TUTORIAL PTTI MEASUREMENT TECHNOLOGY

CHAIRMAN: DR. FRED L. WALLS, NATIONAL INSTITUTE FOR STANDARDS AND TECHNOLOGY

This workshop is divided into three parts. The first part teaches the fundamentals and the basics of AM and PM noise measurements. The second part uses this background in the basic measurements to develop error models for PM and AM measurements. These models are then illustrated by selected practical examples. The emphasis is on identifying parameters to monitor and pitfalls to avoid. A few examples of PM and/or AM noise in selected components are presented. Fractional frequency stability in the time domain is easily calculated from phase noise measurements. This approach is particularly powerful for short measurement times or when there are significant spurious signals.

The third part details some approaches to the measurement problems that extend the frequency range and improve the accuracy and/or speed of the measurements.

I. FUNDAMENTAL CONCEPTS AND DEFINITIONS IN PM AND AM NOISE METROLOGY

Eva Pikal
NIST/University of Colorado

A. Fundamental concepts
B. Simple PM noise measurement systems
C. Simple AM noise measurement systems

II. DISCUSSION OF ERROR MODELS FOR PM AND AM NOISE MEASUREMENTS

Fred L. Walls
NIST

A. Error model for PM noise measurements
B. Error model for AM noise measurements
C. PM and AM noise models
D. Conversion of PM data to $\sigma_y(\tau)$ and mod$\sigma_y(\tau)$

III. STATE-OF-THE-ART MEASUREMENT TECHNIQUES FOR PM AND AM NOISE

Craig W. Nelson
SpectraDynamics

A. Ultra wideband measurements (f = 0.1 Hz to 1 GHz)
B. Integral AM and PM noise standards
C. Ultra low-noise PM and AM measurement systems
S(f) ≤ -190 dBc/Hz)

