	3.500+000	3.984+000	4.472+000	4.963+000	88		
-0.20	1.243+000	1.495+000	1.743+000	1.980+000	2.200+000	2.490+000	2.579+000	2.737+000	2.877+000	3.829+000		
-0.40	1.168+000	1.327+000	1.468+000	1.569+000	1.689+000	1.770+000	1.835+000	1.885+000	1.924+000	2.050+000		
-0.60	1.104+000	1.193+000	1.265+000	1.321+000	1.362+000	1.393+000	1.414+000	1.429+000	1.440+000	1.461+000		
-0.80	1.048+000	1.085+000	1.113+000	1.133+000	1.147+000	1.155+000	1.161+000	1.165+000	1.167+000	1.170+000		
-1.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000		
-1.20	9.599+001	9.331+001	9.160+001	9.056+001	8.995+001	8.960+001	8.940+001	8.929+001	8.924+001	8.917+001		
-1.40	9.286+001	8.840+001	8.573+001	8.419+001	8.333+001	8.287+001	8.262+001	8.249+001	8.242+001	8.235+001		
-1.60	9.098+001	8.565+001	8.264+001	8.098+001	8.009+001	7.963+001	7.938+001	7.926+001	7.920+001	7.913+001		
-1.80	9.156+001	8.682+001	8.425+001	8.288+001	8.217+001	8.181+001	8.162+001	8.153+001	8.148+001	8.143+001		
-2.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000		

$\mu_i(N, R, \mu)$ FOR $R = 1..10$

EPI (N, R, MU) FOR R = 2.00

N									
MU	4	8	16	32	64	128	256	512	1024
2.00	3.333+000	1.200+001	4.533+001	1.760+002	6.933+002	2.752+003	1.097+004	4.378+004	1.749+005
1.80	2.908+000	9.132+000	3.006+001	1.017+002	3.487+002	1.205+003	4.179+003	1.453+004	5.053+004
1.60	2.552+000	7.012+000	2.014+001	5.935+001	1.773+002	5.334+002	1.611+003	4.874+003	1.476+004
1.40	2.255+000	5.439+000	1.366+001	3.512+001	9.142+001	2.395+002	6.299+002	1.659+003	4.374+003
1.20	2.007+000	4.271+000	9.409+000	2.114+001	4.800+001	1.096+002	2.510+002	5.756+002	1.321+003
1.00	1.800+000	3.400+000	6.600+000	1.300+001	2.580+001	5.140+001	1.026+002	2.050+002	0.098+002
0.80	1.628+000	2.750+000	4.733+000	8.215+000	1.431+001	2.494+001	4.347+001	7.576+001	1.320+002
0.60	1.486+000	2.264+000	3.485+000	5.371+000	8.262+000	1.267+001	1.937+001	2.955+001	4.498+001
0.40	1.369+000	1.901+000	2.644+000	3.659+000	5.025+000	6.847+000	9.267+000	1.247+001	1.670+001
0.20	1.273+000	1.629+000	2.075+000	2.615+000	3.256+000	4.005+000	4.876+000	5.882+000	7.042+000
-0.00	1.195+000	1.427+000	1.688+000	1.971+000	2.267+000	2.573+000	2.884+000	3.198+000	3.515+000
-0.20	1.133+000	1.277+000	1.425+000	1.569+000	1.702+000	1.924+000	1.934+000	2.031+000	2.117+000
-0.40	1.084+000	1.168+000	1.246+000	1.315+000	1.373+000	1.420+000	1.457+000	1.487+000	1.509+000
-0.60	1.046+000	1.090+000	1.126+000	1.156+000	1.178+000	1.195+000	1.207+000	1.215+000	1.221+000
-0.80	1.019+000	1.035+000	1.048+000	1.058+000	1.064+000	1.069+000	1.072+000	1.074+000	1.075+000
-1.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-1.20	9.886-001	9.799-001	9.738-001	9.699-001	9.675-001	9.661-001	9.653-001	9.648-001	9.643-001
-1.40	9.837-001	9.719-001	9.641-001	9.593-001	9.565-001	9.550-001	9.541-001	9.537-001	9.534-001
-1.60	9.845-001	9.737-001	9.669-001	9.630-001	9.607-001	9.595-001	9.589-001	9.586-001	9.584-001
-1.80	9.901-001	9.836-001	9.796-001	9.774-001	9.761-001	9.755-001	9.752-001	9.750-001	9.749-001
-2.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000

