THE QUEST TO UNDERSTAND AND REDUCE 1/f NOISE IN AMPLIFIERS AND BAW QUARTZ OSCILLATORS

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ABSTRACT

This paper attempts to trace the highlights in the development of precision oscillators from a historical point of view and speculates on further improvements in their 1/f performance. Using Leeson's model the closed loop phase modulation (PM) noise and frequency stability can be estimated from the stability of the quartz resonator, the resonator quality factor, and the noise of the sustaining amplifier coupled with the stability of the oven and various environmental parameters. The inherent frequency stability of the oscillator is primarily limited by the 1/f PM noise of the BAW quartz resonator and the sustaining amplifier. This paper, therefore, focuses on describing the quest to characterize, understand, and reduce the 1/f PM noise of amplifiers and BAW resonators.

INTRODUCTION

Modern high-stability bulk acoustic wave (BAW) quartz oscillators have evolved through a long and tortuous path. In this paper I trace many of the highlights in the development of these oscillators and speculate on further improvements in their 1/f performance. It is inevitable that some of the important discoveries that have occurred during the past 40 years and some of the people involved, will be left out. For these inadvertent omissions, I ask your indulgence.

Figure 1 shows the basic elements of a quartz oscillator. Using Leeson's model, the closed loop phase modulation (PM) noise and frequency stability can be estimated from the stability of the quartz resonator, the resonator quality factor, and the noise of the sustaining amplifier coupled with the stability of the oven and various environmental parameters [1-6]. The inherent frequency stability of the oscillator is limited primarily by the 1/f PM noise of the BAW quartz resonator and the sustaining amplifier. At 5 MHz, for example, these processes usually dominate the frequency stability for Fourier frequencies from approximately 0.001 Hz to 30 Hz or for measurement times from roughly 0.01 s to 100 s. In several cases it has been shown that the 1/f

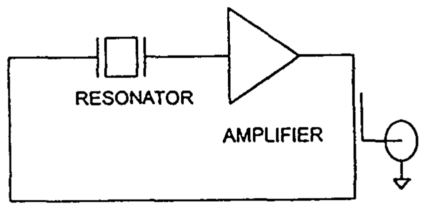


Fig. 1 *Conceptual diagram of an oscillator with the conditions for oscillation.*

frequency modulation (FM) noise of the resonator extends out to at least 10^6 s [4,7]. This paper, therefore, concentrates on the quest to characterize, understand (model), and reduce the 1/f PM noise of BAW resonators and amplifiers. To a large extent the descriptions outlined in this paper will follow the historical order of the developments.

LEESON'S MODEL OF AN OSCILLATOR

Leeson's model is used to estimate the random PM noise of an oscillator from the PM noise of the sustaining amplifier and the resonator. The agreement with experimental results is often better than 3 dB [2,3]. The basis for this model is that the phase around the oscillating loop, ϕ, is given by $\phi = 2N\pi$ with N = 0, 1, 2, 3, ... and the gain around the loop, G, is given by G = 1.00.... The consequence of this is that a phase shift, $d\phi$, anywhere within the oscillation loop causes a frequency change, $d\nu$, from resonance given by

$$\frac{d\nu}{\nu_o} = \frac{1}{2Q_L} d\phi \qquad (1)$$

where ν_0 is the resonance frequency of the resonator and Q_L is the loaded Q-factor of the resonator. Squaring both sides and taking the Fourier transform of both sides yields

$$S_y^{osc}(f) = \frac{1}{4Q_L^2} S_\phi^{loop}(f) \qquad (2)$$

where $S_y^{osc}(f)$ is the PSD of fractional frequency fluctuations of the oscillator due to PM noise of $S_\phi^{loop}(f)$, within the loop. $S_y(f)$ can be expressed in terms of the PM noise of the oscillator as

$$S_y(f) = \frac{f^2}{\nu_0^2} S_\phi(f), \quad S_\phi(f) = 2\mathcal{L}(f). \qquad (3)$$

Equations 1-3 allows us to finally obtain the PM noise of the oscillator within the bandwidth of the resonator as

$$S_\phi^{osc}(f) = \frac{\nu_0^2}{4f^2 Q_L^2} S_\phi^{loop}(f). \qquad (4)$$

The (PM) noise in the amplifier can be modeled by

$$S_\phi^{amp}(f) = \frac{\alpha_E}{f} + \frac{2kFGT}{P}, \qquad (5)$$

where α_E is a phenomenological constant that characterizes the flicker PM noise coefficient for the amplifier, k is Boltzmann's constant, F is the noise figure, G is the gain, T is the temperature in Kelvin, and P the output power of the amplifier.

The PM noise in the resonator can be modeled by

$$S_\phi^R(f) = \frac{\alpha_R}{f} + \frac{2kT}{P_b}, \qquad (6)$$

where α_R is the flicker coefficient for the resonator, and P_b is the total power dissipated in the resonator and load. Combining these yields

$$S_\phi^{osc}(f) = \left(\frac{\nu_0}{2Q_L}\right)^2 \frac{1}{f^2}\left[\frac{\alpha_E}{f} + \frac{2kFGT}{P} + \frac{\alpha_R}{f} + \frac{2kT}{P_b}\right] \quad f<BW, \qquad (7)$$

and

$$S_\phi^{osc}(f) = \frac{\alpha_E}{f} + \frac{2kFGT}{P} \quad f > BW,$$

where the bandwidth of the sustaining loop, BW, is approximately $\nu_0/2Q_L$.

The time-domain frequency stability can be calculated from the phase noise using

$$\sigma_y^2 = \frac{2}{\pi \nu_0 \tau^2} \int_0^{f_h} S_\phi^{osc}(f) \sin^4(\pi f \tau)\, df, \qquad (8)$$

where f_h is the equivalent bandwidth of the measurement system [8]. Flicker PM noise in the resonator leads to a flicker FM in the oscillator and a fractional frequency stability that is independent of averaging time. Flicker PM noise in the amplifier at Fourier frequencies inside the loop bandwidth also leads to flicker FM in the oscillator. Flicker PM noise in the amplifier at Fourier frequencies larger than the loop bandwidth leads to flicker PM in the oscillator and a time domain stability that varies roughly as $1/\tau$, where τ is the measurement time. For flicker FM noise

$$\sigma_y^2 = \left(\frac{2 \ln 2}{\nu_0^2}\right) S_\phi^{osc}(1\text{Hz}). \qquad (9)$$

DEVELOPMENT OF BAW RESONATORS AND OSCILLATORS WITH LOW 1/f NOISE

The first modern resonators with low noise and repeatable temperature performance were the 2.5 and 5 MHz 5th overtone AT-cut resonators developed by Art Warner of AT&T Bell Labs in the early 1950's [9]. Figure 2 shows that the Q-factors of these resonators were virtually as good as present day results. The time-domain frequency stability achieved in 1960 at 2.5 MHz is shown in Figs. 3 and 4 [9-11]. Measurements of frequency stability before the work of David Allan and Jim Barnes in 1966 [12-13] probably were analyzed using the standard deviation; moreover, the bandwidth of the measurement equipment was not often stated.

Allan and Barnes' work was pivotal in providing a clear understanding of the statistics of flicker FM noise and providing clear guidelines for time-domain frequency stability measurements which ensured reproducibility of the measurements. The frequency stability of the oscillators reported in [9-11] was dominated in the short-term by flicker PM noise in the electronics. At very long times temperature fluctuations were probably the dominant drivers.