FUNDAMENTAL CONCEPTS AND DEFINITIONS IN PM AND AM NOISE METROLOGY

Eva F. Pikal
NIST/University of Colorado

FUNDAMENTAL CONCEPTS

SIMPLE PM NOISE MEASUREMENT SYSTEMS

- TWO OSCILLATOR METHOD
- DELAY LINE
- CAVITY DISCRIMINATOR

SIMPLE AM NOISE MEASUREMENT SYSTEMS

1. WHAT IS SPECTRAL PURITY?

SPECTRAL PURITY IS THE RATIO OF SIGNAL-TO-NOISE POWER

2. WHY DO WE CARE?

SPECTRAL PURITY SETS THE FUNDAMENTAL LIMIT FOR ALL FREQUENCY AND TIMING MEASUREMENTS

3. COMMON EXAMPLES

CHANNEL SPACING AND ERROR RATES FOR HIGH SPEED COMMUNICATIONS

CLARITY OF VOICE AND TV SIGNALS

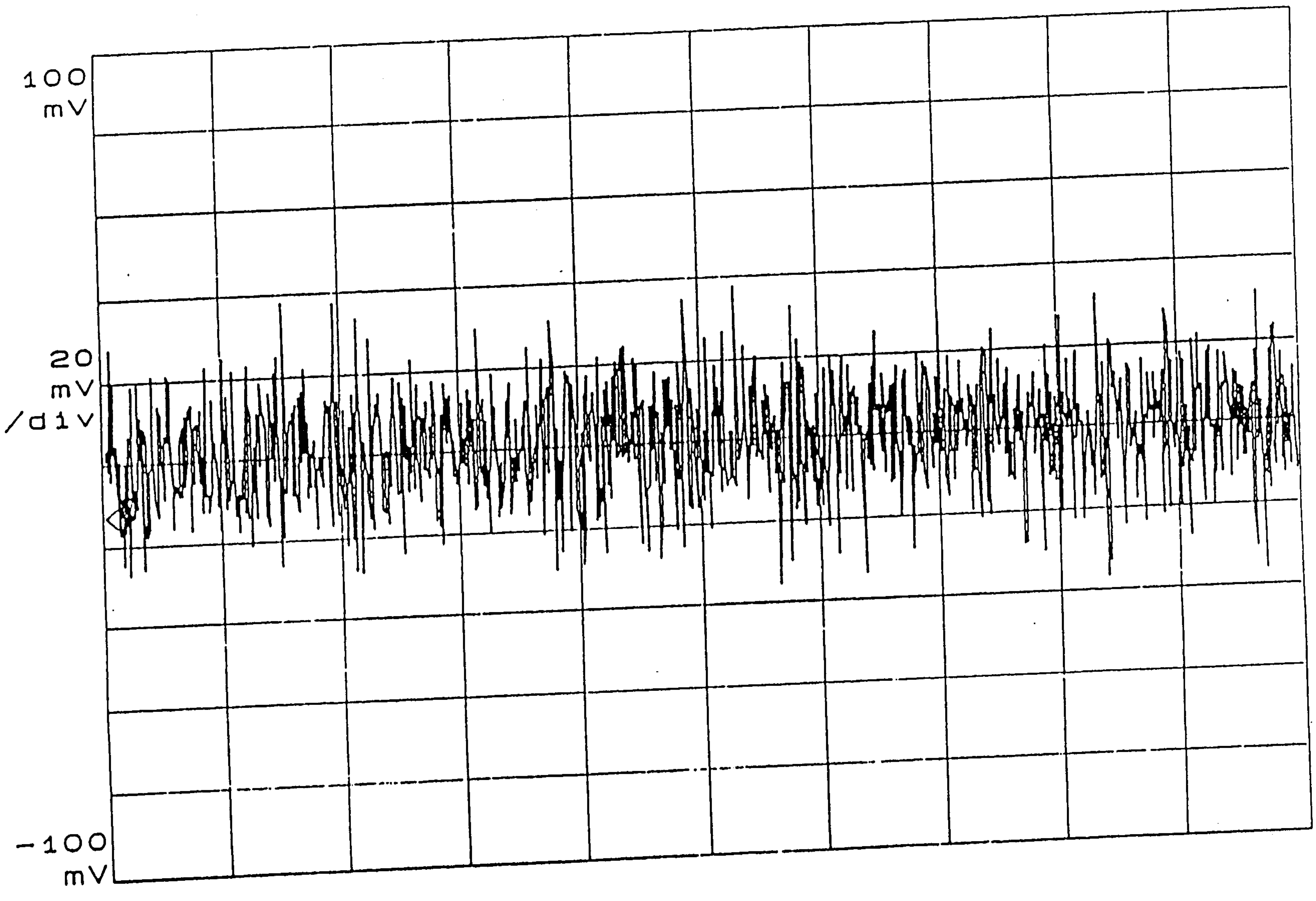
RANGE AND DEFINITION OF RADAR SIGNALS

PRECISION OF ELECTRONIC NAVIGATION

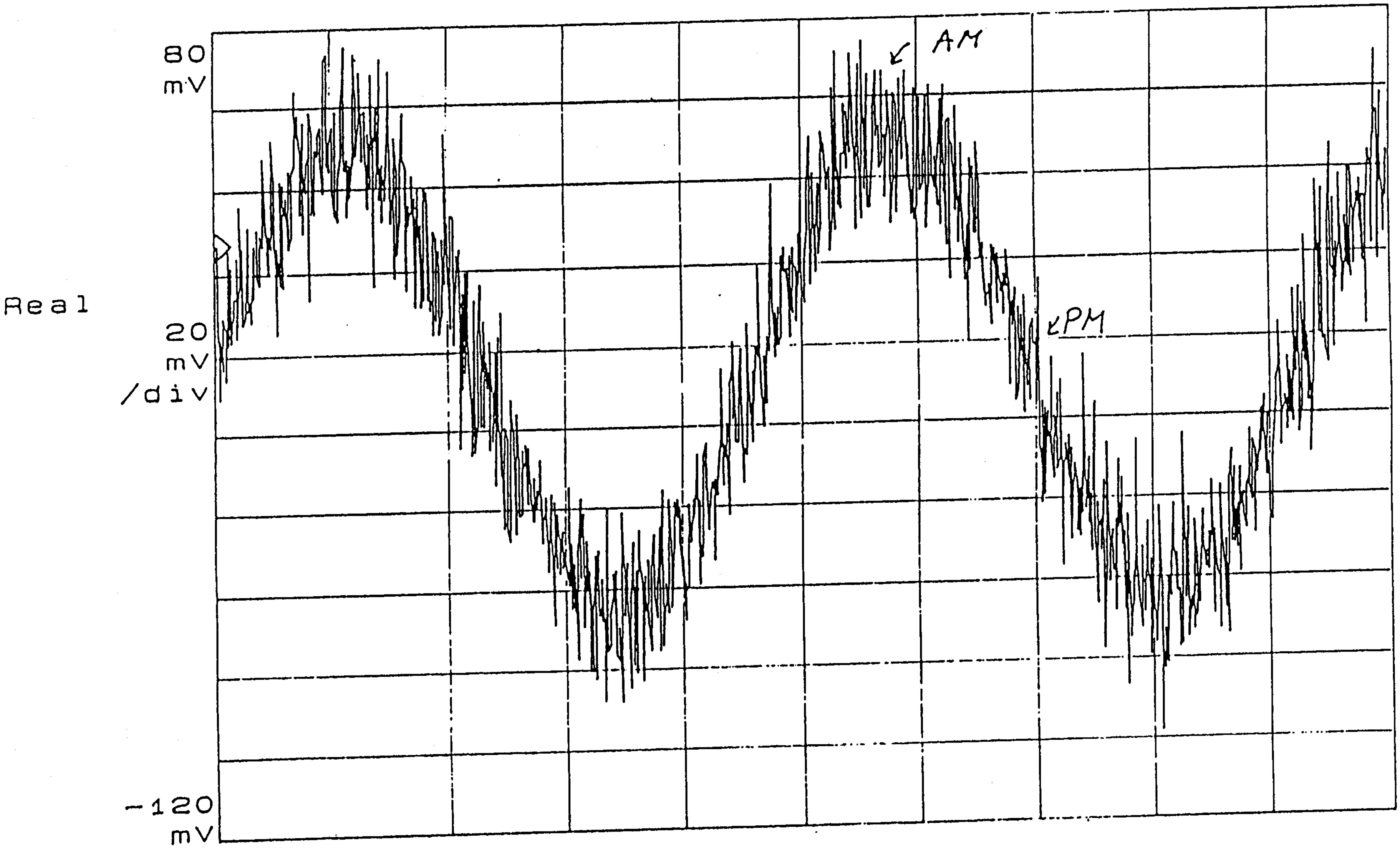
PRECISION AND ACCURACY OF ELECTRONIC MEASUREMENT SYSTEMS

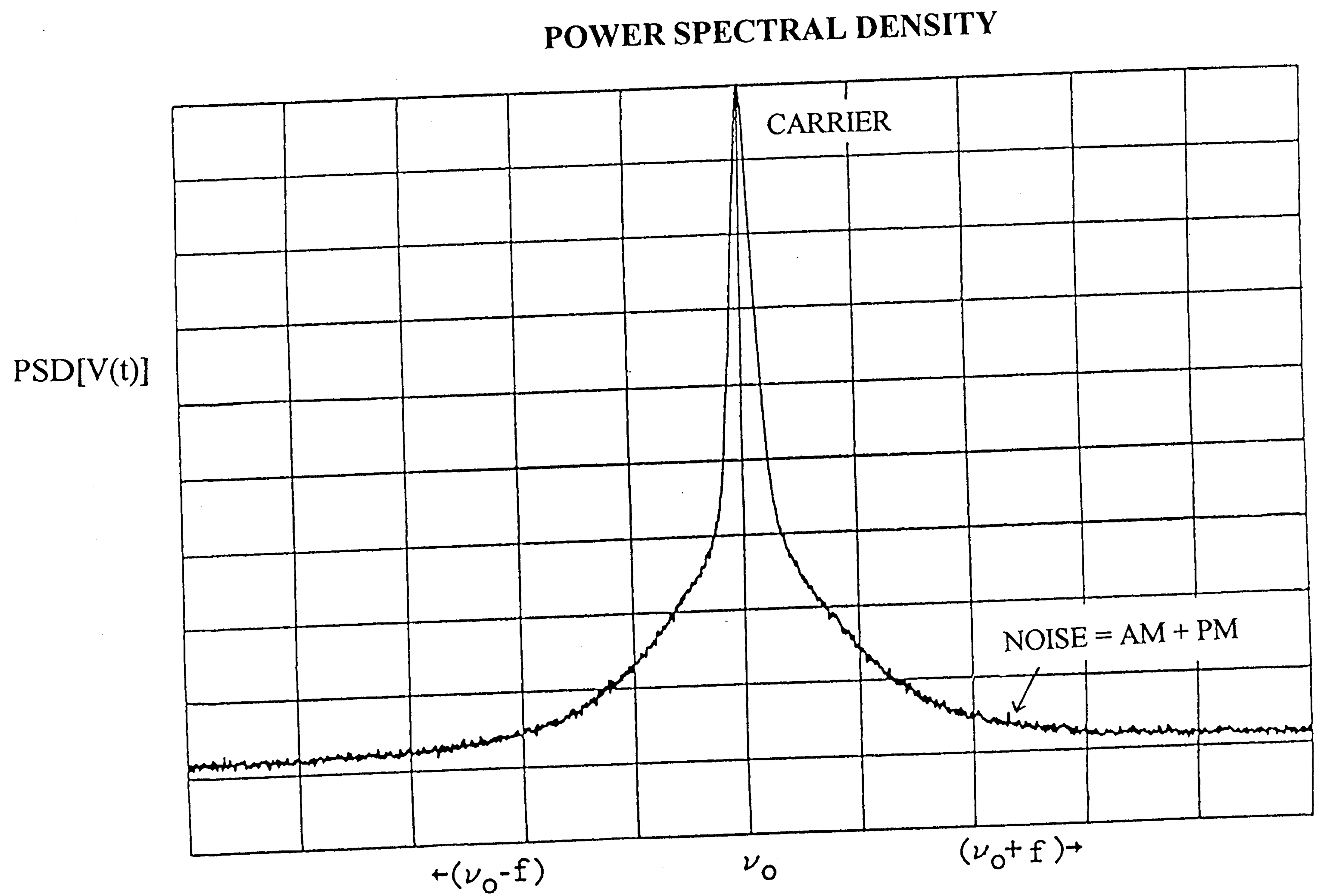
4. WHAT IS THE DIFFERENCE BETWEEN NOISE, AMPLITUDE MODULATION (AM) NOISE, PHASE MODULATION (PM) NOISE, AND FREQUENCY MODULATION FM NOISE?

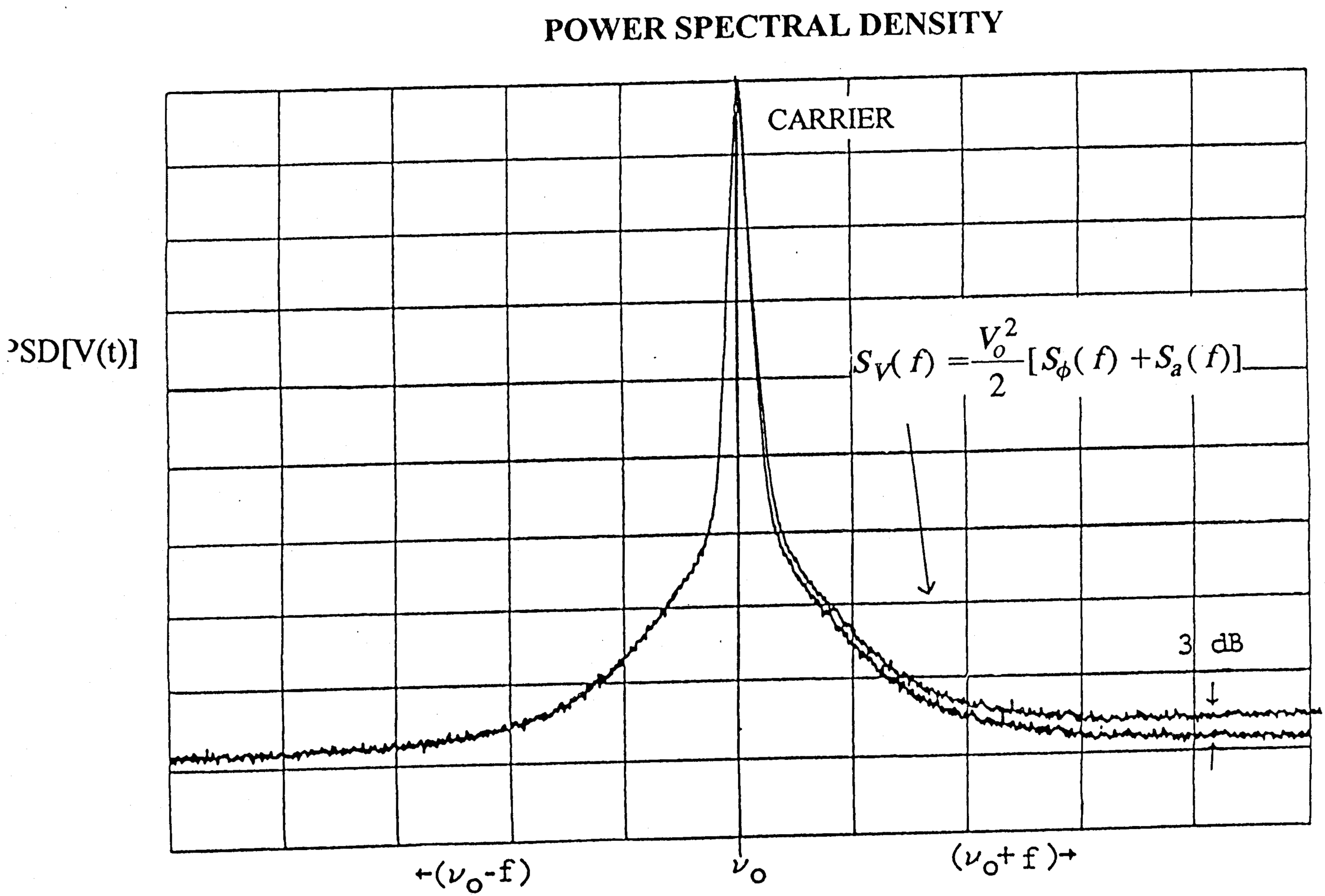
NOISE



NOISY SIGNAL



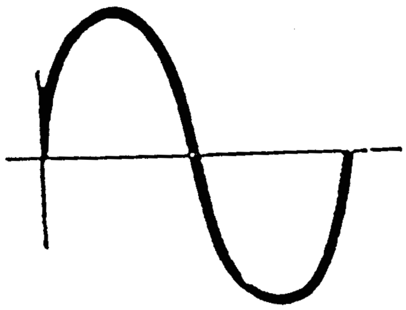

POWER SPECTRAL DENSITY
PSD[V(t)]
CARRIER
NOISE = AM + PM
←(ν_o-f)
ν_o
(ν_o+f)→



POWER SPECTRAL DENSITY
PSD[V(t)]
CARRIER
$S_V(f) = \frac{V_o^2}{2}[S_\phi(f) + S_a(f)]$
3 dB
←(ν_o-f)
ν_o
(ν_o+f)→

WAVE THEORY REVIEW

perfect signal

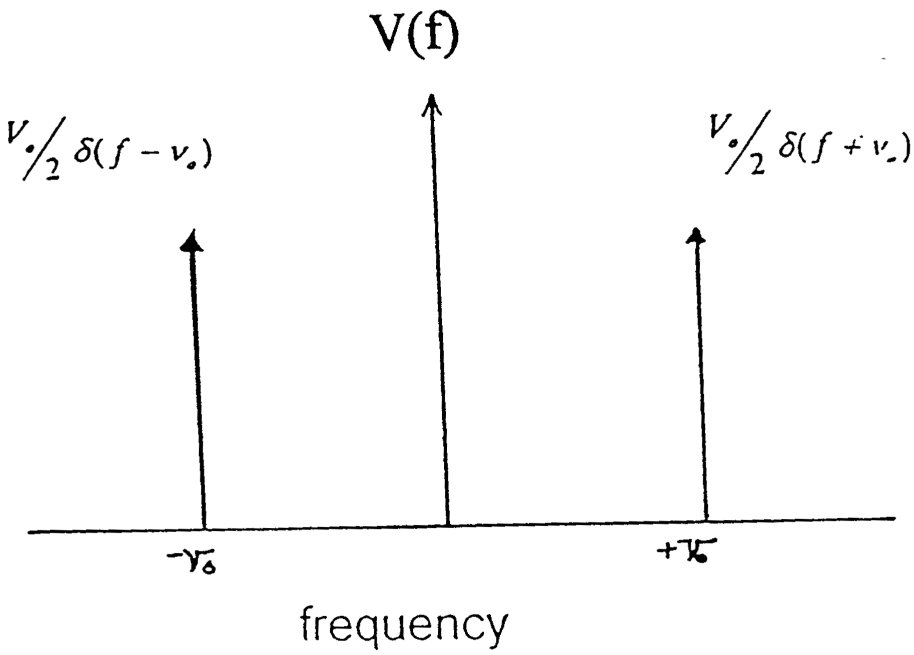


$$V(t) = V_o \cos(2\pi \nu_o t)$$

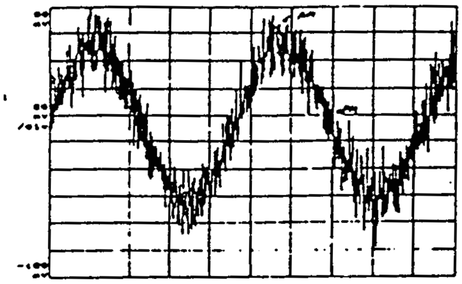
$$phase = 2\pi \nu_o t$$

$$T = period = 1/\nu_o$$

Fourier Transform:



PHASE AND AMPLITUDE FLUCTUATIONS



$$V(t) = [V_o + \varepsilon(t)]\cos(2\pi\nu_o t + \phi(t))$$

$$phase = 2\pi\nu_o t + \phi(t)$$

$$\omega(t) = \frac{d}{dt}[phase]$$

$$\nu(t) = \frac{1}{2\pi}\frac{d}{dt}[2\pi\nu_o t + \phi(t)]$$

$$\nu(t) = \nu_o + \frac{1}{2\pi}\frac{d}{dt}\phi(t)$$

Fractional frequency deviation:

$$y(t) = \frac{\nu(t) - \nu_o}{\nu_o} = \frac{1}{2\pi\nu_o}\frac{d}{dt}\phi(t)$$

PHASE/AMPLITUDE NOISE RELATIONSHIPS

$$S_\phi(f) = [\Delta\phi(f)]^2 \frac{1}{BW} \quad 0 < f < \infty \quad [\frac{rad^2}{Hz}]$$

$$S_a(f) = \frac{[\Delta\varepsilon(f)]^2}{V_o^2} \frac{1}{BW} \quad 0 < f < \infty \quad [\frac{1}{Hz}]$$

$$y(t) = \frac{1}{2\pi\nu_o} \frac{d}{dt} \phi(t)$$

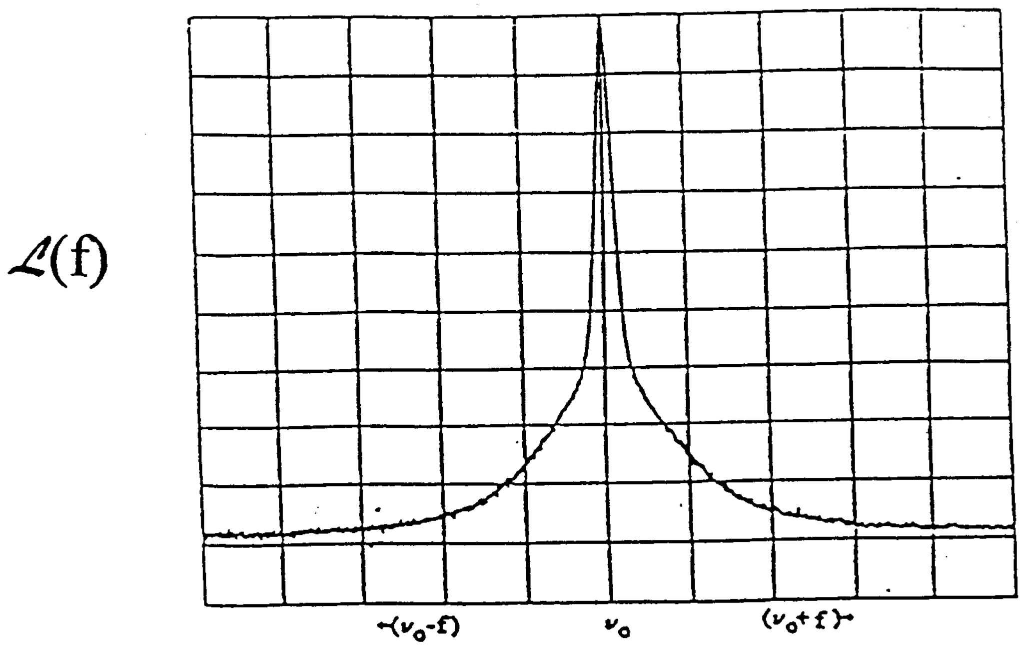
derivative in time = multiplication by ω in freq

= multiplication by ω^2 in spectral density

$$S_y(f) = \frac{1}{[2\pi\nu_o]^2} (2\pi f)^2 S_\phi(f)$$

$$S_\phi(f) = [\frac{\nu_o}{f}]^2 S_y(f) \quad 0 < f < \infty$$

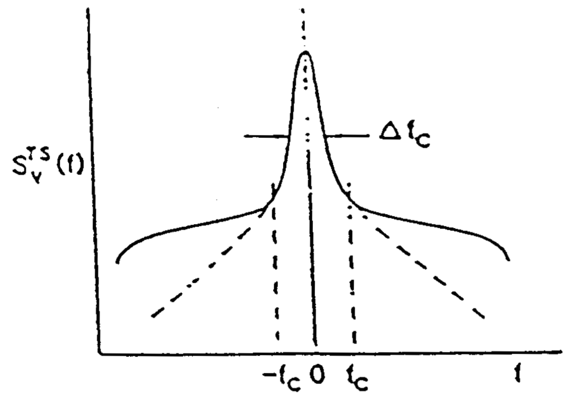
$$S_\phi(f) = \mathcal{L}(\nu_o - f) + \mathcal{L}(\nu_o + f)$$



$$\mathcal{L}(f) = \frac{1}{2} S_\phi(f)$$

$$dBc/_{Hz} = 10\log(\mathcal{L}(f))$$

POWER SPECTRAL DENSITY OF A NOISY SIGNAL



Double side-band spectral density:

$$S_V(f) \cong \frac{V_o^2}{2}[e^{-I(f_c)}\delta(f) + \qquad S_\phi(f) + S_a(f)]$$

$$0 \leq f \leq \infty$$

$$I(f) = \int_{f_c}^{\infty} S_\phi(f)df$$

I(f) = integrated phase modulation due to pedestal.

$\delta(f)$ = carrier with frequency $\pm f_c$

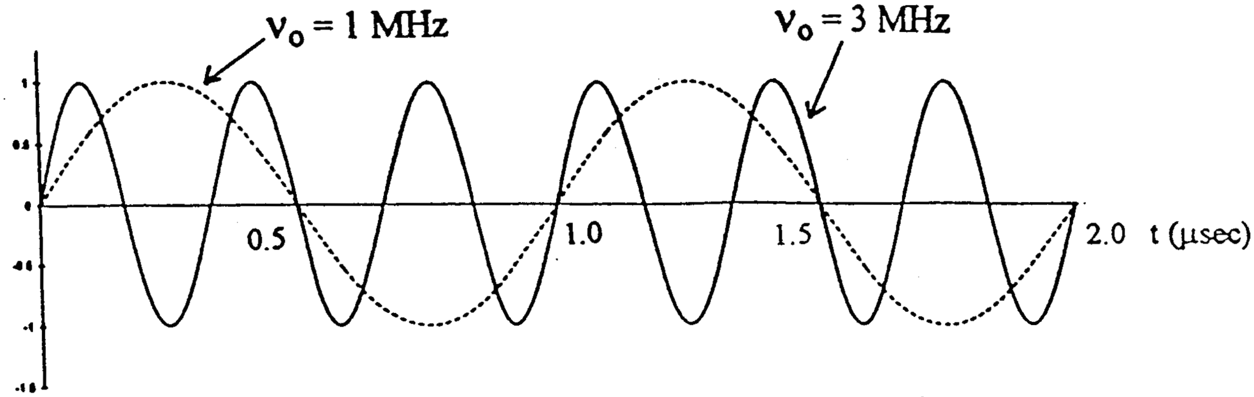
Power in carrier = $\frac{V_o^2}{2}e^{-I(f_c)} \approx \frac{V_o^2}{2}$ for $I(f_c) << 1$

RMS PHASE DEVIATION

$$\varphi^2(f)_{BW} = \int_{f-BW/2}^{f+BW/2} S_\varphi(f)\,df \;\; rad^2$$

FREQUENCY MULTIPLICATION/DIVISION EFFECTS ON PM NOISE

Frequency Multiplication



$$\nu_{o2} = N\nu_{o1} \qquad \Delta\phi_2 = N\Delta\phi_1$$

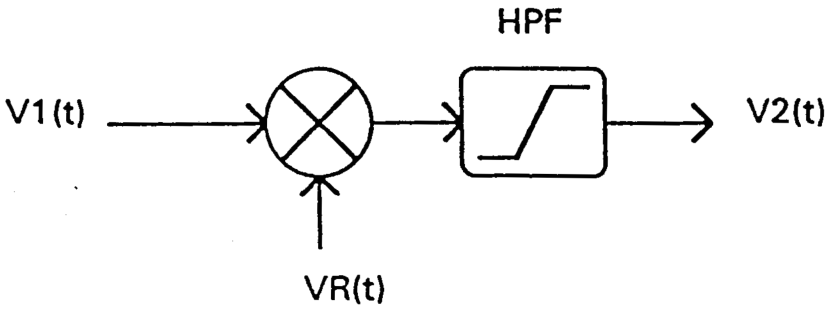
$$S_{\phi_2}(f) = \frac{[\Delta\phi_2]^2}{BW} = \frac{N^2\Delta\phi_1}{BW} = N^2 S_{\phi_1}(f)$$

FREQUENCY DIVISION:

$$\nu_{o2} = \frac{\nu_{o1}}{N}$$

$$S_{\phi_2}(f) = \frac{S_{\phi_1}(f)}{N^2}$$

FREQUENCY TRANSLATION



$$S_\phi(f,\nu_2) = S_\phi(f,\nu_1) + S_\phi(f,\nu_R) + S_{\phi T}(f)$$

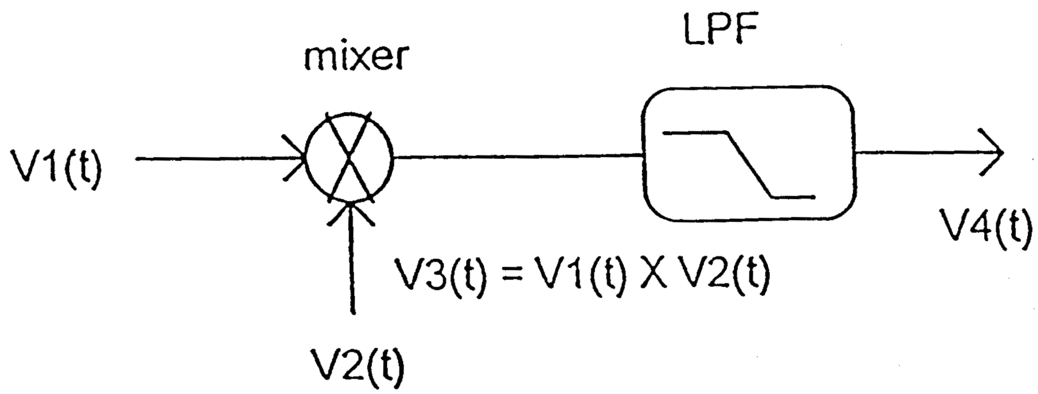
WHERE $\nu_2 = \nu_1 + \nu_R$

$S_\phi(f,\nu_R)$ = PM NOISE OF REFERENCE SIGNAL

$S_{\phi T}(f)$ = PM NOISE ADDED BY THE TRANSLATOR

$S_a(f,\nu_2)$ DEPENDS ON THE DETAILS OF THE TRANSLATION

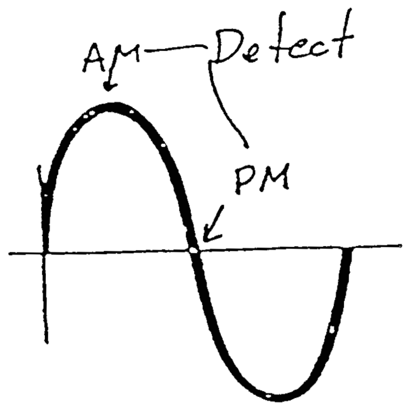
BASIC CONFIGURATIONS OF NOISE MEASUREMENTS