R1(N,RR,MU) FOR R= 4.00

MU	4	8	16	32	64	128	256	512	1024	N
										8
2.00	3.333+000	1.200+001	4.533+001	1.760+002	6.933+002	2.752+003	1.097+004	4.378+004	1.749+005	80
1.80	2.879+000	9.006+000	2.960+001	1.000+002	3.431+002	1.196+003	4.112+003	1.429+004	4.972+004	80
1.60	2.504+000	6.819+000	1.952+001	5.745+001	1.715+002	5.160+002	1.558+003	4.714+003	1.428+004	80
1.40	2.194+000	5.219+000	1.303+001	3.340+001	8.686+001	2.275+002	5.981+002	1.575+003	4.154+003	80
1.20	1.938+000	4.045+000	8.826+000	1.974+001	4.472+001	1.020+002	2.336+002	5.356+002	1.229+003	80
1.00	1.727+000	3.182+000	6.091+000	1.191+001	2.355+001	4.682+001	9.336+001	1.865+002	3.726+002	80
0.80	1.555+000	2.548+000	4.303+000	7.385+000	1.278+001	2.219+001	3.860+001	6.718+001	1.170+002	80
0.60	1.415+000	2.082+000	3.129+000	4.748+000	7.229+000	1.101+001	1.676+001	2.550+001	3.875+001	80
0.40	1.303+000	1.742+000	2.356+000	3.194+000	4.326+000	5.934+000	7.837+000	1.049+001	1.399+001	80
0.20	1.214+000	1.495+000	1.848+000	2.275+000	2.783+000	3.377+000	4.067+000	4.865+000	5.784+000	80
-0.00	1.144+000	1.318+000	1.514+000	1.724+000	1.949+000	2.179+000	2.414+000	2.651+000	2.890+000	80
-0.20	1.092+000	1.194+000	1.298+000	1.399+000	1.494+000	1.580+000	1.658+000	1.727+000	1.788+000	2.204+000
-0.40	1.054+000	1.109+000	1.160+000	1.205+000	1.243+000	1.274+000	1.299+000	1.318+000	1.333+000	1.382+000
-0.60	1.027+000	1.053+000	1.075+000	1.093+000	1.107+000	1.117+000	1.124+000	1.129+000	1.132+000	1.140+000
-0.80	1.010+000	1.019+000	1.026+000	1.031+000	1.035+000	1.037+000	1.039+000	1.040+000	1.041+000	1.042+000
-1.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-1.20	9.953-001	9.916-001	9.890-001	9.873-001	9.862-001	9.856-001	9.853-001	9.851-001	9.850-001	9.849-001
-1.40	9.941-001	9.898-001	9.869-001	9.851-001	9.840-001	9.834-001	9.831-001	9.829-001	9.829-001	9.828-001
-1.60	9.952-001	9.918-001	9.896-001	9.884-001	9.876-001	9.873-001	9.870-001	9.869-001	9.869-001	9.868-001
-1.80	9.974-001	9.957-001	9.946-001	9.940-001	9.936-001	9.935-001	9.934-001	9.933-001	9.933-001	9.933-001
-2.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000

R1(N,0,MU) FOR R= 0.00

H/H	4	8	16	32	64	128	256	512	1024
2.00	3.333•000	1.200•001	6.533•001	1.760•002	6.913•002	2.752•003	1.097•004	4.378•004	1.743•005
1.80	2.970•000	8.963•000	2.945•001	9.951•001	3.413•002	1.179•003	4.990•003	1.421•004	4.945•004
1.60	2.486•000	6.751•000	1.930•001	5.679•001	1.655•002	5.099•002	1.540•003	4.658•003	1.411•004
1.40	2.170•000	5.135•000	1.279•001	3.276•001	8.515•001	2.730•002	5.862•002	1.544•003	4.071•003
1.20	1.910•000	3.953•000	8.590•000	1.917•001	4.341•001	9.998•001	2.265•002	5.195•002	1.192•003
1.00	1.696•000	3.087•000	5.870•000	1.143•001	2.257•001	4.483•001	8.935•001	1.784•002	3.565•002
0.80	1.521•000	2.453•000	4.101•000	6.996•000	1.206•001	2.090•001	3.630•001	6.315•001	1.099•002
0.60	1.389•000	1.991•000	2.950•000	4.433•000	6.706•000	1.017•001	1.544•001	2.345•001	3.559•001
0.40	1.268•000	1.657•000	2.202•000	2.946•000	3.949•000	5.286•000	7.062•000	9.414•000	1.252•001
0.20	1.181•000	1.420•000	1.720•000	2.083•000	2.514•000	3.019•000	3.406•000	4.284•000	5.066•000
-0.00	1.116•000	1.255•000	1.413•000	1.584•000	1.743•000	1.949•000	2.137•000	2.328•000	2.520•000
-0.20	1.069•000	1.145•000	1.223•000	1.299•000	1.371•000	1.436•000	1.494•000	1.546•000	1.592•000
-0.40	1.037•000	1.075•000	1.110•000	1.142•000	1.148•000	1.189•000	1.207•000	1.220•000	1.231•000
-0.60	1.017•000	1.033•000	1.047•000	1.058•000	1.067•000	1.073•000	1.078•000	1.081•000	1.083•000
-0.80	1.006•000	1.011•000	1.014•000	1.017•000	1.020•000	1.021•000	1.022•000	1.023•000	1.023•000
-1.00	1.000•000	1.000•000	1.000•000	1.000•000	1.000•000	1.000•000	1.000•000	1.000•000	1.000•000
-1.20	9.980-001	9.963-001	9.952-001	9.945-001	9.940-001	9.938-001	9.936-001	9.935-001	9.934-001
-1.40	9.978-001	9.961-001	9.950-001	9.944-001	9.940-001	9.937-001	9.936-001	9.935-001	9.935-001
-1.60	9.984-001	9.973-001	9.966-001	9.962-001	9.960-001	9.958-001	9.957-001	9.957-001	9.957-001
-1.80	9.988-001	9.983-001	9.985-001	9.983-001	9.982-001	9.981-001	9.981-001	9.981-001	9.981-001
-2.00	1.000•000	1.000•000	1.000•000	1.000•000	1.000•000	1.000•000	1.000•000	1.000•000	1.000•000