At the time of this work it probably was not realized that the dynamic temperature coefficient of the AT-cut resonator was very large, typically

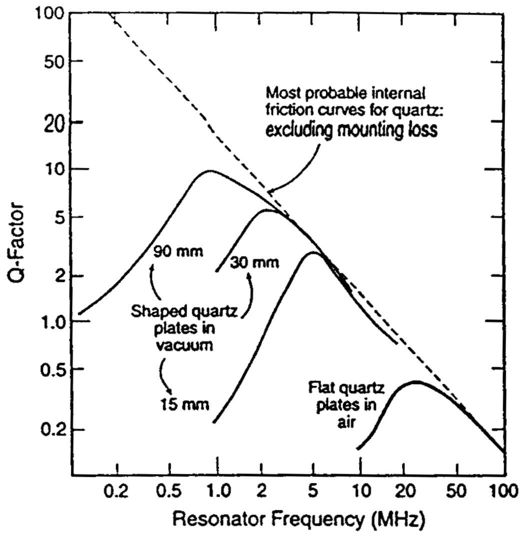


Fig. 2 *Resonator Q factor versus frequency for 15, 30, and 90 mm quartz plates in vacuum. The dashed line shows the product of Q V where V is the resonance frequency. Adapted from Warner, 1960 [9].*

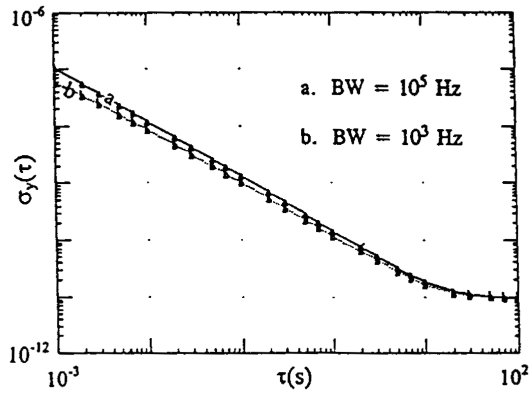


Fig. 3 *Fractional frequency stability versus measurement time for a 2.5 MHz 5th overtone AT-cut resonator for a measurement bandwidth of both 10^5 and 10^3 Hz. From Warner, 1960 [9].*

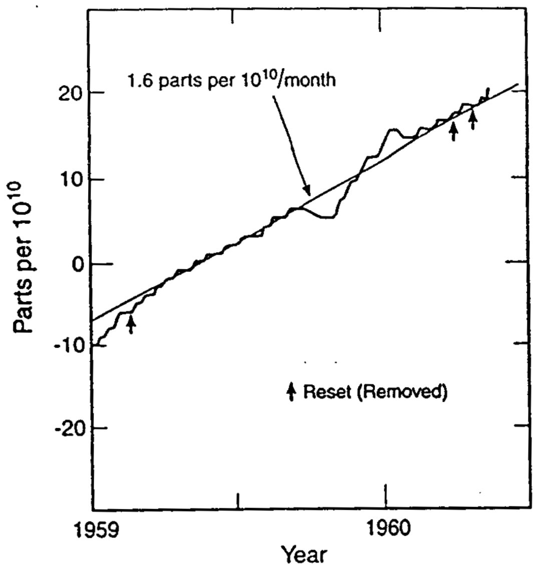


Fig. 4 *Fractional frequency stability of a 2.5 MHz oscillator vs. a cesium standard. Adapted from Anderson and Merrill, 1960 [11].*

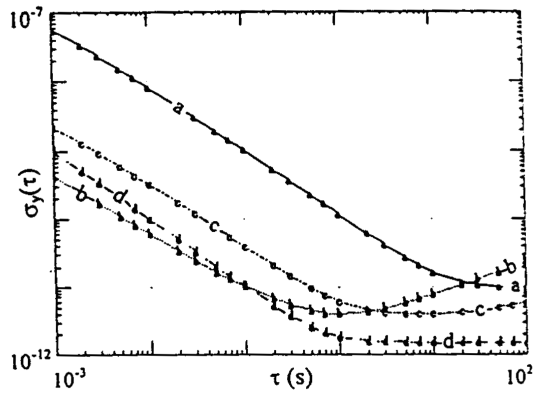


Fig. 5 *Fractional frequency stability of quartz oscillators in a 1 kHz bandwidth. Curve a from Warner, 1960 [9] is for a 2.5 MHz oscillator. Curve b from Stratemeyer, 1964 [15], curve c from Cutler and Searle, 1966 [16], and curve d from Pustarfi, 1966 [17] are 5 MHz oscillators.*

100 times larger than the static coefficients near turnover. Nevertheless, the long-term frequency stability was very good, as demonstrated by the results in Fig. 4 [11].

The next few years saw a steady improvement of about a factor of 20 dB in the phase noise and a factor of nearly 10 in the time-domain frequency stability as shown in Fig. 5 [14-17]. Virtually all of these oscillators used the new AT-cut resonators designed by Art Warner.

Another major milestone was the development of the Sulzer oscillators first using 2.5 and then 5 MHz 5th overtone resonators around 1968. It is now clear from the circuit diagrams that Peter Sulzer understood much more about the PM noise in the electronics and the role of the dynamic temperature coefficient than any of his predecessors. His ovens had very high thermal gains, good insulation, and high thermal mass around the resonator. The thermal stability achieved at the resonator was approximately 20 nK/s. The result was another factor of 10 improvement in the short-term frequency stability as illustrated in Fig. 6 [18]. The drift of these oscillators after running for many months was typically parts in 10^{12}/day. The primary differences between these extraordinary oscillators and many present day oscillators are their size, power, and higher PM flicker noise.

The pioneering work of Don Halford and Art Wainwright [19], who introduced an unbypassed resistor in the emitter leg of a transistor to reduce the phase modulation, was incorporated into an oscillator by Helmut Brandenberger [20]. The major advantage over the Sulzer oscillator was that the phase noise from approximately 0.5 Hz to 1 kHz was much lower as shown in Fig. 7. This yielded much better short-term frequency stability than the Sulzer oscillator for measurement times from about 0.001 to 3 s as shown in Fig. 8.

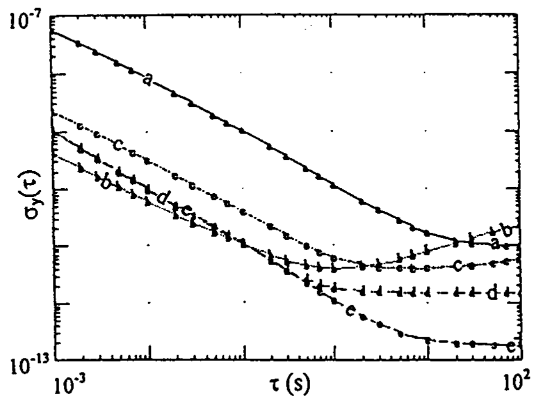


Fig. 6 *Fractional frequency stability of quartz oscillators in a 1 kHz bandwidth. Curve a from Warner, 1960 [9] is for a 2.5 MHz oscillator. Curve b from Stratemeyer, 1964 [15], curve c from Cutler and Searle, 1966 [16], curve d from Pustarfi, 1966 [17], and curve e from Sulzer, 1968 [18] are 5 MHz oscillators*

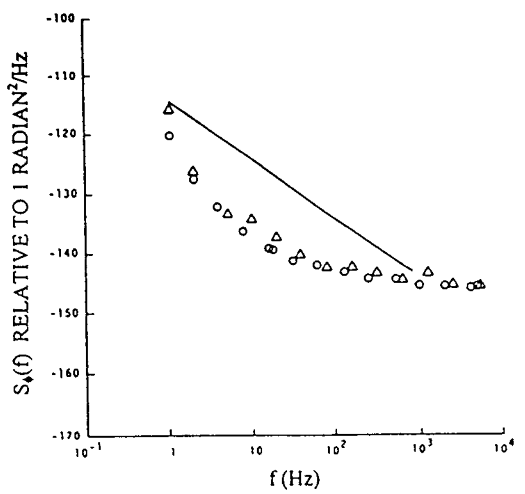


Fig. 7 *The data labeled Δ and O are the PM noise of the 5 MHz Brandenberger oscillator, 1971 [20]. The solid line is the estimated PM noise of a 5 MHz Sulzer oscillator [18].*

The wide band PM noise of approximately $S_\phi(f)$ = -142 dB rel to 1 rad^2/Hz and the long-term stability of 2-5 x 10^{-13} remained about the same.

I joined the Time and Frequency Division of the National Bureau of Standards in the fall of 1973 after the departure of Don Halford. Art Wainwright showed me many of the tricks of the trade. He and I collaborated on the development of the phase bridge method, schematically shown in Fig. 9, for measuring the noise of a resonator pair without the use of the oscillator electronics [21]. Our results showed that the 1/f PM noise of the resonators extended to much higher Fourier frequencies than was observed in quartz oscillators of the day. (See Figs. 10 and 11.) These results suggested that the wide band PM noise of oscillators could be reduced by at least 20 dB without increasing the power dissipated in the resonator.