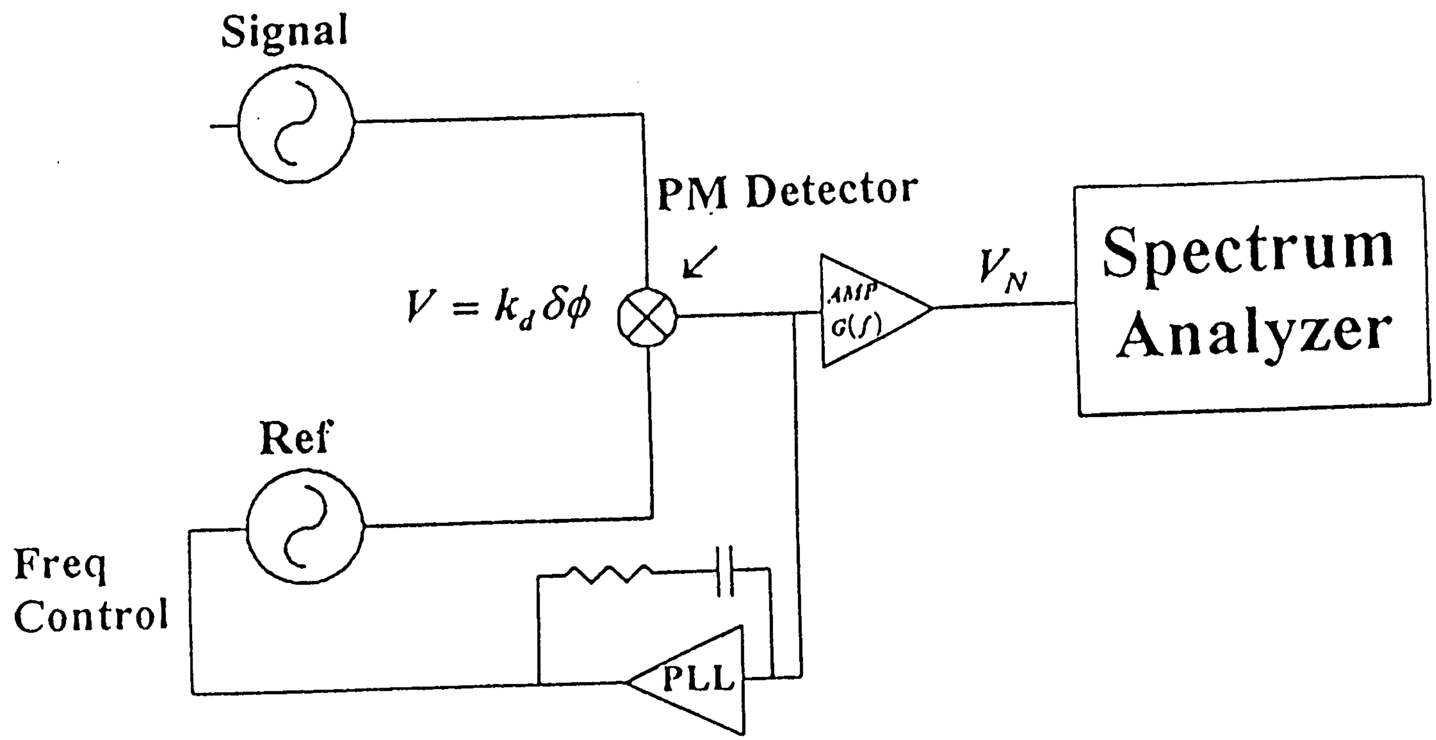
$$V_1(t) = [V_1 + \varepsilon_1(t)]\cos[2\pi\nu_o t + \phi_1]$$
$$V_2(t) = [V_2 + \varepsilon_2(t)]\cos[2\pi\nu_o t + \phi_2]$$

$$V_3(t) = \frac{A_1 A_2}{2}\{\cos[2\pi(2\nu_o)t + \phi_1 + \phi_2] + \cos[\phi_1 - \phi_2]\}$$

$$V_4(t) = \frac{A_1 A_2}{2}[\cos(\phi_1 - \phi_2)]$$
$$AM \Rightarrow \phi_1 - \phi_2 = \pi n$$
$$PM \Rightarrow \phi_1 - \phi_2 = \frac{\pi}{2} + \pi n$$



Simple PM Measurements



$\dfrac{PSDV_N}{[k_d G(f)]^2}$ measures $S_\phi(f)$ of the Signal plus the system noise.

It is difficult to seperate the system noise from a signal with low PM noise. Results uncorrected for PLL and gain variations with Fourier frequency.

NOISE TERMS INCLUDED IN $\frac{PSD(V_n)}{K_d^2 G(f)^2}$

$$S_\phi(f) = \frac{[\Delta\phi_A(f) - \Delta\phi_B(f)]^2}{BW} + \frac{V_n(f)^2_{mixer}}{K_d^2 BW} + \frac{V_n(f)^2_{amp}}{K_d^2 BW}$$

$$+ \frac{V_n(f)^2_{SA}}{K_d^2 G(f)^2 BW} + S_{aA}(f)\beta_A^2 + S_{aB}(f)\beta_B^2$$

$$S_\phi(f)_{pair} = S_{\phi A}(f) + S_{\phi B}(f) + \frac{V_n(f)^2_{system}}{K_d^2 BW} + S_{aA}(f)\beta_A^2 + S_{aB}(f)\beta_B^2$$

TO GET NOISE FLOOR SET A = B

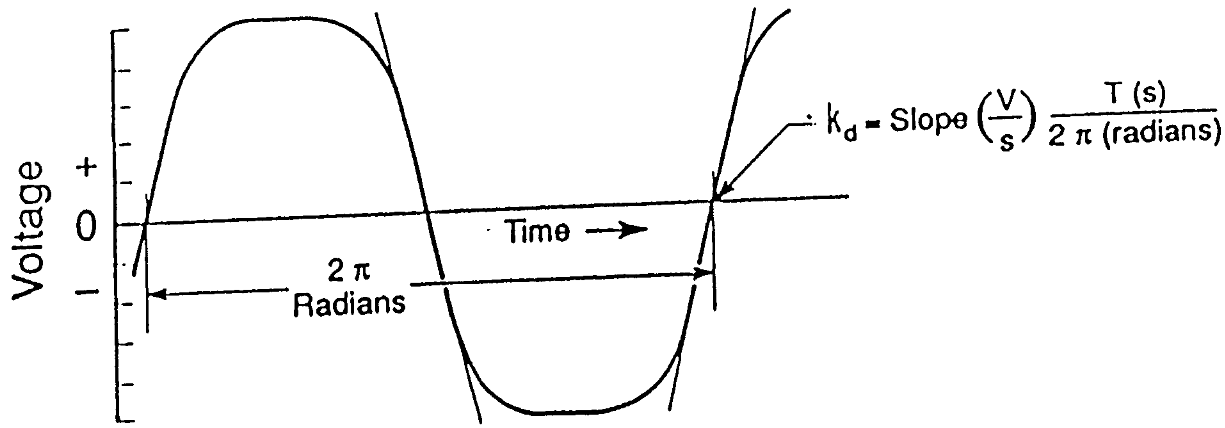
$$S_\phi(f)_{Noise\ Floor} = (2\pi f \tau_{delay})^2 S_{\phi A}(f) + \frac{V_n(f)^2{}_{system}}{K_d{}^2 BW} + S_{aA}(f)\beta_A{}^2 + S_\phi(f)_{power\ splitter}$$

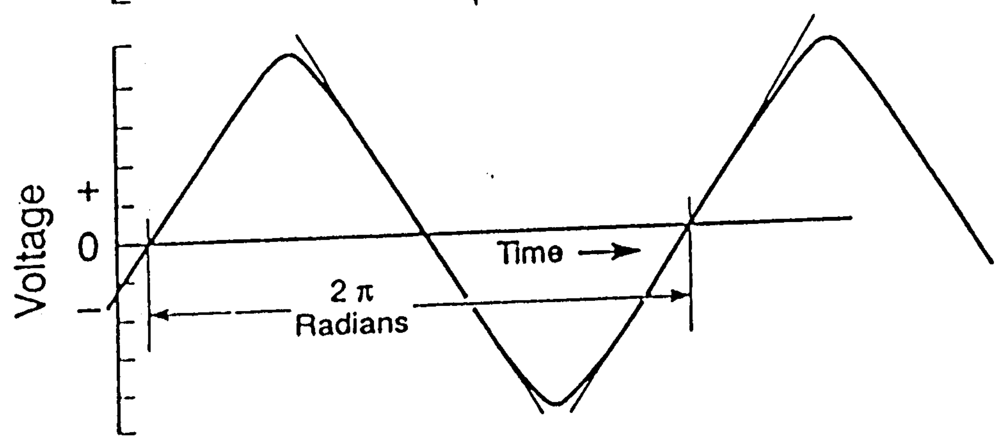
$$\tau_{delay} = \frac{n\pi}{2}\frac{1}{\upsilon_o}$$

TO CALCULATE INDIVIDUAL PM NOISE FOR AN OSCILLATOR

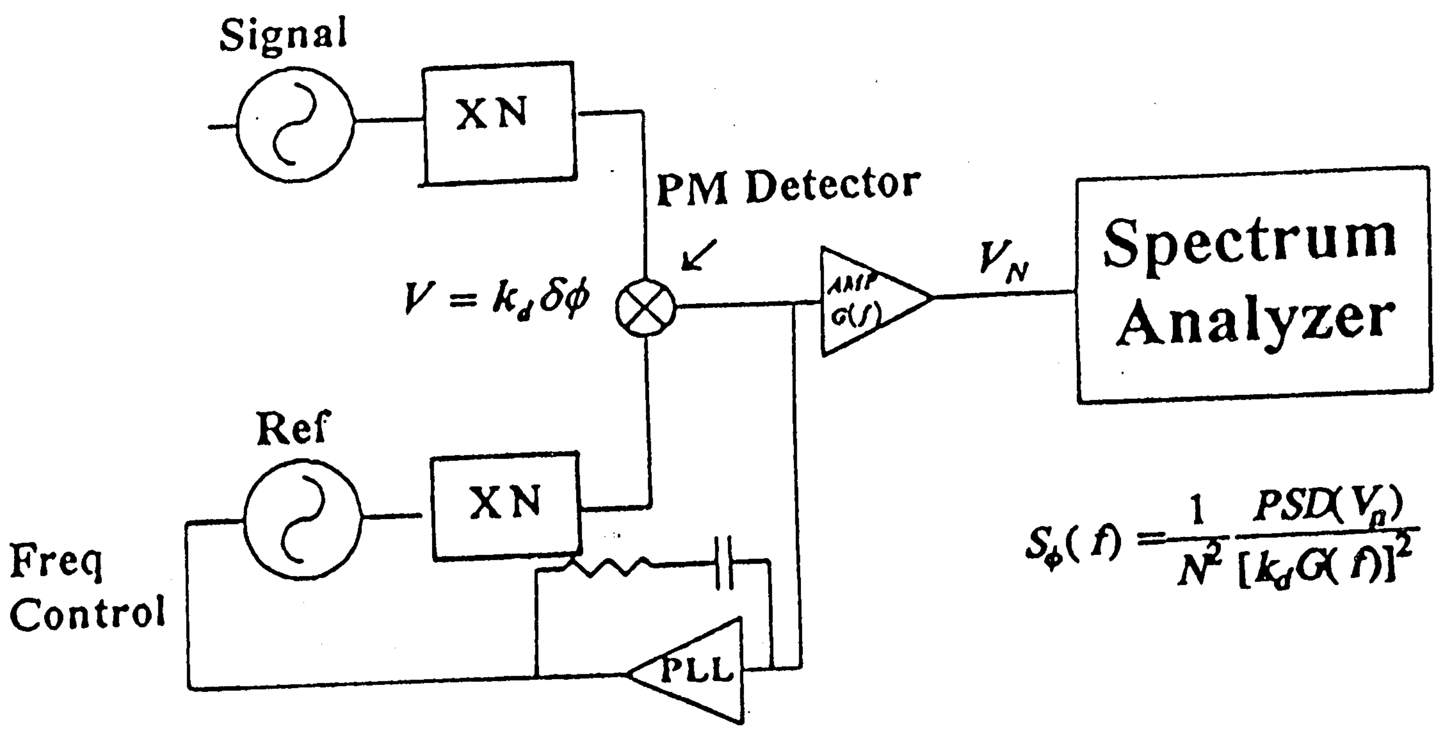
$$S_\phi(f)_{AB} + S_\phi(f)_{AC} - S_\phi(f)_{BC} = 2S_{\phi A}(f) + \frac{V_n{}^2}{K_d{}^2 BW} + 2S_{aA}(f)\beta_A{}^2$$