R1(N,R,MU) FOR R= 16.00

MU	N						1024
	4	8	16	32	64	128	
2.00	3.3333+000	1.2000+001	4.5333+001	1.7600+002	6.9333+002	2.752+003	1.097+004
1.80	2.8666+000	8.949+000	2.940+001	9.934+001	3.406+002	1.177+003	4.083+003
1.60	2.4800+000	6.727+000	1.923+001	5.654+001	1.688+002	5.077+002	1.533+003
1.40	2.161+000	5.104+000	1.270+001	3.251+001	8.450+001	2.213+002	5.817+002
1.20	1.898+000	3.914+000	8.491+000	1.894+001	4.285+001	9.749+001	2.236+002
1.00	1.681+000	3.043+000	5.766+000	1.121+001	2.211+001	4.389+001	8.747+001
0.80	1.504+000	2.404+000	3.997+000	6.794+000	1.149+001	2.023+001	3.512+001
0.60	1.360+000	1.939+000	2.848+000	4.254+000	6.409+000	9.696+000	1.469+001
0.40	1.247+000	1.606+000	2.108+000	2.794+000	3.717+000	4.950+000	6.487+000
0.20	1.160+000	1.372+000	1.637+000	1.958+000	2.340+000	2.787+000	3.307+000
-0.00	1.097+000	1.214+000	1.346+000	1.489+000	1.639+000	1.794+000	1.952+000
25	-0.20	1.054+000	1.113+000	1.174+000	1.233+000	1.249+000	1.339+000
-0.40	1.026+000	1.053+000	1.078+000	1.101+000	1.119+000	1.135+000	1.147+000
-0.60	1.011+000	1.021+000	1.030+000	1.037+000	1.043+000	1.047+000	1.050+000
-0.80	1.003+000	1.006+000	1.008+000	1.010+000	1.011+000	1.012+000	1.013+000
-1.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-1.20	9.991-001	9.984-001	9.979-001	9.974-001	9.973-001	9.972-001	9.972-001
-1.40	9.992-001	9.985-001	9.981-001	9.979-001	9.977-001	9.976-001	9.976-001
-1.60	9.995-001	9.991-001	9.989-001	9.987-001	9.986-001	9.986-001	9.986-001
-1.80	9.998-001	9.997-001	9.996-001	9.995-001	9.995-001	9.995-001	9.995-001
-2.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000

R1(N,R,MU) FOR R= 32.00

MU	4	N						80
		8	16	32	64	128	256	
2.00	3.3333+000	1.2000+001	4.5333+001	1.7600+002	6.9333+002	2.7524+003	1.0974+004	4.3784+004
1.80	2.8654+000	8.9454+000	2.9384+001	9.9284+001	3.4054+002	1.1764+003	4.0814+003	1.4184+004
1.60	2.4784+000	6.7194+000	1.9204+001	5.6464+001	1.6854+002	5.0704+002	1.5314+003	4.6324+003
1.40	2.1584+000	5.0914+000	1.2664+001	3.2424+001	8.4254+001	2.2064+002	5.7994+002	1.5274+003
1.20	1.8934+000	3.8984+000	8.4484+000	1.8834+001	4.2614+001	9.7144+001	2.2234+002	5.0974+002
1.00	1.6744+000	3.0214+000	5.7164+000	1.1114+001	2.1884+001	4.3444+001	8.6564+001	1.7284+002
0.80	1.4944+000	2.3784+000	3.9404+000	6.6854+000	1.1494+001	1.9864+001	3.4474+001	5.9924+001
0.60	1.3484+000	1.9084+000	2.7874+000	4.1474+000	6.2314+000	9.4084+000	1.4244+001	2.1584+001
0.40	1.2334+000	1.5724+000	2.0464+000	2.6934+000	3.5654+000	4.7294+000	6.2754+000	8.3204+000
0.20	1.1464+000	1.3384+000	1.5794+000	1.8714+000	2.2184+000	2.6254+000	3.0974+000	3.6434+000
-0.00	1.0834+000	1.1844+000	1.2974+000	1.4204+000	1.5504+000	1.6834+000	1.8194+000	1.9574+000
-0.20	1.0434+000	1.0904+000	1.1394+000	1.1844+000	1.2304+000	1.2714+000	1.3074+000	1.3394+000
-0.40	1.0194+000	1.0394+000	1.0574+000	1.0734+000	1.0874+000	1.0984+000	1.1074+000	1.1144+000
-0.60	1.0074+000	1.0144+000	1.0194+000	1.0244+000	1.0284+000	1.0304+000	1.0324+000	1.0334+000
-0.80	1.0024+000	1.0034+000	1.0054+000	1.0064+000	1.0064+000	1.0074+000	1.0074+000	1.0074+000
-1.00	1.0004+000	1.0004+000	1.0004+000	1.0004+000	1.0004+000	1.0004+000	1.0004+000	1.0004+000
-1.20	9.9964-001	9.9934-001	9.9914-001	9.9904-001	9.9894-001	9.9884-001	9.9884-001	9.9884-001
-1.40	9.9974-001	9.9944-001	9.9934-001	9.9924-001	9.9914-001	9.9914-001	9.9914-001	9.9914-001
-1.60	9.9984-001	9.9974-001	9.9964-001	9.9944-001	9.9964-001	9.9954-001	9.9954-001	9.9954-001
-1.80	9.9994-001	9.9994-001	9.9994-001	9.9994-001	9.9994-001	9.9984-001	9.9984-001	9.9984-001
-2.00	1.0004+000	1.0004+000	1.0004+000	1.0004+000	1.0004+000	1.0004+000	1.0004+000	1.0004+000