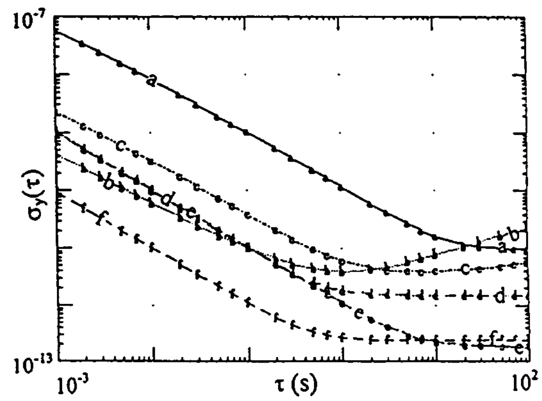


Fig. 8 *Fractional frequency stability of quartz oscillators in a 1 kHz bandwidth. Curve a from Warner, 1960 [9] is for a 2.5 MHz oscillator. Curve b from Stratemeyer, 1964 [15], curve c from Cutler and Searle, 1966 [16], curve d from Pustarfi, 1966 [17], curve e from Sulzer, 1968 [18], and curve f from Brandenberger, 1971 [20] are 5 MHz oscillators.*

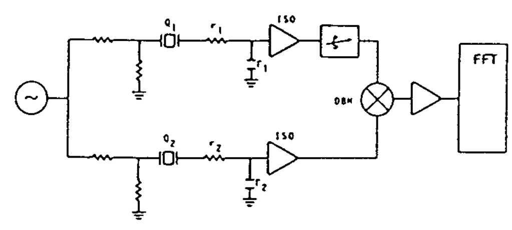


Fig. 9 *Block diagram of phase bridge for measuring the PM noise in resonator pairs. By equalizing the resonator frequencies and Q-factors, the contribution of PM noise in the oscillator to the measurement can be reduced by ~20 dB. From Walls and Wainwright, 1975 [21].*

This challenge of producing oscillators with reduced wide band PM noise was first taken up by Charles Stone around 1975. Over the course of the next several years he produced a series of oscillators from 5 MHz to 100 MHz using AT-cut resonators in which the flicker FM noise of the resonator dominated to much higher frequencies. Typically these oscillators had PM noise floors of less than -175 dBc/Hz. This was about 30 dB lower than previous oscillators and actually much better than the projections as shown in Fig. 12. The time-domain performance of a modern version of this design using SC-cut resonators is shown in Fig. 13 [22]. Oscillators with PM noise floors below -163 dBc/Hz are now commonly available.

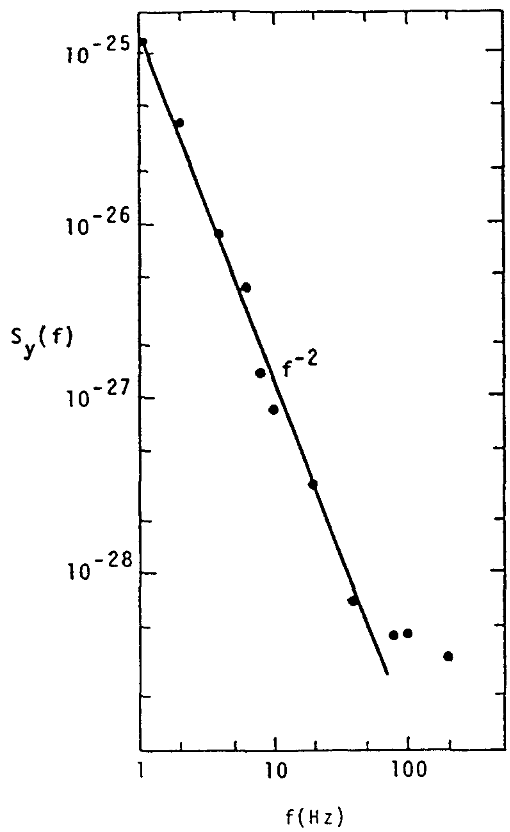


Fig. 10 *Direct phase bridge measurements of fractional frequency noise $S_y(f)$ in a pair of 5 MHz resonators with a Q-factor of 2.5 million. No correction for resonator bandwidth has been applied.*

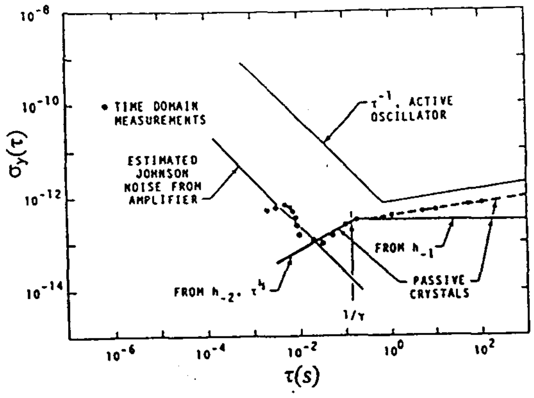


Fig. 11 *Comparison of measured $\sigma_y(\tau)$ vs. the value calculated from the measurement of $S_y(f)$ of the resonators from Fig. 10. From Walls and Wainwright, 1975 [21].*

A major innovation in the design of quartz resonators was the development of the BVA resonator by Raymond Besson [23]. In this new resonator, the electrodes were separated from the quartz by a few micrometers, and the oscillating portion was isolated from the mounting structure by very thin quartz bridges. The goal was to reduce the perturbations from the electrodes and the mounting structure on the frequency stability of the resonator. The idea of separating the electrodes from the resonator had been pursued earlier by Don Hammond but was not followed through to commercial production [24].

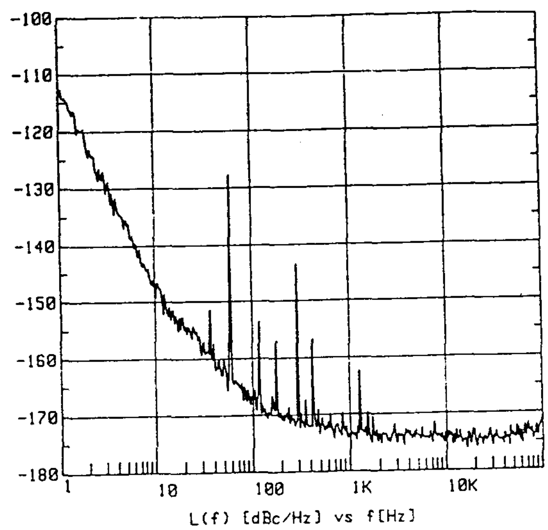


Fig. 12 *Phase noise of a high drive level 5 MHz oscillator. From Stone, ~1977.*

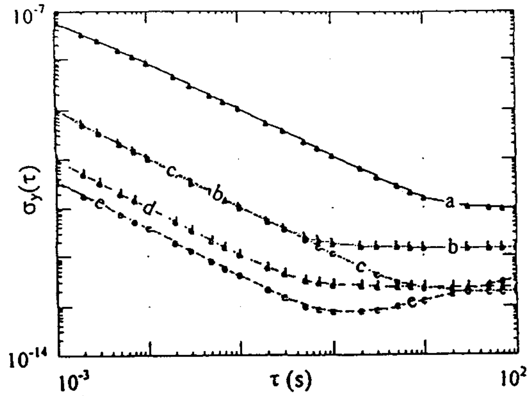


Fig. 13 *Fractional frequency stability of quartz oscillators in a 1 kHz bandwidth. Curve a from Warner, 1960 [9] is for a 2.5 MHz oscillator. Curve b from Pustarfi, 1966 [17], curve c from Sulzer, 1968 [18], curve d from Brandenberger, 1971 [20], and curve e from Stone, 1989 [22] are 5 MHz oscillators.*

In 1978 we were successful at NBS in demonstrating a frequency stability of approximately 8×10^{-14} using a first generation BVA resonator [25]. The approach, shown in Fig. 14, used the resonator in a reflectometer mode with the short-term stability provided by a Charles Stone oscillator. The BVA resonators of this vintage were troubled by mechanical hysteresis due to thermally induced stress between the three plates comprising the resonator. These results hinted at the potential for significant advances but were not practical due to resonator hysteresis and system complexity. Figure 15 shows a modern 10 MHz high-performance BVA resonator from Besson where the points of contact of the three plates are polished so flat that they become bonded by molecular forces [26]. This design eliminated the mechanical problems of the previous models. Unfortunately this resonator was not available at the time the reflectometer experiments were done.