CALIBRATION FACTOR k_d





NOISE FLOOR IMPROVEMENT USING FREQUENCY MULTIPLIERS



$$S_\phi(f) = \frac{1}{N^2}\frac{PSD(V_n)}{[k_d G(f)]^2}$$

DISCUSSION OF DIRECT PHASE COMPARISON

ADVANTAGES

HIGHEST RESOLUTION (LOWEST NOISE FLOOR)

NOISE FLOOR MEASURED WITH INFERIOR OSCILLATOR

VERY WIDE BAND PERFORMANCE

INEXPENSIVE

DISADVANTAGES

REQUIRES A REFERENCE OF COMPARABLE STABILITY

REQUIRES PHASE-LOCKED-LOOP (PLL) TO MAINTAIN $\delta\phi < 0.1$ rad

CALIBRATION DIFFICULT FOR $f \ll$ PLL BW

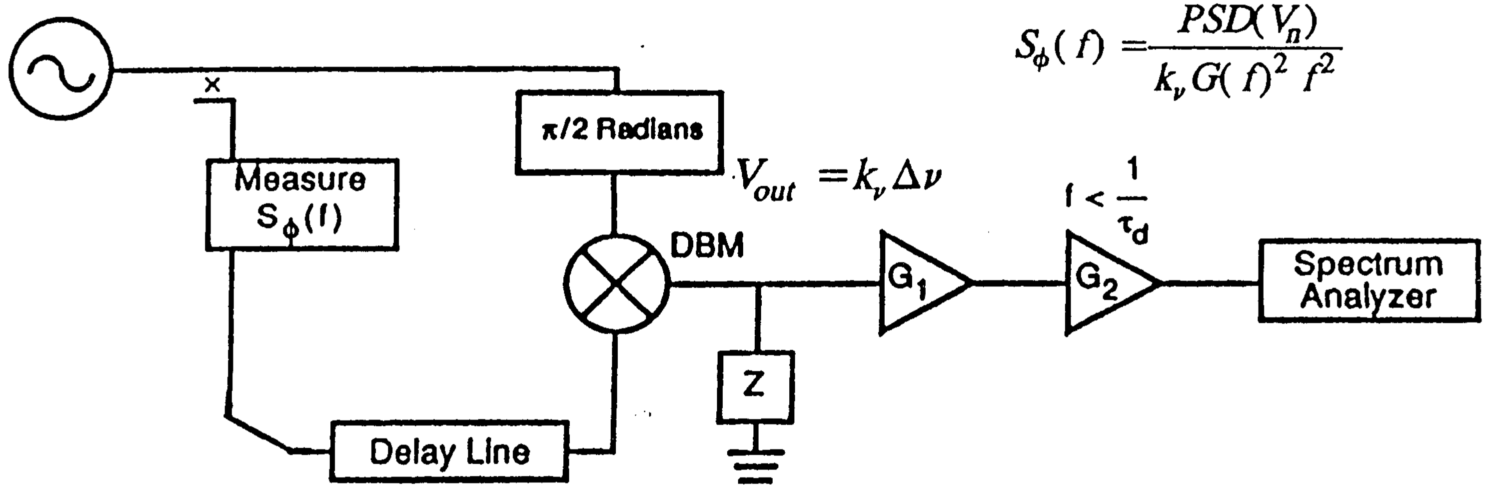
SENSITIVE TO HARMONIC DISTORTION

FREQUENCY RESPONSE DEPENDS ON POWER & LOAD

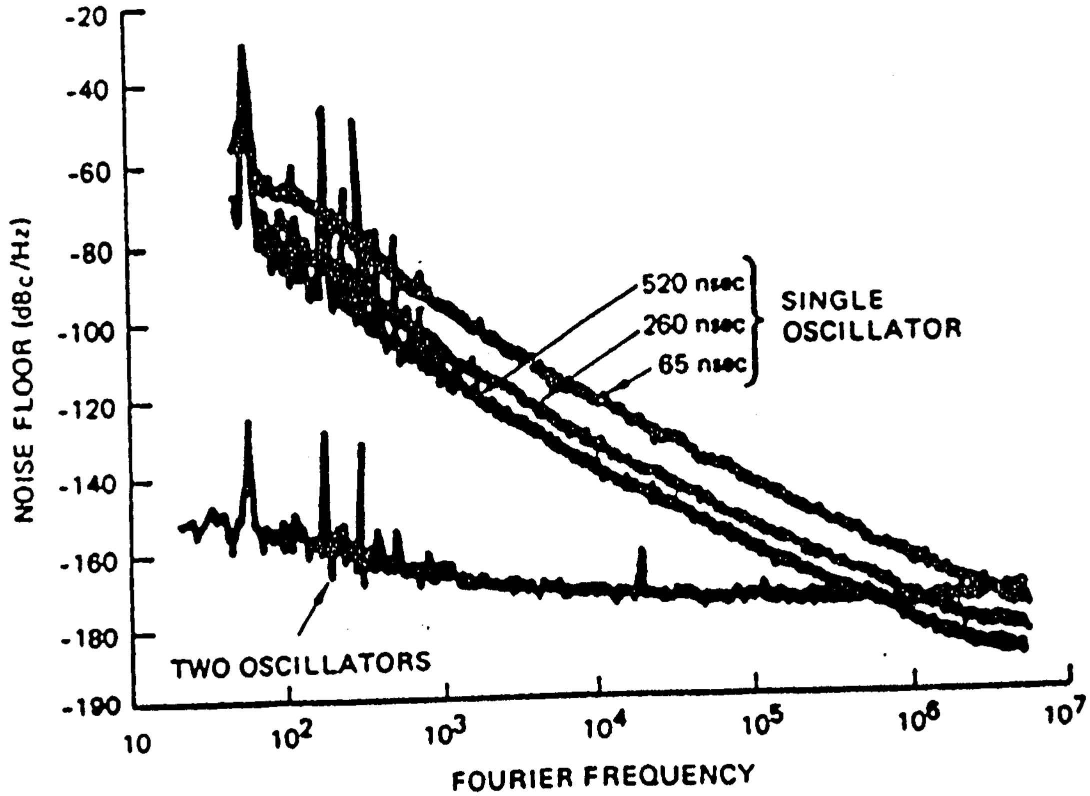
MEASUREMENT OF $S_\phi(f)$ USING A DELAY LINE