R1(N,R,MU) FOR R= 64.00

NU	4	N						1024
		8	16	32	64	128	256	
2.00	3.333+000	1.200+001	4.533+001	1.760+002	6.933+002	2.752+003	1.097+004	4.378+004
1.80	2.865+000	8.943+000	2.938+001	9.927+001	3.404+002	1.176+003	4.080+003	1.418+004
1.60	2.477+000	6.716+000	1.919+001	5.644+001	1.685+002	5.067+002	1.530+003	4.630+003
1.40	2.156+000	5.087+000	1.265+001	3.238+001	8.415+001	2.204+002	5.793+002	1.526+003
1.20	1.890+000	3.891+000	8.429+000	1.879+001	4.251+001	9.690+001	2.218+002	5.085+002
1.00	1.670+000	3.010+000	5.691+000	1.105+001	2.177+001	4.322+001	8.611+001	1.719+002
0.80	1.489+000	2.363+000	3.909+000	6.624+000	1.137+001	1.966+001	3.411+001	5.929+001
0.60	1.341+000	1.889+000	2.749+000	4.080+000	6.119+000	9.229+000	1.396+001	2.114+001
0.40	1.224+000	1.548+000	2.003+000	2.624+000	3.461+000	4.578+000	6.060+000	8.023+000
0.20	1.135+000	1.313+000	1.536+000	1.808+000	2.129+000	2.506+000	2.944+000	3.450+000
-0.00	1.073+000	1.161+000	1.261+000	1.369+000	1.482+000	1.600+000	1.719+000	1.840+000
-0.20	1.035+000	1.073+000	1.112+000	1.151+000	1.187+000	1.220+000	1.249+000	1.275+000
-0.40	1.014+000	1.028+000	1.042+000	1.054+000	1.044+000	1.072+000	1.078+000	1.083+000
-0.60	1.005+000	1.009+000	1.013+000	1.016+000	1.018+000	1.020+000	1.021+000	1.022+000
-0.80	1.001+000	1.002+000	1.003+000	1.004+000	1.004+000	1.004+000	1.004+000	1.004+000
-1.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-1.20	9.998-001	9.997-001	9.996-001	9.995-001	9.995-001	9.995-001	9.995-001	9.995-001
-1.40	9.999-001	9.998-001	9.997-001	9.997-001	9.997-001	9.997-001	9.996-001	9.996-001
-1.60	9.999-001	9.999-001	9.999-001	9.999-001	9.999-001	9.999-001	9.998-001	9.998-001
-1.80	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-2.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000

R1(N,R,MU) FOR R=128.00

MU	4	N						8	
		8	16	32	64	128	256		
2.00	3.333+000	1.200+001	4.533+001	1.760+002	6.933+002	2.752+003	1.097+004	4.378+004	1.749+005
1.80	2.865+000	8.943+000	2.938+001	9.926+001	3.404+002	1.176+003	4.080+003	1.410+004	4.932+004
1.60	2.477+000	6.715+000	1.919+001	5.643+001	1.684+002	5.067+002	1.530+003	4.629+003	1.402+004
1.40	2.156+000	5.085+000	1.265+001	3.237+001	8.411+001	2.203+002	5.790+002	1.525+003	0.021+003
1.20	1.889+000	3.887+000	8.421+000	1.877+001	4.246+001	9.680+001	2.215+002	5.079+002	1.166+003
1.00	1.668+000	3.005+000	5.679+000	1.103+001	2.172+001	4.311+001	8.589+001	1.714+002	3.426+002
0.80	1.486+000	2.354+000	3.891+000	6.589+000	1.131+001	1.955+001	3.391+001	5.893+001	1.025+002
0.60	1.336+000	1.877+000	2.725+000	4.037+000	6.048+000	9.115+000	1.378+001	2.086+001	3.160+001
0.40	1.217+000	1.532+000	1.973+000	2.576+000	3.388+000	4.471+000	5.909+000	7.813+000	1.033+001
0.20	1.127+000	1.294+000	1.504+000	1.759+000	2.062+000	2.416+000	2.827+000	3.303+000	3.851+000
-0.00	1.065+000	1.144+000	1.232+000	1.329+000	1.430+000	1.534+000	1.640+000	1.768+000	1.856+000
-0.20	1.029+000	1.060+000	1.092+000	1.124+000	1.153+000	1.181+000	1.205+000	1.226+000	1.245+000
-0.40	1.010+000	1.021+000	1.031+000	1.040+000	1.047+000	1.053+000	1.058+000	1.062+000	1.065+000
-0.60	1.003+000	1.006+000	1.008+000	1.010+000	1.012+000	1.013+000	1.014+000	1.014+000	1.015+000
-0.80	1.001+000	1.001+000	1.002+000	1.002+000	1.002+000	1.002+000	1.002+000	1.002+000	1.002+000
-1.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-1.20	9.999-001	9.999-001	9.998-001	9.998-001	9.998-001	9.998-001	9.998-001	9.998-001	9.998-001
-1.40	1.000+000	9.999-001	9.999-001	9.999-001	9.999-001	9.999-001	9.999-001	9.999-001	9.999-001
-1.60	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	9.999-001	9.999-001	9.999-001
-1.80	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-2.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000