The first oscillators to routinely achieve a fractional frequency stability below 1×10^{-13} were

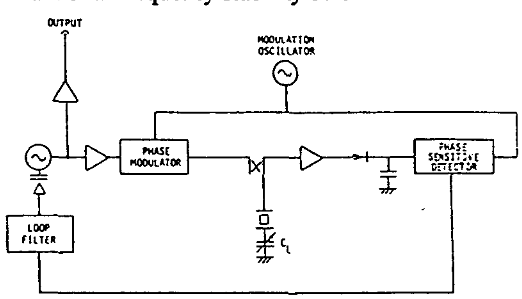


Fig. 14 *Phase modulation system for locking an oscillator to a passive quartz resonator. Adapted from Stein et. al., 1978 [25].*

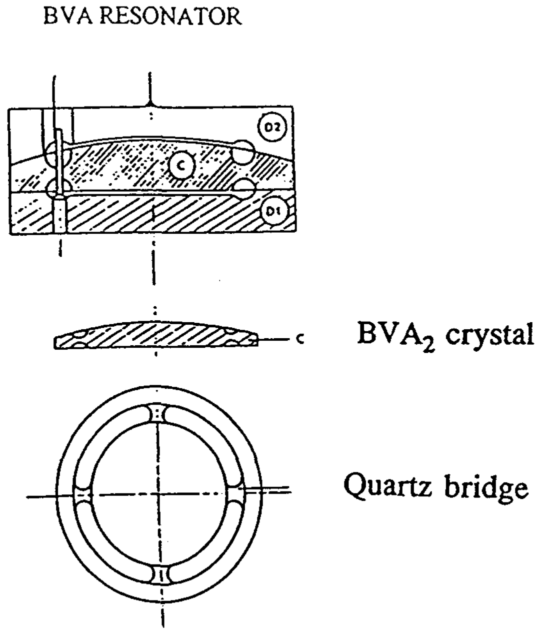


Fig. 15 *Schematic diagram of BVA2 quartz resonator. From Besson 1990.*

those designed by Jerry Norton for various space flights [27-29]. His designs were in many ways similar to those of Peter Sulzer. The oscillator circuits used the parallel mode resonance and very special attention was paid to the thermal design. The oven design of the Norton oscillators and their vacuum environment lead to a thermal performance that significantly outperformed previous oscillators. Table I shows the flicker FM performance of a number of 5 MHz AT-cut and BVA resonators in Norton oscillators. Note that oscillators using resonators with the same Q-factor differ significantly in their flicker FM noise. Still, quite a few of the resonators achieve a fractional frequency stability below 1×10^{-13}. In my opinion, these unique results are the direct result of a better temperature stability than that which was available in other oscillator designs. Obviously, the electronic design was also carefully done. Figure 16 shows the performance of 3 oscillators using BVA resonators [28,29].

The approximate thermal behavior of a traditional 5th overtone AT-cut resonator is given by

$$\frac{d\nu}{\nu_0} \cong 10^{-9}\ \Delta T^2\ 10^{-5}\ dT/dt, \qquad (10)$$

where ΔT is the temperature difference in K from turnover and dT/dt is the rate of change of temperature in K/s. To achieve a frequency stability of 3×10^{-14} requires that the temperature of the resonator change no faster than 3 nK/s.

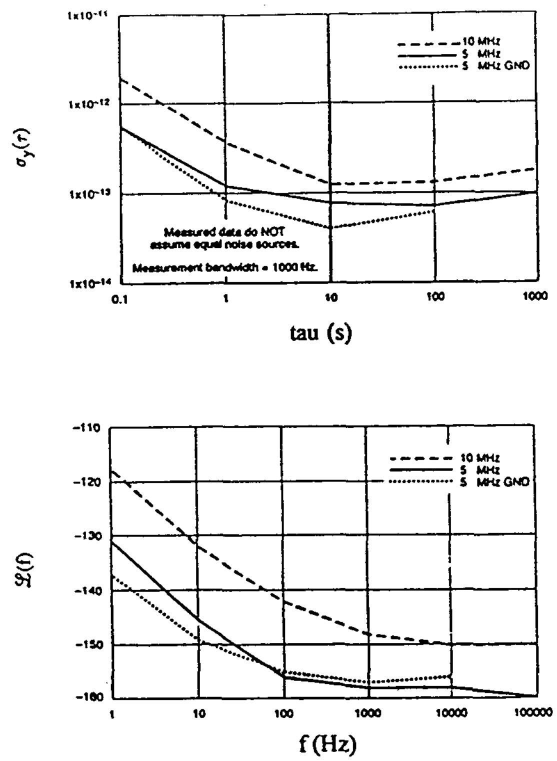


Fig. 16 *Frequency stability of BVA quartz oscillators. From Norton, 1994 [28].*

Table II shows the approximate frequency offsets because of temperature cycling of the oven due to the static thermal sensitivity of AT-cut resonators versus temperature offset of the oven from turnover. Unfortunately the dynamic temperature coefficient dominates these static temperature effects in most ovens, and in my opinion, explains the inability of many oscillator designs to achieve sub 1 x 10^{-13} frequency stability. It is not an accident that the advances of Sulzer and Norton both incorporated significant improvements in thermal design. Analysis and experiments show that the thermal stability of crystal ovens can be greatly improved [30-32]. The fundamental limit to thermal stability with present technology is of the order of a few nK/s and mK/year [33]. In benign thermal environments thermal fluctuations can be eliminated as a driver of instability in quartz oscillators.

One of the most important advances in resonator design was the prediction by Errol EerNisse in 1975 of the existence of a resonator cut that was much less sensitive to in-plane stresses and yet still maintained good temperature turnover characteristics [34]. The new stress compensated (SC)-cut resonator turned out to be the same as the thermal transient compensated (TTC)-cut of Jack Kusters. It was much less sensitive to stress and had a dynamic temperature sensitivity that was about a factor of 100 times smaller than the traditional AT-cut, when the angles were properly chosen [35,36]. Table III shows the approximate thermal performance of a 5 MHz SC-cut resonator. There was another very important advantage of this new design: it was much less sensitive to "activity dips." An activity dip occurs when harmonics of the primary mode couple with higher order modes. These interactions reduce the Q and disturb the phase of the primary mode [37]. The development of the SC-cut meant that the temperature fluctuations of traditional ovens had a much smaller effect on the frequency stability of the oscillator. Suddenly, many manufacturers could achieve a frequency stability of the order of parts in 10^{-13}.

Table IV shows the flicker FM noise achieved with SC-cut resonators by Stone [22], Norton [28], Besson [33], and Wenzel [38]. These results are, to my knowledge, the state-of-the-art for quartz crystal oscillators between 5 and 10 MHz.

How much further can the flicker FM noise of oscillators be reduced? To attempt an answer it is necessary to formulate a model for 1/f noise in resonators and the amplifiers used in the sustaining stage.

ANALYSIS OF 1/f NOISE IN RESONATORS

Jean-Jacques Gagnepain et al., [39] used the resonator phase bridge to study the intrinsic 1/f noise in resonators [21]. Their 1981 paper was a major milestone in attempting to develop theoretical models of 1/f noise in resonators. By varying temperature to change the Q-factor, they were able to eliminate the effects of many other parameters and show that the 1/f noise was somehow associated with the unloaded Q-factor of the resonator. (See Figs. 17-18.) They proposed that

$$S_y(f) = K_v/Q^4, \qquad (11)$$

where K_v is a phenomenological constant that would need to be explained by a theory of 1/f noise. Since the product of intrinsic Q-factor and carrier frequency is a constant (see Fig. 2) [8], this would predict that $S_y(f)$ should scale as v^4, where v is the oscillator frequency.

Tom Parker compared the 1/f noise in BAW and surface acoustic wave (SAW) resonators and suggested that the 1/f noise in all quartz resonators could be explained by

$$S_\phi(f) = K_\phi/Q^4, \tag{12}$$

where K_ϕ is a phenomenological constant that would need to be explained by a theory of 1/f noise. See Fig. 19 [40]. In this case, $S_y(f)$ should scale as ν^2.

Our work with Peter Handel in 1992 suggested that the intrinsic noise in the resonator might be due to the loss of phonons out of the main mode due to scattering [41]. This quantum process leads naturally to a dependence on the volume of the resonating mode

$$S_y(f) = \beta \mathrm{Vol}/Q^4, \tag{13}$$

where β is of order 1 if the volume is expressed in cm^3. This would imply $S_y(f)$ scaling as υ^3 for constant diameter electrodes, and as υ for the diameter of the electrodes scaling as $1/\upsilon$.

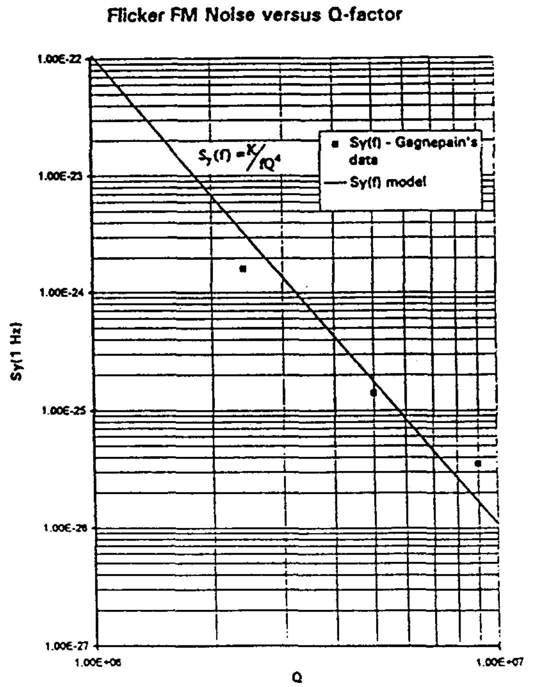


Fig. 17 *$S_y(f)$ versus Q-factor for a particular resonator. The Q was varied by changing temperature. From Gagnepain, et al., 1981 [39].*