$$S_\phi(f) = \frac{\upsilon_o^2}{f^2} S_y(f)$$

$$S_\phi(f) = \frac{PSD(V_n)}{k_\nu G(f)^2 f^2}$$

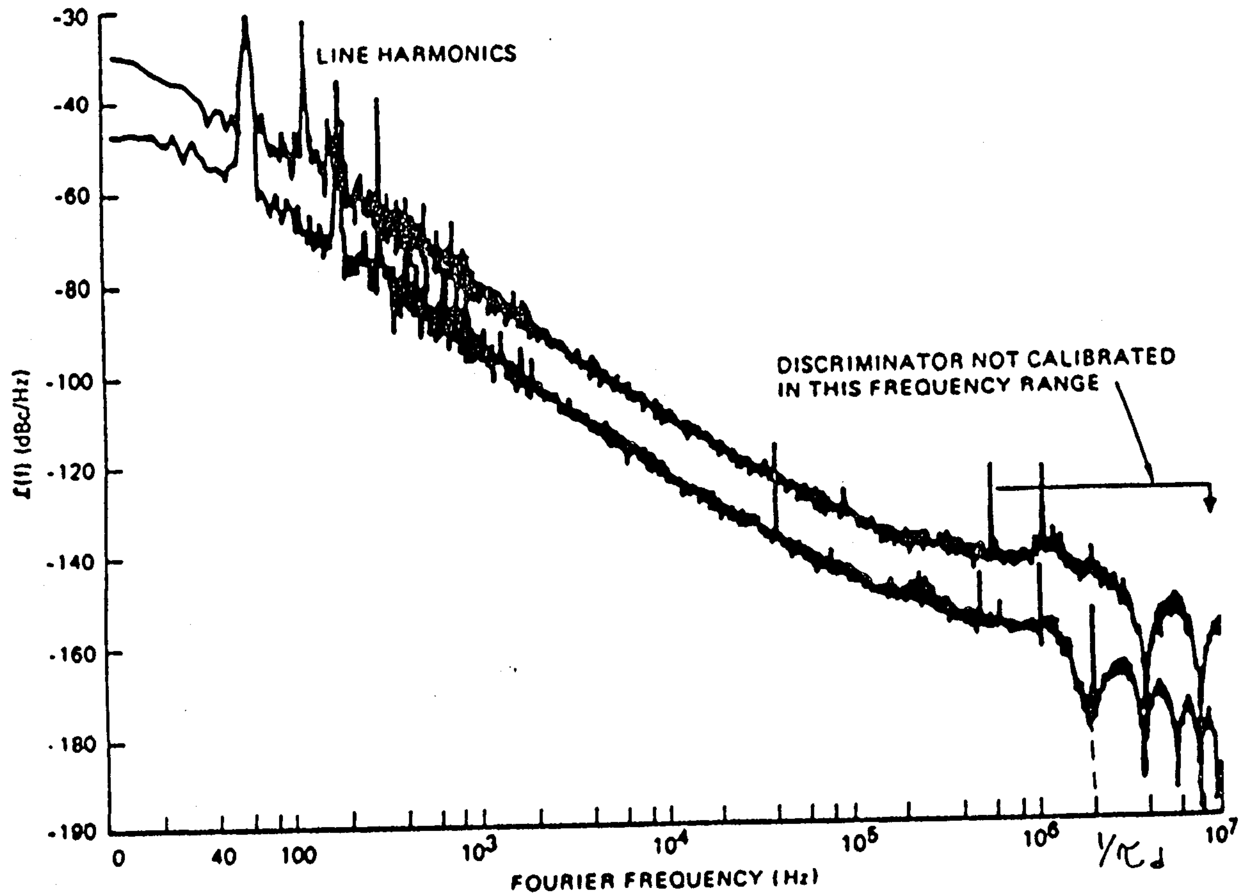


NOISE FLOOR COMPARISON FOR TWO MEASUREMENT SYSTEMS: DELAY LINE SYSTEM VS. TWO OSCILLATOR SYSTEM



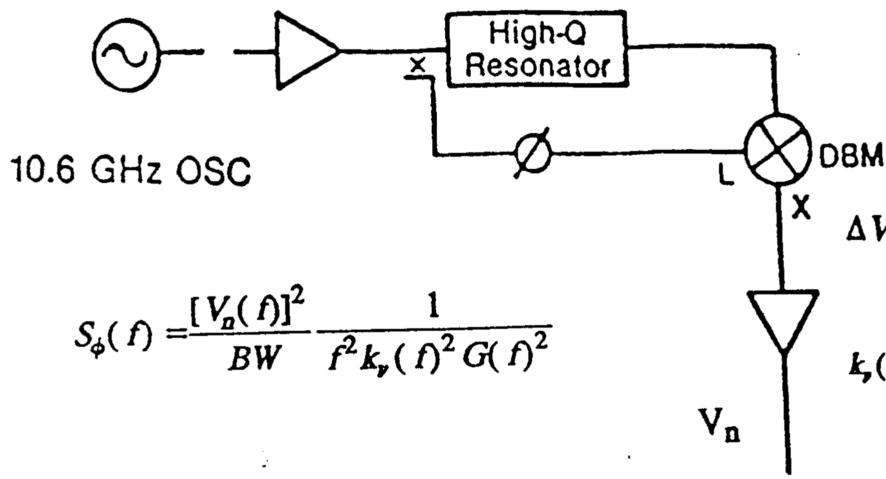
From: Infrared and Millimeter Waves, Vol. 11, pp. 239-289, 1984 (also in NIST Technical Note 1337).

DETERMINATION OF τ_d



From: Infrared and Millimeter Waves, Vol. 11, pp. 239-289, 1984 (also in NIST Technical Note 1337).

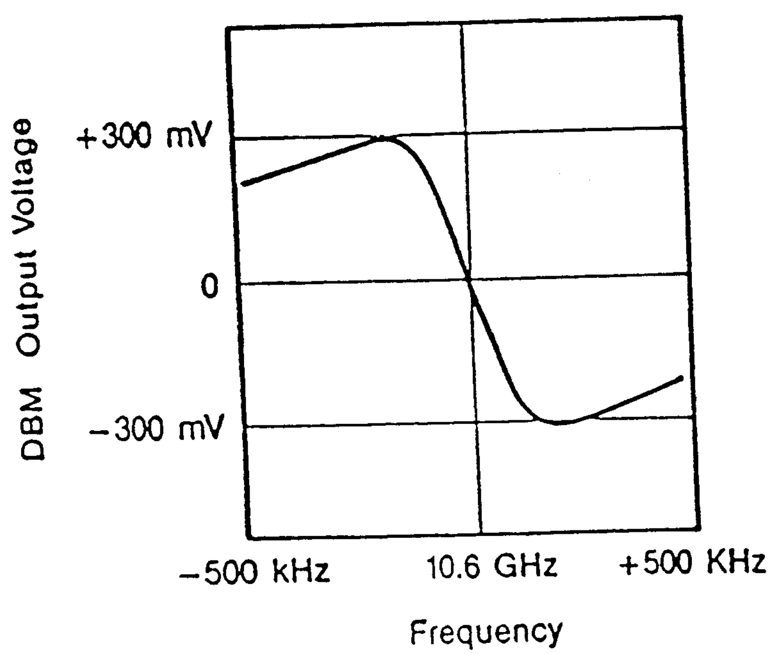
MEASUREMENT OF PHASE NOISE USING A HIGH-Q CAVITY



$$\Delta V = k_v(f)\Delta \nu$$

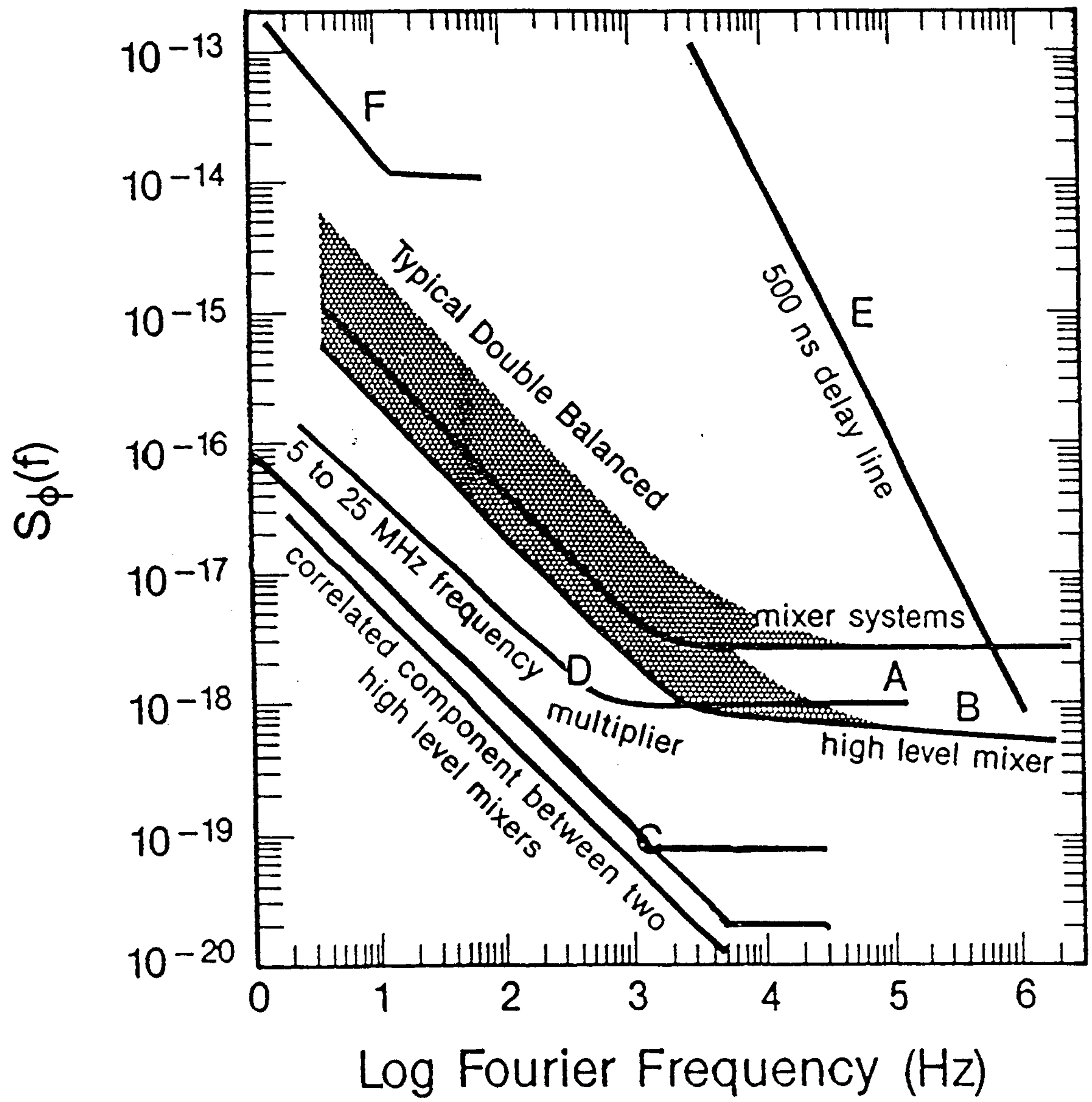
$$S_\phi(f) = \frac{[V_n(f)]^2}{BW} \frac{1}{f^2 k_v(f)^2 G(f)^2}$$