R1(N,R,MU) FOR R = 256.00

MU	N						
	4	8	16	32	64	128	256
2.00	3.333+000	1.200+001	4.533+001	1.760+002	6.933+002	2.752+003	1.097+004
1.80	2.865+000	8.943+000	2.938+001	9.924+001	3.404+002	1.176+003	4.080+003
1.60	2.477+000	6.715+000	1.919+001	5.642+001	1.684+002	5.066+002	1.530+003
1.40	2.156+000	5.084+000	1.264+001	3.236+001	8.10+001	2.202+002	5.789+002
1.20	1.889+000	3.886+000	8.418+000	1.874+001	4.204+001	9.675+001	2.214+002
1.00	1.668+000	3.003+000	5.673+000	1.101+001	2.169+001	4.305+001	8.578+001
0.80	1.484+000	2.350+000	3.881+000	6.570+000	1.127+001	1.948+001	3.380+001
0.60	1.333+000	1.869+000	2.709+000	4.010+000	6.003+000	9.042+000	1.366+001
0.40	1.212+000	1.520+000	1.952+000	2.541+000	3.335+000	4.394+000	5.800+000
0.20	1.121+000	1.280+000	1.479+000	1.722+000	2.009+000	2.346+000	2.737+000
-0.00	1.059+000	1.130+000	1.210+000	1.296+000	1.388+000	1.482+000	1.577+000
-0.20	1.024+000	1.050+000	1.077+000	1.103+000	1.127+000	1.150+000	1.170+000
-0.40	1.008+000	1.016+000	1.023+000	1.029+000	1.035+000	1.039+000	1.043+000
-0.60	1.002+000	1.004+000	1.005+000	1.007+000	1.008+000	1.009+000	1.009+000
-0.80	1.000+000	1.001+000	1.001+000	1.001+000	1.001+000	1.001+000	1.001+000
-1.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-1.20	1.000+000	9.999-001	9.999-001	9.999-001	9.999-001	9.999-001	9.999-001
-1.40	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	9.999-001
-1.60	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-1.80	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-2.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000

R1(N•R, MU) FOR Q = 512.00

MU	N							00
	4	8	16	32	64	128	256	
2.00	3.3333+000	1.2000+001	4.5333+001	1.7600+002	6.9333+002	2.7520+003	1.0970+004	4.3780+004
1.80	2.8650+000	8.9430+000	2.9380+001	9.9240+001	3.4040+002	1.1760+003	4.0800+003	1.4180+004
1.60	2.4770+000	6.7140+000	1.9190+001	5.6420+001	1.6840+002	5.0660+002	1.5300+003	4.6280+003
1.40	2.1560+000	5.0840+000	1.2640+001	3.2360+001	8.4100+001	2.2020+002	5.7890+002	1.5250+003
1.20	1.8890+000	3.8850+000	8.4160+000	1.8760+001	4.2430+001	9.6730+001	2.2140+002	5.0760+002
1.00	1.6667+000	3.0010+000	5.6700+000	1.1010+001	2.1680+001	4.3030+001	8.5720+001	1.7110+002
0.80	1.4830+000	2.3470+000	3.8750+000	6.5580+000	1.1250+001	1.9050+001	3.3730+001	5.8620+001
0.60	1.3310+000	1.8640+000	2.6990+000	3.9920+000	5.9730+000	8.9940+000	1.3590+001	2.0560+001
0.40	1.2090+000	1.5120+000	1.9360+000	2.5160+000	3.2960+000	4.3380+000	5.7210+000	7.5530+000
0.20	1.1160+000	1.2680+000	1.4600+000	1.6920+000	1.9670+000	2.2900+000	2.6650+000	3.0980+000
-0.00	1.0540+000	1.1180+000	1.1910+000	1.2700+000	1.3530+000	1.4380+000	1.5260+000	1.6140+000
-0.20	1.0200+000	1.0420+000	1.0640+000	1.0860+000	1.1070+000	1.1250+000	1.1420+000	1.1570+000
-0.40	1.0060+000	1.0120+000	1.0170+000	1.0220+000	1.0260+000	1.0300+000	1.0320+000	1.0340+000
-0.60	1.0010+000	1.0030+000	1.0040+000	1.0040+000	1.0050+000	1.0060+000	1.0060+000	1.0060+000
-0.80	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0001+000	1.0010+000
-1.00	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000
-1.20	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000
-1.40	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000
-1.60	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000
-1.80	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000
-2.00	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000	1.0000+000

R1(N,R,MU) FOR R=1024.00

MU	4	N								80
		8	16	32	64	128	256	512	1024	
2.00	3.333+000	1.200+001	4.533+001	1.760+002	6.933+002	2.752+003	1.097+004	4.378+004	1.749+005	80
1.80	2.865+000	8.943+000	2.938+001	9.926+001	3.404+002	1.176+003	4.080+003	1.418+004	4.932+004	80
1.60	2.477+000	6.714+000	1.919+001	5.642+001	1.684+002	5.066+002	1.530+003	4.628+003	1.402+004	80
1.40	2.156+000	5.084+000	1.264+001	3.236+001	8.409+001	2.202+002	5.789+002	1.525+003	4.020+003	80
1.20	1.889+000	3.885+000	8.416+000	1.874+001	4.243+001	9.672+001	2.213+002	5.075+002	1.165+003	80
1.00	1.667+000	3.001+000	5.668+000	1.100+001	2.167+001	4.301+001	8.569+001	1.711+002	3.418+002	80
0.80	1.482+000	2.345+000	3.872+000	6.552+000	1.124+001	1.942+001	3.369+001	5.855+001	1.018+002	80
0.60	1.330+000	1.860+000	2.693+000	3.980+000	5.953+000	8.963+000	1.354+001	2.049+001	3.102+001	80
0.40	1.206+000	1.505+000	1.924+000	2.497+000	3.268+000	4.297+000	5.663+000	7.472+000	9.862+000	80
0.20	1.112+000	1.259+000	1.444+000	1.668+000	1.934+000	2.245+000	2.407+000	3.025+000	3.507+000	80
-0.00	1.049+000	1.108+000	1.175+000	1.249+000	1.324+000	1.402+000	1.483+000	1.564+000	1.645+000	80
-0.20	1.017+000	1.035+000	1.054+000	1.073+000	1.090+000	1.106+000	1.120+000	1.132+000	1.144+000	1.220+000
-0.40	1.004+000	1.009+000	1.013+000	1.017+000	1.020+000	1.022+000	1.024+000	1.026+000	1.027+000	1.031+000
-0.60	1.001+000	1.002+000	1.003+000	1.003+000	1.004+000	1.004+000	1.004+000	1.004+000	1.004+000	1.004+000
-0.80	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-1.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-1.20	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-1.40	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-1.60	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-1.80	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-2.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000