Figure 20 shows the attempt to fit the data for a number of AT- and SC-cut resonators to the three models expressed by Eqs. (11-13). The data for each resonator are the best that have been reported for the particular frequency, manufacturer, and

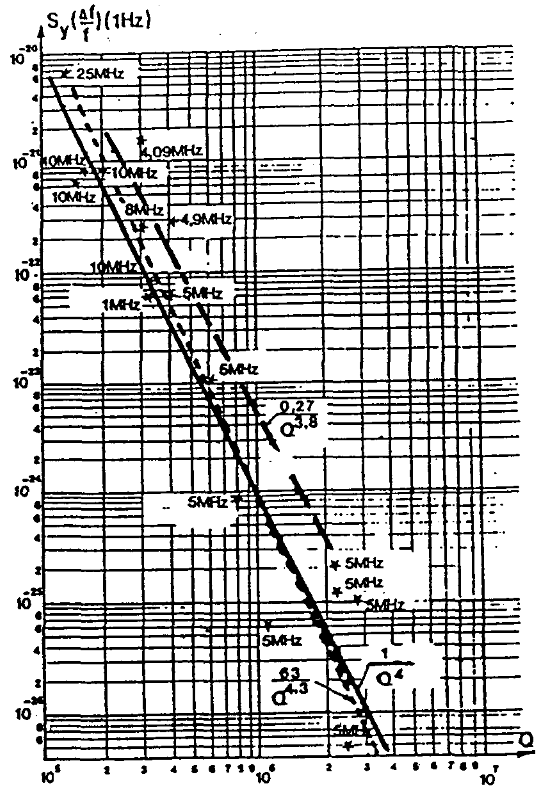


Fig. 18 *1/f frequency noise, 1 Hz from the carrier vs Q-factor. From Gagnepain, et al., 1981 [39].*

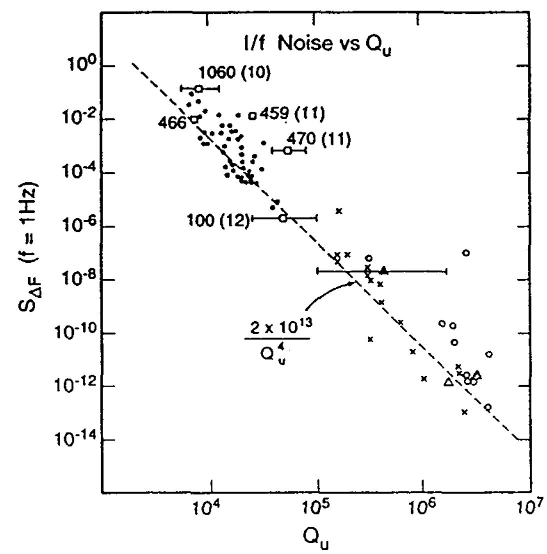


Fig. 19 *1/f noise levels of quartz Rayleigh wave and bulk acoustic wave resonators as a function of unloaded Q. Adapted from Parker, 1985 [40].*

resonator design. Only resonators where the unloaded Q-factor was within 20% of the material limit were considered. The data of Fig. 20 cover the frequency range from 2.5 MHz to 100 MHz for

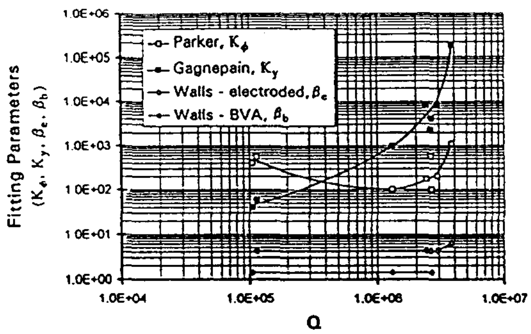


Fig. 20 *Fitting parameters for three proposed models of BAW quartz resonator noise versus unloaded Q-factor. From Walls, et. al [41].*

AT- and SC-cuts. It is very interesting that the data seems to fit the volume dependence model much better than any of the other models. This is in direct contrast to the results of Parker who found that the 1/f noise in SAW devices scaled as 1/vol [42]. The noise in the two types of resonators appears to originate from different mechanisms.

An interesting question is exactly how to calculate the volume. Is it the volume between the diameter of the electrodes or is it the weighted volume of the oscillating mode? Mike Driscoll and William Hansen attempted to address this problem in their 1993 paper by looking at the 1/f noise in several AT-cut, BT-cut, and SC-cut resonators in the frequency range from 40 to 160 MHz [43]. Their results on AT and SC-cut resonators seem to indicate that the acoustic volume is a better choice for use with Eq. (12), although even then the agreement was not very satisfactory. (See Fig. 21 and Tables V and VI.) Driscoll and Hansen's data fits Eq. (12) better than Eq. (13). The 1/f noise they measured on a BT-cut resonator was much higher than predicted by using either the electrode or the acoustic volume.

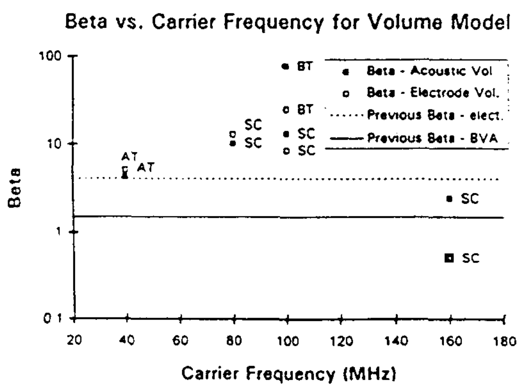


Fig. 21 *Fitting parameter for AT, SC, and BT-cut resonators. Adapted from Driscoll and Hansen, 1993 [43].*

Eva Ferre-Pikal measured the 1/f noise of 5th overtone 100 MHz SC-cut resonators. These resonators were made from the same quartz material by the same manufacturer for electrode areas differing by a factor of 2 above and below the standard size. Some of her unpublished 1993 results are shown in Fig. 22. The two resonators with the lowest 1/f noise have the smallest size electrodes, which is consistent with the volume dependence, or at least a dependence on electrode diameter. The wide dispersion between resonators with the same value of Q, however, clearly indicates that the 1/f noise must also depend on at least one other parameter besides Q-factor. The data of Table I lead to similar conclusions. This other factor must have a very large contribution to the 1/f noise and little effect on the Q-factor. Perhaps this other factor is non-linear coupling to the multitude of higher order modes mediated by stress due to crystal imperfections, the electrodes, and the mounting structure. In the resonators included in Fig. 20, there is a

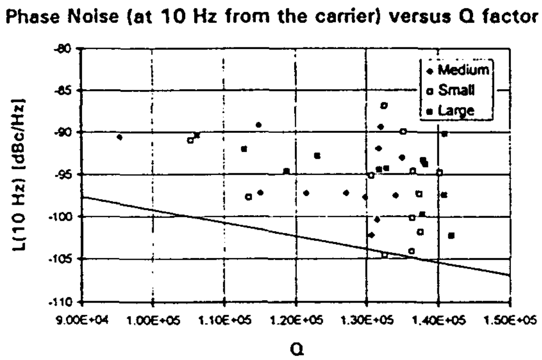


Fig. 22 *L(10 Hz) for 5th overtone 100 MHz resonators with small, medium, or large electrodes as a function of Q-factor. Unpublished data from Ferre-Pikal.*

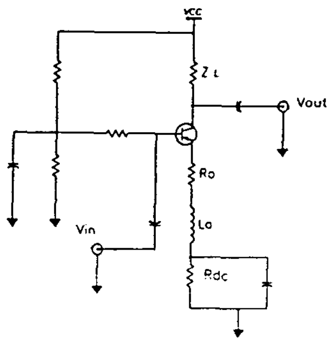
substantial difference between the 1/f noise in the electroded resonators and the BVA designs, which have no electrode on the resonator and greatly reduced mounting stress. Is this just an accident due to the small number of resonators studied and our inability to know the details of manufacturing, or is it an indication that the surface interface and/or the mounting stress contribute to the 1/f noise? We have additional work in progress to compare the 1/f noise in electroded resonators with the 1/f noise in BVA resonators with similar electrode areas and with resonators made with different manufacturing techniques to attempt to explore these additional noise effects further.

The bottom line at this point is that although there is some uncertainty, we can still assert that in general the 1/f noise improves as Q^4 and that the 1/f noise is better at lower oscillation frequencies than at higher oscillation frequencies. We do not understand the origin of 1/f noise in quartz crystal resonators, nor do we even understand how the 1/f noise should scale with frequency. We do, however, have a number of new questions, and we expect that the pursuit of answers to these questions will shed additional light on the problem.