$$k_v(f) \propto \frac{1}{1+\left(\frac{2Qf}{\nu_o}\right)^2}$$

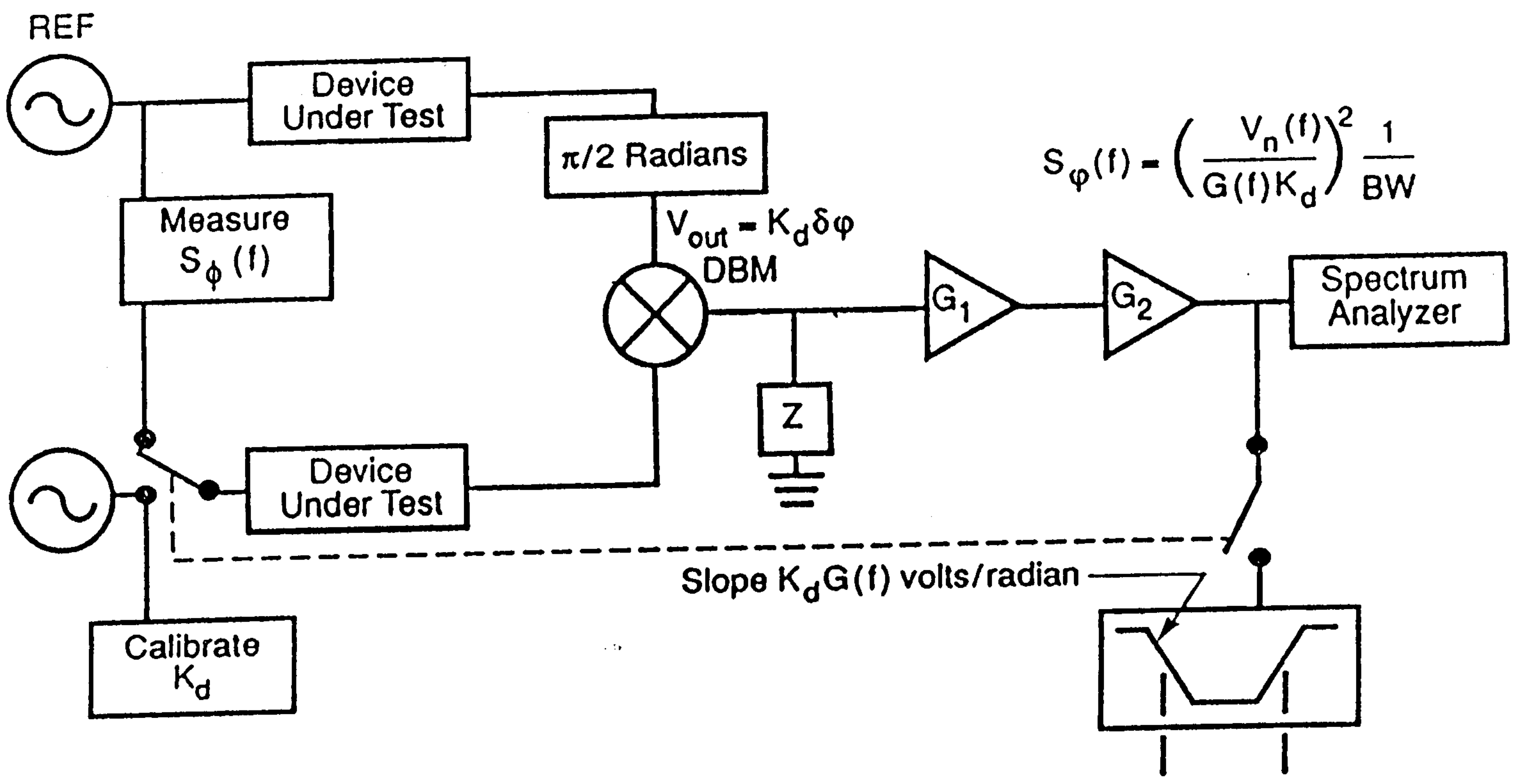


DIRECT FREQUENCY COMPARISONS	DIRECT PHASE COMPARISONS
ADVANTAGES:	
DOES NOT REQUIRE SECOND SOURCE.	LOWEST NOISE FLOOR
SYSTEM TRACKS FREQUENCY CHANGES IN SOURCE	NOISE FLOOR MEASURED WITH INFERIOR OSCILLATOR
NO PLL EFFECTS	VERY WIDE BAND PERFORMANCE
SIMPLE CALIBRATION $V = G k \Delta\upsilon$	INEXPENSIVE
DISADVANTAGES:	
NOISE FLOOR SCALES AS 1/f2 NEAR CARRIER	REFERENCE OF COMPARABLE STABILITY
NOISE FLOOR DIFFICULT TO MEASURE	REQUIRES PLL TO MANTAIN $\Delta\phi < 0.1$ rad
DIFF. CAVITIES REQUIRED FOR EACH υ	CALIBRATION DIFFICULT FOR f << PLL BW
DIFF. CAVITIES/DELAY LINES FOR DIFF. f	SENSITIVE TO HARMONIC DISTORTION
DIFFICULT TO MEASURE BEYOND CAVITY BW OR BEYOND DELAY LINE TIME	FREQUENCY RESPONSE DEPENDS ON POWER & LOAD

Comparison of Noise Floor for Different Techniques



Measurement of $S_\varphi(f)$ for Two Amplifiers

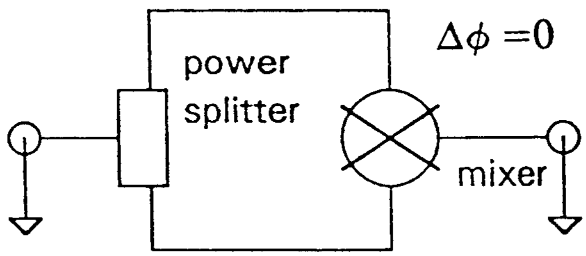


$$S_\varphi(f) = \left(\frac{V_n(f)}{G(f)K_d}\right)^2 \frac{1}{BW}$$

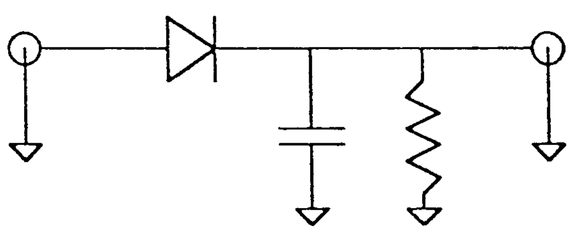
AM NOISE DEFINITION

$$S_a(f) = \left(\frac{\Delta\varepsilon}{V_o}\right)^2 \frac{1}{BW}$$

AM DETECTORS

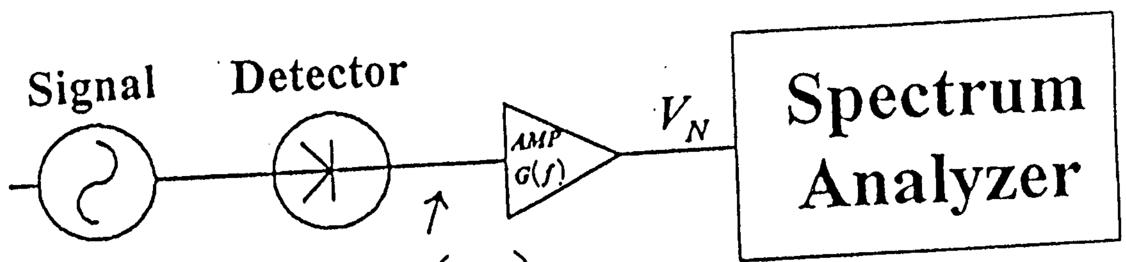


Mixer Detector



Diode Detector

Simple AM Measurement



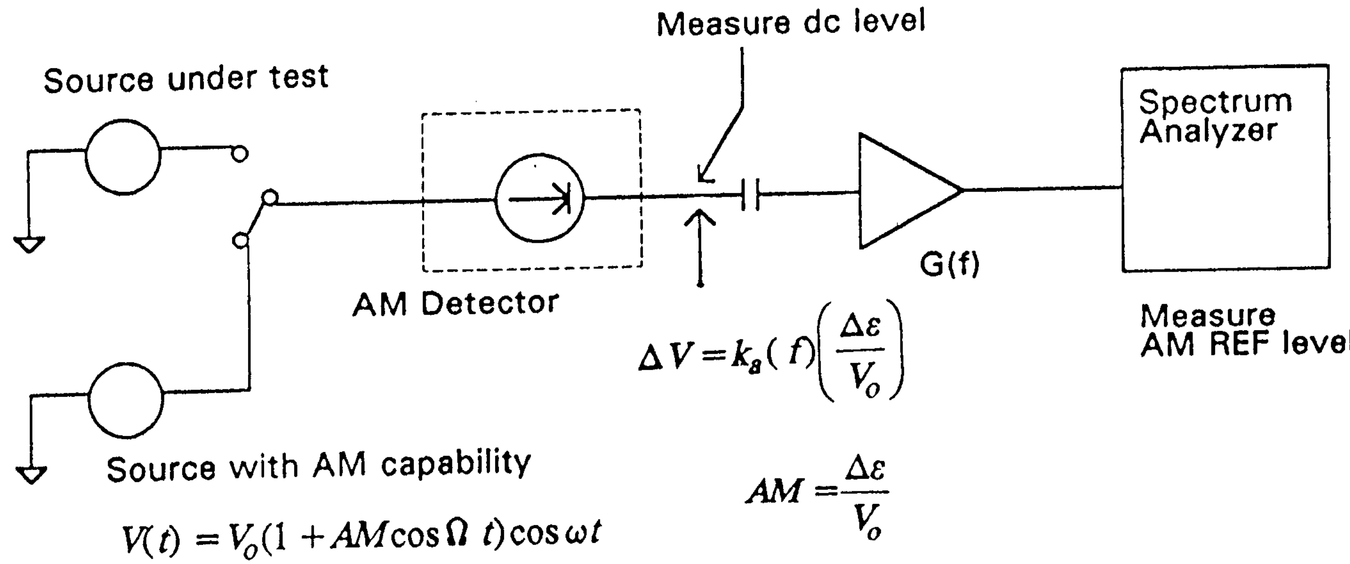
$$V(t) = k_a \delta\left(\frac{\varepsilon}{V_O}\right)$$

$$PSD(V_n) = \frac{V_n(f)^2}{BW} = [k_a(f)G(f)]^2 \left(\frac{\Delta\varepsilon}{V_o}\right)^2 \frac{1}{BW}$$

$\dfrac{PSD V_N}{[k_a G(f)]^2}$ measures $S_a(f)$ of the Signal plus the system noise.

It is difficult to seperate the system noise from a signal with low AM noise.

Determination of $[k_a(f)G(f)]$ for AM Measurement Systems



AM at the input signal: $\frac{1}{2}\left(\frac{\% AM}{100}\right)^2$

AM at output: AM REF level

$$[k_a(f)G(f)]^2 = \frac{\text{AM at Output}}{\text{AM at Input}} = \frac{(AM\ REF\ \text{level})^2}{\frac{1}{2}\left(\frac{\% AM}{100}\right)^2}$$

II. DISCUSSION OF ERROR MODELS FOR PM AND AM NOISE MEASUREMENTS

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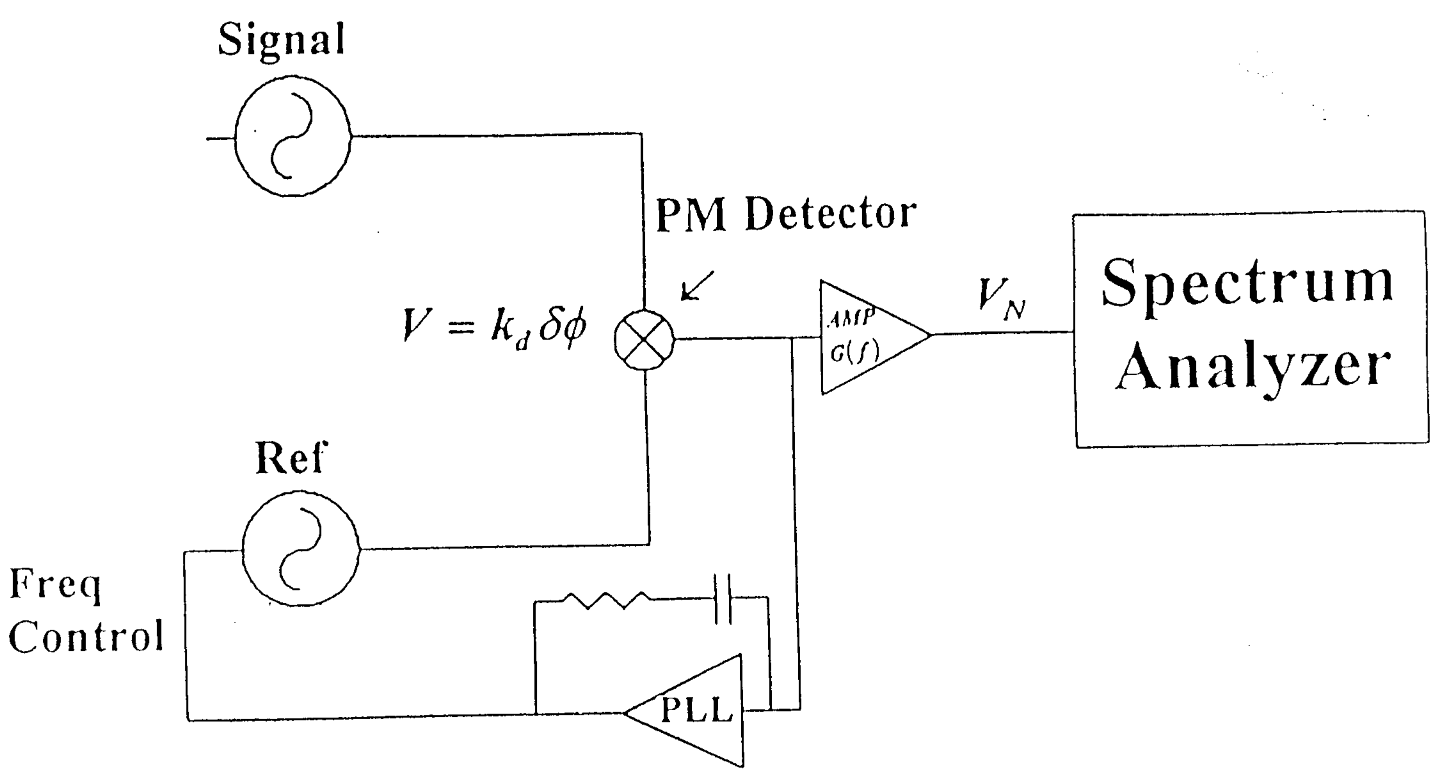
A. Error model for PM noise measurements

B. Error model for AM noise measurements

C. PM and AM noise models

D. Conversion of PM data to $\sigma_y(\tau)$ and mod$\sigma_y(\tau)$

Simple PM Measurements



$\frac{PSDV_N}{[k_d G(f)]^2}$ measures $S_\phi(f)$ of the Signal plus the system noise.