R1(N,R,MU) FOR R=2048.00

MU	N							-	
	4	8	16	32	64	128	256		
2.00	3.333+000	1.200+001	4.533+001	1.760+002	6.933+002	2.752+003	1.097+004	4.378+004	1.749+005
1.80	2.865+000	8.943+000	2.938+001	9.924+001	3.404+002	1.176+003	4.080+003	1.418+004	0.932+000
1.60	2.077+000	6.714+000	1.919+001	5.642+001	1.684+002	5.066+002	1.530+003	4.628+003	1.402+004
1.40	2.156+000	5.084+000	1.264+001	3.236+001	8.409+001	2.202+002	5.789+002	1.525+003	4.020+003
1.20	1.889+000	3.885+000	8.415+000	1.875+001	4.243+001	9.672+001	2.213+002	5.075+002	1.165+003
1.00	1.667+000	3.000+000	5.667+000	1.100+001	2.167+001	4.301+001	8.568+001	1.710+002	3.417+002
0.80	1.482+000	2.345+000	3.870+000	6.548+000	1.123+001	1.901+001	3.367+001	5.851+001	1.018+002
0.60	1.329+000	1.858+000	2.688+000	3.972+000	5.941+000	8.902+000	1.351+001	2.044+001	3.095+001
0.40	1.204+000	1.501+000	1.916+000	2.483+000	3.247+000	4.266+000	5.620+000	7.412+000	9.780+000
0.20	1.108+000	1.251+000	1.431+000	1.649+000	1.906+000	2.209+000	2.560+000	2.966+000	3.430+000
-0.00	1.045+000	1.100+000	1.162+000	1.229+000	1.299+000	1.372+000	1.446+000	1.521+000	1.596+000
-0.20	1.014+000	1.030+000	1.046+000	1.061+000	1.076+000	1.089+000	1.101+000	1.112+000	1.122+000
-0.40	1.003+000	1.007+000	1.010+000	1.012+000	1.015+000	1.017+000	1.018+000	1.019+000	1.020+000
-0.60	1.001+000	1.001+000	1.002+000	1.002+000	1.002+000	1.003+000	1.003+000	1.003+000	1.003+000
-0.80	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-1.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-1.20	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-1.40	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-1.60	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-1.80	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000
-2.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000

R1(N,R,MU) FOR R=oo

MU	4	8	16	32	64	128	256	512	1024	N
										8
2.00	3.3333+000	1.2000+001	4.5333+001	1.760+002	6.9333+002	2.752+003	1.097+004	4.378+004	1.749+005	8
1.80	2.8655+000	8.943+000	2.938+001	9.924+001	3.400+002	1.176+003	4.080+003	1.418+003	4.932+004	8
1.60	2.477+000	6.714+000	1.919+001	5.642+001	1.684+002	5.066+002	1.530+003	4.628+003	1.402+004	8
1.40	2.156+000	5.084+000	1.264+001	3.236+001	8.409+001	2.202+002	5.789+002	1.525+003	4.020+003	8
1.20	1.889+000	3.885+000	8.415+000	1.875+001	4.243+001	9.672+001	2.213+002	5.075+002	1.165+003	8
1.00	1.667+000	3.000+000	5.657+000	1.100+001	2.167+001	4.300+001	8.567+001	1.710+002	3.417+002	8
0.80	1.482+000	2.343+000	3.867+000	6.543+000	1.123+001	1.940+001	3.364+001	5.846+001	1.017+002	8
0.60	1.327+000	1.854+000	2.680+000	3.958+000	5.916+000	8.903+000	1.344+001	2.034+001	3.080+001	8
0.40	1.198+000	1.487+000	1.890+000	2.441+000	3.184+000	4.174+000	5.489+000	7.231+000	9.533+000	8
0.20	1.091+000	1.210+000	1.350+000	1.541+000	1.757+000	2.009+000	2.303+000	2.642+000	3.033+000	8
-0.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	8
-0.20	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	8
-0.40	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	8
-0.60	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	8
-0.80	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	8
-1.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	8
-1.20	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	8
-1.40	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	8
-1.60	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	8
-1.80	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	8
-2.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	8