ANALYSIS OF 1/f NOISE IN AMPLIFIERS

The first work that I am aware of, describing the 1/f PM noise in amplifiers was that of Don Halford and colleagues in 1968 [19]. They showed that the 1/f PM noise in amplifiers was due to an intrinsic modulation of the signal that could be reduced by up to 40 dB when negative feedback was used to reduce the rf gain. The most common method was to use an unbypassed resistor in the emitter of a BJT or source lead of a FET. (See Fig. 23.) The work of Svein Andresen showed the dependence of PM noise in doublers on emitter resister [44]. This work was, and still is, the basis for virtually all of the low PM noise designs in use today.

Common Emitter Amplifier



$$\text{Gain} = \frac{Z_L}{r_e' + R_o + \omega L_o}\ ; \qquad r_e' = \frac{1}{g_m}$$

Fig. 23 *Schematic diagram of a common emitter BJT amplifier. Both PM and AM noise depend on the base current noise, base to collector voltage noise, bias, DC gain, and RF gain. In general, changes in gain lead to AM noise while impedance changes lead to PM noise.*

In 1992 we started a program at NIST to understand the relationship, if any, between AM and PM noise in amplifiers and in oscillators. Figures 24-26 show the results of some of our measurements [45,46]. These data show that the AM noise in amplifiers is not just the thermal noise that was previously assumed, but it also had a 1/f character that was remarkably similar to the 1/f PM noise. The AM and PM noise in many oscillators is similar outside the bandwidth of the resonator. Could both the AM and PM noise be due to the same process? (See Figs. 25-26.) We then embarked on a systematic study of the up conversion of the baseband noise of bipolar junction transistors (BJTs) to produce PM and AM noise in linear amplifiers. We have found that the AM noise can be higher than, equal to, or lower than the PM noise, depending on the circuit configuration. Our studies, undertaken primarily by Eva Ferre-Pikal, measured the 1/f components of current and voltage in the transistors and the resulting 1/f PM

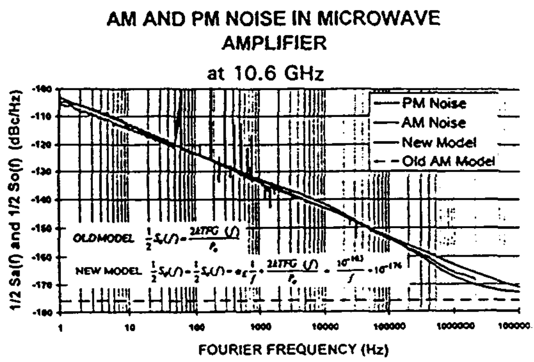


Fig. 24 *AM and PM noise of an x-band amplifier. From Ascarrunz, et. al., 1993 [45].*

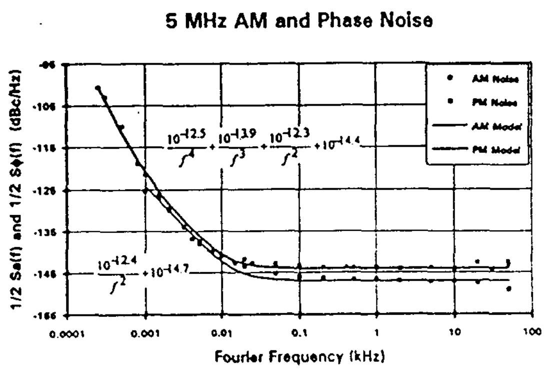


Fig. 25 *AM and PM noise of 5 MHz quartz oscillator. From Nelson, et al., 1994 [46].*

and AM noise for different circuit configurations [47,48]. From these data we have constructed a model for the dependence of both PM and AM noise in linear BJT amplifiers on baseband current and voltage noise and circuit configuration. Our results show that baseband noise modulation of the gain generates the AM noise, while baseband noise modulation of the impedance at various terminals generates the PM noise. We find that the PM noise has a dependence on carrier frequency that is often higher than that of the AM noise.

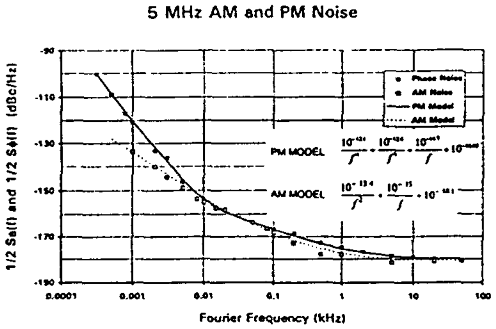


Fig. 26 *AM and PM noise of a 5 MHz oscillator. From Nelson, et. al., 1994 [46].*

Using this new model for PM and AM noise, we have modified a 5 channel distribution amplifier and achieved PM noise of L(10 Hz) = - 169 dBc/Hz ,which is within 2 dB of thermal noise [48,49]. (See Fig. 27.) The AM noise in the distribution amplifier is below our measurement noise. Ferre-Pikal has also made 5 MHz amplifiers with a gain of about 24 dB and PM noise of L(10 Hz) = -158 dBc/Hz. As in the previous example, the PM noise is within 1-2 dB of thermal noise, and the AM noise is below our measurement noise. Additional details are found in [47,48]. A low noise 100 MHz is discussed in [50].

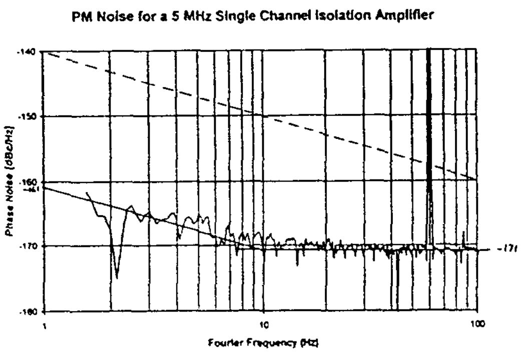


Fig. 27 *L(f) of a 5 MHz distribution amplifier [46,47]. The basic design of Nelson, et. al., 1994 [49] has been modified slightly to reduce the 1/f portion of the PM noise.*

What impact, if any, will amplifiers with lower 1/f noise have on quartz oscillators? Going back to Eq. (7), we see that as the 1/f noise in the resonator improves, the 1/f noise in the amplifier becomes more important.

$$\sigma_y^{osc2} = \frac{2\ln 2}{4Q_L^2} S_\phi^{amp}(1\text{Hz}), \qquad (14)$$

when flicker PM noise in the amplifier is the dominant noise process.

For example consider the phase noise data for the 5 MHz ground based Norton oscillator in Fig. 16. The bandwidth of this oscillator can be estimated to be approximately ± 0.75 to 1 Hz from the data on the resonator. The PM noise outside this range appears to be flicker PM. Using a value of L(10 Hz) = -149 dBc/Hz as a measure of the flicker PM noise in the amplifier, Eq. (14) indicates an oscillator flicker FM of 4.6×10^{-14}, assuming all of the 1/f PM noise originates in the sustaining stage. This result is within 1-2 dB of the observed value and indicates that flicker PM in the sustaining amplifier probably dominates the flicker FM of the oscillator. To achieve a 1/f FM noise floor in the oscillator of 1×10^{-14} requires that the 1/f PM noise in the sustaining amplifier be less than L(1 Hz) = -153 dBc/Hz for a 5 MHz carrier. Using our new model of AM and PM noise in BJT amplifiers it should be possible to reduce the contribution of the electronics well below this value, perhaps as low as L(1) = - 159 dBc/Hz which would be sufficient to reduce the amplifier contribution to approximately 5×10^{-15} at 5 MHz.

It is likely that the performance observed for the 10 MHz SC-cut tactical BVA based oscillators is also limited by the PM noise of the sustaining stage. If this is correct, reducing the PM noise in the sustaining stage should immediately make possible 10 MHz oscillators with a frequency stability floor of less than 1×10^{-13}.

DISCUSSION

Figure 28 shows that the short-term frequency stability of BAW crystal oscillators has improved dramatically over the past 35 years. These improvements have been due to significant reductions in the 1/f PM noise in the sustaining amplifiers, the introduction of the SC-cut resonator and the BVA resonators, and the development of better ovens. These improvements now make possible BAW crystal oscillators with a flicker FM noise of at least 4×10^{-14}. Further improvements in frequency stability should be possible using sustaining amplifiers with still lower 1/f noise. Amplifiers with much lower 1/f PM noise than previously available have been demonstrated within just the past few months [47,51].

A breakthrough in understanding 1/f noise in resonators will, in my opinion, require a reexamination of the effects of stress and nonlinear coupling to the various modes of the resonator. Thermal effects limit the frequency stability of many oscillators; however, the technology to remove temperature effects on quartz oscillators in benign environments is presently available [6, 30-32].