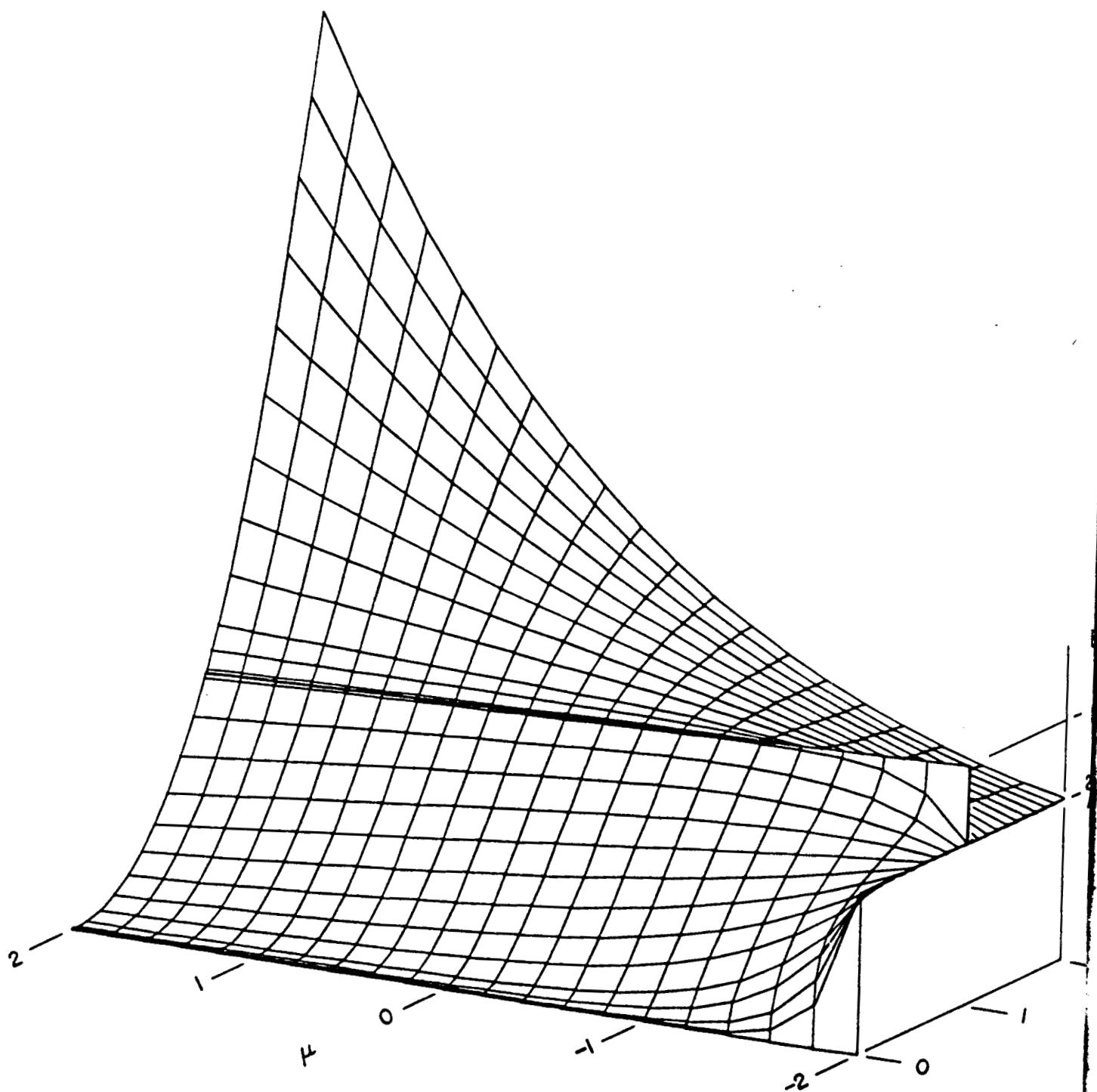


Fig. 2 The bias function, $B_2(r, \mu)$

B2(R,MU)

MU	R	R	R	R	R	R	R
0.001	0.003	0.010	0.030	0.100	0.200	0.400	0.800
2.00	1.000-006	9.000-006	1.000-004	9.000-004	1.000-002	4.000-002	1.400-001
1.80	1.072-006	9.605-006	1.072-004	9.642-004	1.070-002	4.263-002	1.686-001
1.60	1.152-006	1.037-005	1.152-004	1.036-003	1.147-002	4.507-002	1.776-001
1.40	1.245-006	1.120-005	1.244-004	1.118-003	1.233-002	4.860-002	1.871-001
1.20	1.356-006	1.221-005	1.355-004	1.216-003	1.333-002	5.207-002	1.972-001
1.00	1.500-006	1.349-005	1.495-004	1.337-003	1.450-002	5.600-002	2.080-001
0.80	1.698-006	1.524-005	1.683-004	1.493-003	1.593-002	6.052-002	2.196-001
0.60	2.001-006	1.788-005	1.955-004	1.708-003	1.773-002	6.583-002	2.321-001
0.40	2.530-006	2.228-005	2.381-004	2.020-003	2.006-002	7.219-002	2.457-001
0.20	3.594-006	3.048-005	3.100-004	2.494-003	2.316-002	7.996-002	2.607-001
-0.00	6.065-006	4.745-005	4.404-004	3.250-003	2.742-002	8.962-002	2.773-001
35	-0.20	1.260-005	8.606-005	6.921-004	4.507-003	3.340-002	1.018-001
-0.40	3.174-005	1.809-004	1.204-003	6.664-003	4.195-002	1.175-001	3.168-001
-0.60	9.231-005	4.280-004	2.288-003	1.047-002	5.438-002	1.379-001	3.407-001
-0.80	2.949-004	1.101-003	4.662-003	1.735-002	7.271-002	1.646-001	3.482-001
-1.00	1.000-003	3.000-003	1.000-002	3.000-002	1.000-001	2.000-001	4.000-001
-1.20	3.525-003	8.089-003	2.225-002	5.362-002	1.410-001	2.472-001	4.371-001
-1.40	1.276-002	2.467-002	5.081-002	9.828-002	2.032-001	3.104-001	4.808-001
-1.60	4.708-002	7.306-002	1.183-001	1.836-001	2.979-001	3.956-001	5.324-001
-1.80	1.762-001	2.195-001	2.793-001	3.479-001	4.431-001	5.107-001	5.936-001
-2.00	6.667-001	6.667-001	6.667-001	6.667-001	6.667-001	6.667-001	6.667-001