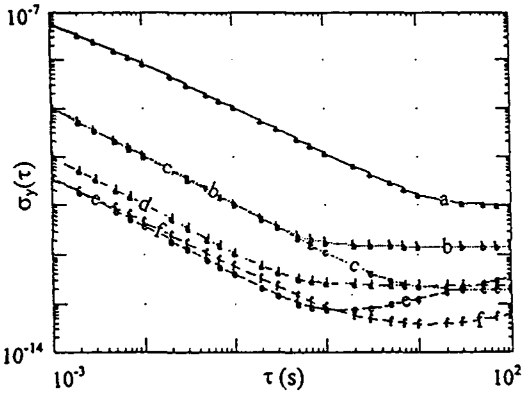


Fig. 28 *Summary of advances in 1/f frequency stability of BAW quartz oscillators from 1960 to 1995. Fractional frequency stabilities shown are for a bandwidth of 1 kHz bandwidth. Curve a from Warner, 1960 [9] is for a 2.5 MHz oscillator. Curve b from Pustarfi, 1966 [17], curve c from Sulzer, 1968 [18], curve d from Brandenberger, 1971 [20], curve e from Stone, 1989 [22], and curve f from and Norton, 1994, [28] are 5 MHz oscillators.*

ACKNOWLEDGMENTS

Over the past 22 years I have benefited greatly from interactions with a large number of colleagues. Discussions with Jean-Jacques Gagnepain, Raymond Besson, Jean Ubersfeld, Charles Stone, Art Wainwright, Andrea DeMarchi, Tom Parker, John Vig, David Allan, Jerry Norton, Eva Ferre-Pikal, Steve Jefferts, and Peter Handel have been especially helpful in my personal quest to understand 1/f noise. I am also grateful to Eva Ferre-Pikal for the use of her unpublished work and Gwen Bennett who helped prepare the manuscript.

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Table. I. 1/f frequency stability (column 10) along with frequency, Q-factors, and other parameters of resonators tested in oscillators, from Norton [26-29]. Beta (column 11) is from the model of Walls, et. al., Eq 13.

Nominal Freq	Q	Type	Overtone	Diam	Elec Diam	Thick	Volume	SQR(V/Q^4)(2ln2)	stab	Beta
(MHz)	(x10^6)			(cm)	(cm)	(cm)	(cm3)		(1/f)	
5	2.0	elec	3rd	1.5	0.79	0.114	0.055	6.90E-14	7.00E-14	1.03
5	1.9	elec	3rd	1.5	0.79	0.114	0.055	7.65E-14	2.80E-13	13.40
5	2.35	elec	3rd	1.5	0.79	0.114	0.055	5.00E-14	1.70E-13	11.56
5	2.22	elec	3rd	1.5	0.79	0.114	0.055	5.60E-14	1.40E-13	6.24
5	2.92	elec	3rd	1.5	0.79	0.114	0.055	3.24E-14	6.60E-14	4.15
5	2.89	elec	3rd	1.5	0.79	0.114	0.055	3.31E-14	2.20E-13	44.28
5	3.09	elec	3rd	1.5	0.79	0.114	0.055	2.89E-14	1.00E-13	11.96
5	2.93	elec	3rd	1.5	0.79	0.114	0.055	3.22E-14	1.70E-13	27.94
5	2.86	elec	3rd	1.5	0.79	0.114	0.055	3.38E-14	1.20E-13	12.64
5	2.78	elec	3rd	1.5	0.79	0.114	0.055	3.57E-14	1.50E-13	17.63
5	2.75	elec	3rd	1.5	0.79	0.114	0.055	3.65E-14	1.20E-13	10.80
5	2.88	elec	3rd	1.5	0.79	0.114	0.055	3.33E-14	8.90E-14	7.15
5	2.7	elec	3rd	1.5	0.79	0.114	0.055	3.79E-14	2.00E-13	27.88
5	2.7	elec	3rd	1.5	0.79	0.114	0.055	3.79E-14	1.10E-13	8.43
5	2.7	elec	3rd	1.5	0.79	0.114	0.055	3.79E-14	1.40E-13	13.66
5	2.7	elec	3rd	1.5	0.79	0.114	0.055	3.79E-14	8.40E-14	4.92
5	2.81	elec	3rd	1.5	0.79	0.114	0.055	3.50E-14	2.20E-13	39.58
5	2.81	elec	3rd	1.5	0.79	0.114	0.055	3.50E-14	1.40E-13	16.03
5	2.81	elec	3rd	1.5	0.79	0.114	0.055	3.50E-14	1.20E-13	11.78
5	2.6	BVA	3rd	1.5	1.00	0.1089	0.055	5.08E-14	1.20E-13	5.58
5	2.58	BVA	3rd	1.5	1.00	0.1089	0.085	5.16E-14	7.10E-14	1.90
5	2.63	BVA	3rd	1.5	1.00	0.1089	0.085	4.96E-14	5.90E-14	1.41
5	2.6	BVA	3rd	1.5	1.00	0.1089	0.085	5.08E-14	6.90E-14	1.85
5	2.47	BVA	3rd	1.5	1.00	0.1089	0.085	5.63E-14	1.90E-13	11.40
5	2.61	BVA	3rd	1.5	1.00	0.1089	0.085	5.04E-14	1.50E-13	8.86
10	1.31	BVA	3rd				0.021	9.94E-14	1.20E-13	1.46
10	1.37	BVA	3rd				0.021	9.09E-14	1.40E-13	2.37

Table II. Typical frequency versus temperature coefficients for an AT-cut resonator with a turnover temperature of 85 °C as a function of oven parameters. Frequency stability of 1 x 10^{-13} requires $\Delta T/dt < 10$ nK/s.

$$\Delta v/v \cong 3 \times 10^{-8}\ \Delta T^2 + 10^{-5}\ dT/dtx$$

Oven Offset mK	±100 +mK	Oven Change ± 10 mK	± 1 mK	±0.1 mK
0	3×10^{-10}	3×10^{-12}	3×10^{-14}	3×10^{-15}
1	3×10^{-10}	5×10^{-12}	2×10^{-13}	6×10^{-15}
10	5×10^{-10}	2×10^{-11}	6×10^{-13}	6×10^{-14}
100	2×10^{-9}	6×10^{-11}	6×10^{-12}	6×10^{-13}

Table III. Typical frequency versus temperature coefficients for a SC-cut resonator with a turnover temperature of 85°C as a function of oven parameters. Frequency stability of 1 x 10^{-13} requires $\Delta T/dt$ < 500 nK/s.

$$\Delta v/V \cong \times 10^{-9} + 2 \times 10^{-7}\ dT/dt$$

Oven Offset mK	±100 mK	Oven Change ± 10 mK	± 1 mK	± 0.1 mK
0	4×10^{-11}	4×10^{-13}	2×10^{-15}	4×10^{-17}
1	4×10^{-11}	6×10^{-13}	2×10^{-14}	8×10^{-16}
10	6×10^{-11}	2×10^{-12}	8×10^{-14}	8×10^{-15}
100	2×10^{-10}	8×10^{-12}	8×10^{-13}	8×10^{-14}

Table IV. Frequency stability of selected quartz oscillators. The oscillators reported in Besson [33] and Norton [28] use BVA resonators while those reported in Stone [22] and Wenzel [30] use electroded resonators.

	Resonator Type	Frequency	$\sigma_y(\tau)$ τ 0.1 s	$\sigma_y(\tau)$ 0.1s	$\sigma_y(\tau)$ 10 s	$\sigma_y(\tau)$ 100 s
Besson [33]	BVA 3rd SC	10 MHz			9×10^{-13}	3×10^{-13}
Norton [28]	BVA 3rd SC	10 MHz	2×10^{-12}	3.8×10^{-13}	1.3×10^{-13}	1.8×10^{-13}
Norton [28]	BVA 3rd SC	5 MHz	5.7×10^{-13}	8.4×10^{-14}	3.7×10^{-14}	6×10^{-14}
Stone [22]	Electroded 5th SC	5 MHz	4×10^{-13}	7×10^{-14}	1.2×10^{-13}	3×10^{-13}
Wenzel [38]	Electroded 3rd SC	5 MHz	2×10^{-13}	3×10^{-13}	6×10^{-13}	

Table V. VHF Crystal Flicker-of-Frequency Noise: Best Measured Results in Comparison to Walls' Volume Model Predictions Using Electroded Volume