B2(R,MU)

MU	1.00	1.01	1.10	R	2.00	4.00	8.00	16.00	32.00
2.00	1.000+000	1.020+000	1.210+000	4.000+000	1.600+001	6.400+001	2.560+002	1.024+003	
1.80	1.000+000	1.019+000	1.198+000	3.642+000	1.289+001	4.513+001	1.574+002	5.485+002	
1.60	1.000+000	1.018+000	1.186+000	3.316+000	1.039+001	3.188+001	9.706+001	2.947+002	
1.40	1.000+000	1.017+000	1.174+000	3.018+000	8.389+000	2.259+001	6.008+001	1.590+002	
1.20	1.000+000	1.016+000	1.162+000	2.747+000	6.784+000	1.607+001	3.741+001	8.644+001	
1.00	1.000+000	1.015+000	1.150+000	2.500+000	5.500+000	1.150+001	2.350+001	4.750+001	
0.80	1.000+000	1.014+000	1.138+000	2.275+000	4.475+000	8.297+000	1.495+001	2.653+001	
0.60	1.000+000	1.013+000	1.126+000	2.071+000	3.658+000	6.051+000	9.673+000	1.516+001	
0.40	1.000+000	1.012+000	1.114+000	1.886+000	3.007+000	4.473+000	6.404+000	8.951+000	
0.20	1.000+000	1.011+000	1.102+000	1.718+000	2.489+000	3.364+000	4.365+000	5.514+000	
-0.00	1.000+000	1.010+000	1.089+000	1.566+000	2.078+000	2.581+000	3.082+000	3.582+000	
-0.20	1.000+000	1.009+000	1.075+000	1.429+000	1.752+000	2.027+000	2.265+000	2.472+000	
-0.40	1.000+000	1.007+000	1.060+000	1.304+000	1.494+000	1.633+000	1.738+000	1.817+000	
-0.60	1.000+000	1.006+000	1.043+000	1.192+000	1.290+000	1.351+000	1.392+000	1.418+000	
-0.80	1.000+000	1.004+000	1.024+000	1.091+000	1.128+000	1.148+000	1.159+000	1.166+000	
-1.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	
-1.20	1.000+000	9.929+001	9.693+001	9.181+001	8.990+001	8.912+001	8.879+001	8.865+001	
-1.40	1.000+000	9.776+001	9.281+001	8.446+001	8.192+001	8.103+001	8.071+001	8.058+001	
-1.60	1.000+000	9.429+001	8.708+001	7.787+001	7.561+001	7.494+001	7.472+001	7.465+001	
-1.80	1.000+000	8.614+001	7.883+001	7.196+001	7.062+001	7.028+001	7.018+001	7.015+001	
-2.00	1.000+000	6.667+001	6.667+001	6.667+001	6.667+001	6.667+001	6.667+001	6.667+001	

B2(R,MU)

MU	64.00	128.00	256.00	R	512.00	1024.00	2048.00	∞
2.00	4.096+003	1.638+004	6.554+004	2.621+005	1.049+006	4.194+006	16.783+005	∞
1.80	1.910+003	6.653+003	2.317+004	8.067+004	2.809+005	9.783+005	35.114+004	∞
1.60	8.937+002	2.710+003	8.215+003	2.490+004	7.550+004	2.288+005	8.955+004	∞
1.40	4.201+002	1.109+003	2.928+003	7.727+003	2.039+004	5.381+004	18.777+004	∞
1.20	1.991+002	4.579+002	1.052+003	2.418+003	5.555+003	1.277+004	5.988+003	∞
1.00	9.550+001	1.915+002	3.835+002	7.675+002	1.536+003	3.071+003	14.822+003	∞
0.80	4.669+001	8.179+001	1.429+002	2.493+002	4.346+002	7.570+002	33.500+002	∞
0.60	2.348+001	3.609+001	5.521+001	8.418+001	1.281+002	1.946+002	10.000+002	∞
0.40	1.231+001	1.674+001	2.260+001	3.031+001	4.050+001	5.395+001	18.000+001	∞
0.20	6.834+000	8.351+000	1.009+001	1.209+001	1.439+001	1.704+001	10.000+001	∞
-0.00	4.082+000	4.582+000	5.082+000	5.582+000	6.082+000	6.582+000	10.000+000	∞
-0.20	2.652+000	2.809+000	2.945+000	3.064+000	3.167+000	3.258+000	3.863+000	∞
-0.40	1.877+000	1.923+000	1.957+000	1.983+000	2.003+000	2.018+000	2.065+000	∞
-0.60	1.436+000	1.447+000	1.455+000	1.460+000	1.463+000	1.465+000	1.470+000	∞
-0.80	1.170+000	1.172+000	1.173+000	1.174+000	1.174+000	1.174+000	1.175+000	∞
-1.00	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	1.000+000	∞
-1.20	8.859-001	8.856-001	8.855-001	8.854-001	8.854-001	8.854-001	8.854-001	8.854-001
-1.40	8.053-001	8.052-001	8.051-001	8.051-001	8.051-001	8.051-001	8.051-001	8.051-001
-1.60	7.462-001	7.462-001	7.461-001	7.461-001	7.461-001	7.461-001	7.461-001	7.461-001
-1.80	7.015-001	7.014-001	7.014-001	7.014-001	7.014-001	7.014-001	7.014-001	7.014-001
-2.00	6.667-001	6.667-001	6.667-001	6.667-001	6.667-001	6.667-001	6.667-001	6.667-001