CRYSTAL CUT	SC	SC	SC	BT	AT
FREQUENCY	80 MHz	100 MHz	160 MHz	100 MHz	40 MHz
OVERTONE	3rd	3rd	5th	5th	5th
UNLOADED Q	125K	119K	75K	300K	250K
BLANK THICKNESS (cm)	0.00678	0.00543	0.00566	0.013	0.021
ELECTRODE DIAMETER (cm)	0.162	0.140	0.254	0.317	0.366
ELECTRODED VOLUME (cm^3)	.00014	.000083	.00028	.001	.0022
BEST MEASURED Sy(f=100 Hz)*	$7.4x10^{-26}$	$3.6x10^{-26}$	$4.1x10^{-26}$	$3.0x10^{-26}$	$1.25x10^{-26}$
RESONATOR Sy(f=100 Hz)**	$6.4x10^{-26}$	$2.6x10^{-26}$	$2.5x10^{-26}$	$2.0x10^{-26}$	$1.0x10^{-26}$
MODEL PREDICTED Sy(f= 100 Hz) (using β_e=4 om eq. 1)	$2.3x10^{-26}$	$1.7x10^{-26}$	$3.3x10^{-26}$	$5.0x10^{-27}$	$2.2x10^{-26}$

* Including contribution of 4-crystal reference oscillator

** Removing contribution of 4-crystal reference oscillator

Table VI. VHF Crystal Flicker-of-Frequency Noise: Best Measured Results in Comparison to Walls' Volume Model Predictions Using Acoustic Energy Volume

CRYSTAL CUT	SC	SC	SC	BT	AT
FREQUENCY	80 MHz	100 MHz	160 MHz	100 MHz	40 MHz
OVERTONE	3rd	3rd	5TH	5TH	5TH
UNLOADED Q	125K	119K	75K	300K	250K
BLANK THICKNESS (cm)	0.00678	0.00543	0.00566	0.013	0.021
ELECTRODE DIAMETER (cm)	0.162	0.140	0.254	0.317	0.366
ELECTRODED VOLUME (cm^3)	.00018	.000056	.00054	.00034	.0012
BEST MEASURED Sy(f=100 Hz)*	$7.4x10^{-26}$	$3.6x10^{-26}$	$4.1x10^{-26}$	$3.0x10^{-26}$	$1.25x10^{-26}$
RESONATOR Sy(f=100 Hz)**	$6.4x10^{-26}$	$2.6x10^{-26}$	$2.5x10^{-26}$	$2.0x10^{-26}$	$1.0x10^{-26}$
MODEL PREDICTED Sy(f= 100 Hz) (using β_e=4 om eq. 1)	$2.9x10^{-26}$	$1.1x10^{-26}$	$6.8x10^{-26}$	$1.6x10^{-27}$	$1.2x10^{-26}$

Table. I. 1/f frequency stability (column 10) along with frequency, Q-factors, and other parameters of resonators tested in oscillators, from Norton [26-29]. Beta (column 11) is from the model of Walls, et. al., Eq 13.

Nominal Freq (MHz)	Q (x10^6)	Type	Overtone	Diam (cm)	Elec Diam (cm)	Thick (cm)	Volume (cm3)	SQR(V/Q^4)(2ln2)	stab (1/f)	Beta
5	2.0	elec	3rd	1.5	0.79	0.114	0.055	6.90E-14	7.00E-14	1.03
5	1.9	elec	3rd	1.5	0.79	0.114	0.055	7.65E-14	2.80E-13	13.40
5	2.35	elec	3rd	1.5	0.79	0.114	0.055	5.00E-14	1.70E-13	11.56
5	2.22	elec	3rd	1.5	0.79	0.114	0.055	5.60E-14	1.40E-13	6.24
5	2.92	elec	3rd	1.5	0.79	0.114	0.055	3.24E-14	6.60E-14	4.15
5	2.89	elec	3rd	1.5	0.79	0.114	0.055	3.31E-14	2.20E-13	44.28
5	3.09	elec	3rd	1.5	0.79	0.114	0.055	2.89E-14	1.00E-13	11.96
5	2.93	elec	3rd	1.5	0.79	0.114	0.055	3.22E-14	1.70E-13	27.94
5	2.86	elec	3rd	1.5	0.79	0.114	0.055	3.38E-14	1.20E-13	12.64
5	2.78	elec	3rd	1.5	0.79	0.114	0.055	3.57E-14	1.50E-13	17.63
5	2.75	elec	3rd	1.5	0.79	0.114	0.055	3.65E-14	1.20E-13	10.80
5	2.88	elec	3rd	1.5	0.79	0.114	0.055	3.33E-14	8.90E-14	7.15
5	2.7	elec	3rd	1.5	0.79	0.114	0.055	3.79E-14	2.00E-13	27.88
5	2.7	elec	3rd	1.5	0.79	0.114	0.055	3.79E-14	1.10E-13	8.43
5	2.7	elec	3rd	1.5	0.79	0.114	0.055	3.79E-14	1.40E-13	13.66
5	2.7	elec	3rd	1.5	0.79	0.114	0.055	3.79E-14	8.40E-14	4.92
5	2.81	elec	3rd	1.5	0.79	0.114	0.055	3.50E-14	2.20E-13	39.58
5	2.81	elec	3rd	1.5	0.79	0.114	0.055	3.50E-14	1.40E-13	16.03
5	2.81	elec	3rd	1.5	0.79	0.114	0.055	3.50E-14	1.20E-13	11.78
5	2.6	BVA	3rd	1.5	1.00	0.1089	0.055	5.08E-14	1.20E-13	5.58
5	2.58	BVA	3rd	1.5	1.00	0.1089	0.085	5.16E-14	7.10E-14	1.90
5	2.63	BVA	3rd	1.5	1.00	0.1089	0.085	4.96E-14	5.90E-14	1.41
5	2.6	BVA	3rd	1.5	1.00	0.1089	0.085	5.08E-14	6.90E-14	1.85
5	2.47	BVA	3rd	1.5	1.00	0.1089	0.085	5.63E-14	1.90E-13	11.40
5	2.61	BVA	3rd	1.5	1.00	0.1089	0.085	5.04E-14	1.50E-13	8.86
10	1.31	BVA	3rd				0.021	9.94E-14	1.20E-13	1.46
10	1.37	BVA	3rd				0.021	9.09E-14	1.40E-13	2.37

Table II. Typical frequency versus temperature coefficients for an AT-cut resonator with a turnover temperature of 85 °C as a function of oven parameters. Frequency stability of 1×10^{-13} requires $\Delta T/dt < 10$ nK/s.

$$\Delta\nu/\nu \cong 3 \times 10^{-8}\ \Delta T^2 + 10^{-5}\ dT/dt$$

Oven Offset mK	±100 +mK	Oven Change ± 10 mK	± 1 mK	±0.1 mK
0	$3x10^{-8}$	$3x10^{-8}$	$3x10^{-8}$	$3x10^{-8}$
1	$3x10^{-8}$	$5x10^{-8}$	$2x10^{-8}$	$6x10^{-8}$
10	$5x10^{-8}$	$2x10^{-8}$	$6x10^{-8}$	$6x10^{-8}$
100	$2x10^{-8}$	$6x10^{-8}$	$6x10^{-8}$	$6x10^{-8}$

Table V. VHF Crystal Flicker-of-Frequency Noise: Best Measured Results in Comparison to Walls' Volume Model Predictions Using Electroded Volume

CRYSTAL CUT	SC	SC	SC	BT	AT
FREQUENCY	80 MHz	100 MHz	160 MHz	100 MHz	40 MHz
OVERTONE	3rd	3rd	5th	5th	5th
UNLOADED Q	125K	119K	75K	300K	250K
BLANK THICKNESS (cm)	0.00678	0.00543	0.00566	0.013	0.021
ELECTRODE DIAMETER (cm)	0.162	0.140	0.254	0.317	0.366
ELECTRODED VOLUME (cm^3)	.00014	.000083	.00028	.001	.0022
BEST MEASURED Sy(f=100 Hz)*	$7.4x10^{-26}$	$3.6x10^{-26}$	$4.1x10^{-26}$	$3.0x10^{-26}$	$1.25x10^{-26}$
RESONATOR Sy(f=100 Hz)**	$6.4x10^{-26}$	$2.6x10^{-26}$	$2.5x10^{-26}$	$2.0x10^{-26}$	$1.0x10^{-26}$
MODEL PREDICTED Sy(f= 100 Hz) (using β_e=4 om eq. 1)	$2.3x10^{-26}$	$1.7x10^{-26}$	$3.3x10^{-26}$	$5.0x10^{-27}$	$2.2x10^{-26}$

* Including contribution of 4-crystal reference oscillator

** Removing contribution of 4-crystal reference oscillator

Table VI. VHF Crystal Flicker-of-Frequency Noise: Best Measured Results in Comparison to Walls' Volume Model Predictions Using Acoustic Energy Volume

CRYSTAL CUT	SC	SC	SC	BT	AT
FREQUENCY	80 MHz	100 MHz	160 MHz	100 MHz	40 MHz
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