It is difficult to seperate the system noise from a signal with low PM noise. Results uncorrected for PLL and gain variations with Fourier frequency.

ERROR MODEL FOR PM MEASUREMENTS

1. DETERMINATION OF K
2. DETERMINATION OF AMPLIFIER G(f)
3. PLL EFFECTS (IF ANY)
4. CONTRIBUTION OF AM NOISE
5. HARMONIC DISTORTION
6. CONTRIBUTION OF SYSTEM NOISE FLOOR
7. CONTRIBUTION OF REFERENCE NOISE
8. STATISTICAL CONFIDENCE OF DATA
9. LINEARITY OF SPECTRUM ANALYZERS
10. ACCURACY OF PSD FUNCTION

1. DETERMINATION OF K

TRANSDUCER SENSITIVITY DEPENDS ON

A. Frequency

B. Signal power and impedance, reference power and impedance

C. Mixer termination at all three ports

D. Cable lengths

ACCURACY OF DETERMINATION DEPENDS ON DEGREE ABOVE PARAMETERS HELD CONSTANT PLUS

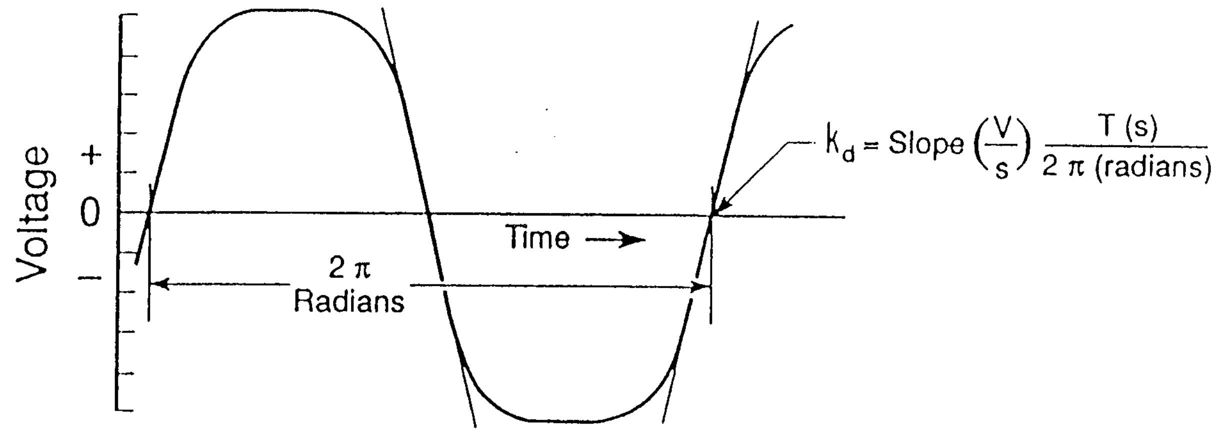
A. Symmetry of waveform

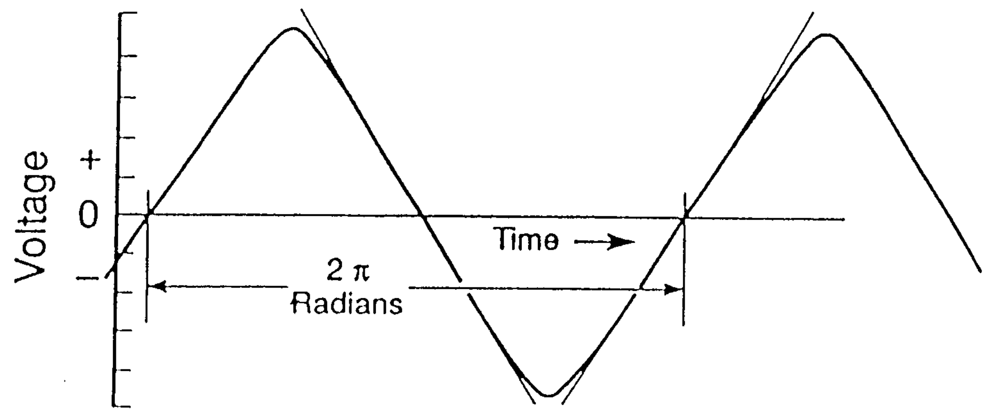
B. Signal-to-noise-ratio

C. Phase deviation from 90°-depends on noise level, dc offset-may depend on f

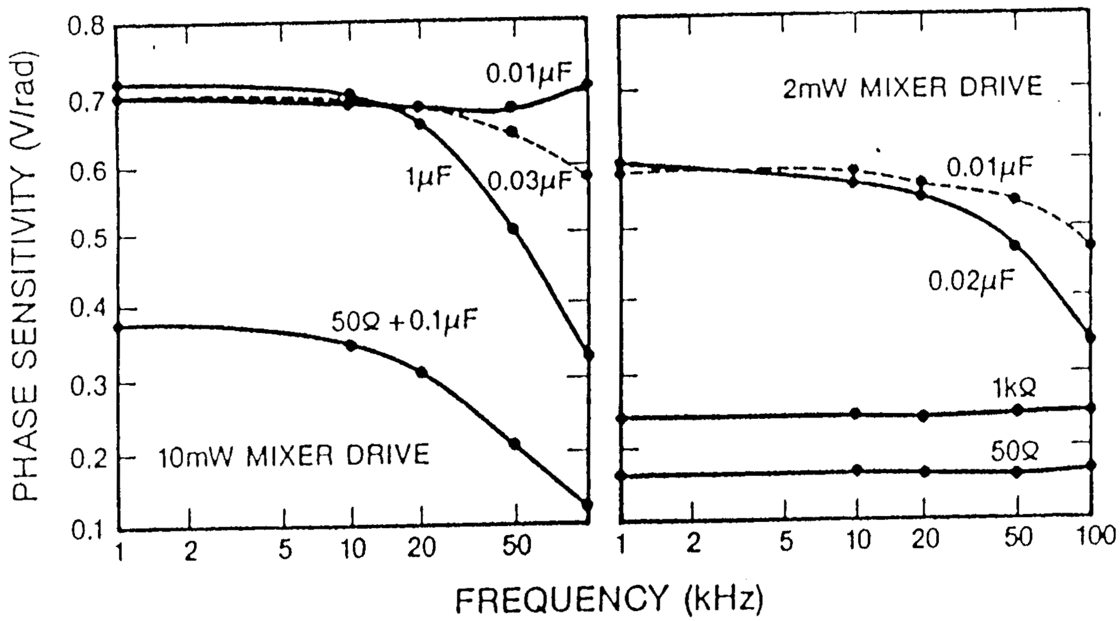
CALIBRATION CONDITION MUST REPLICATE THE MEASUREMENT CONDITION AS CLOSELY AS POSSIBLE

MIXER OUTPUT VOLTAGE VERSUS PHASE (TIME)

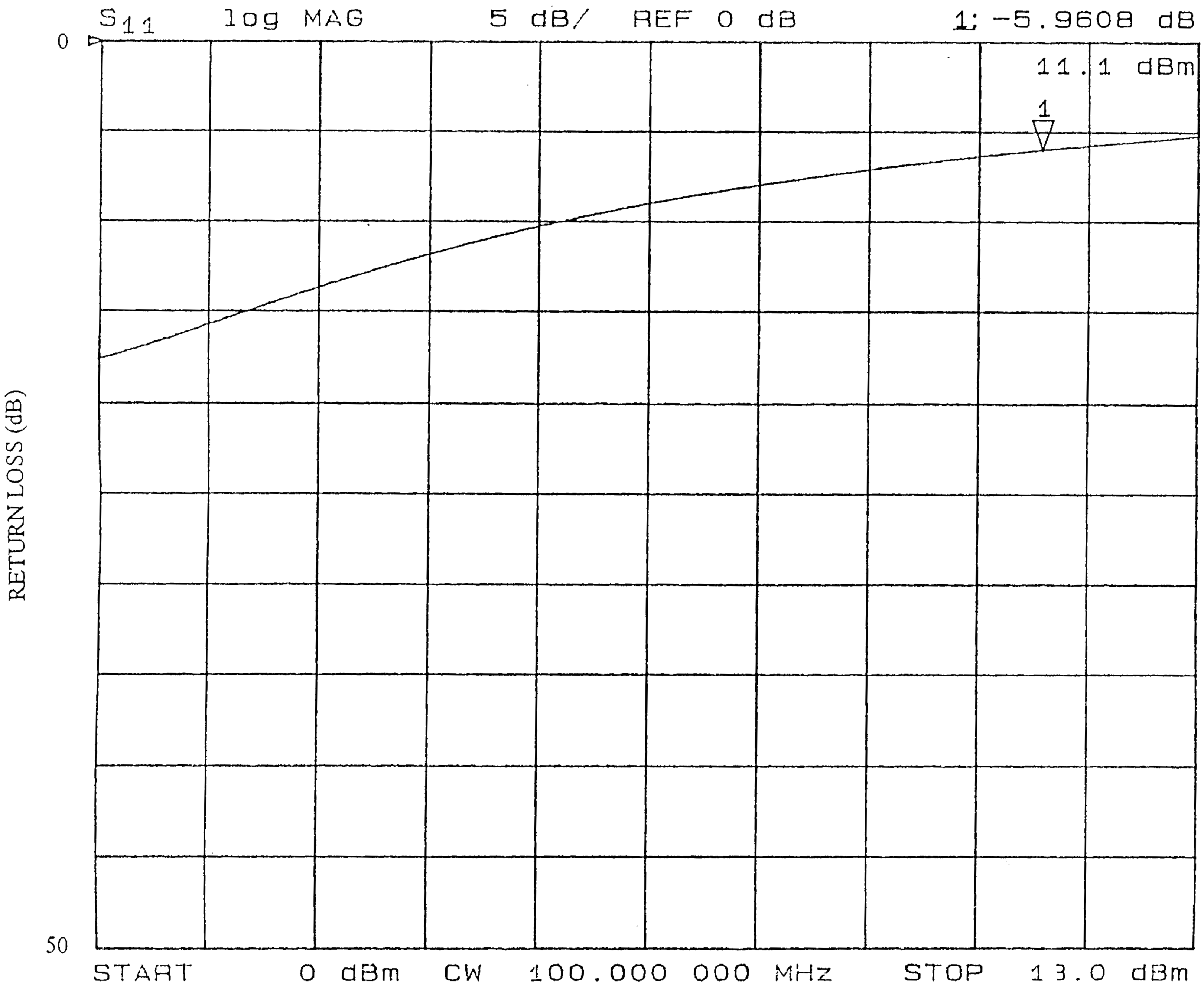




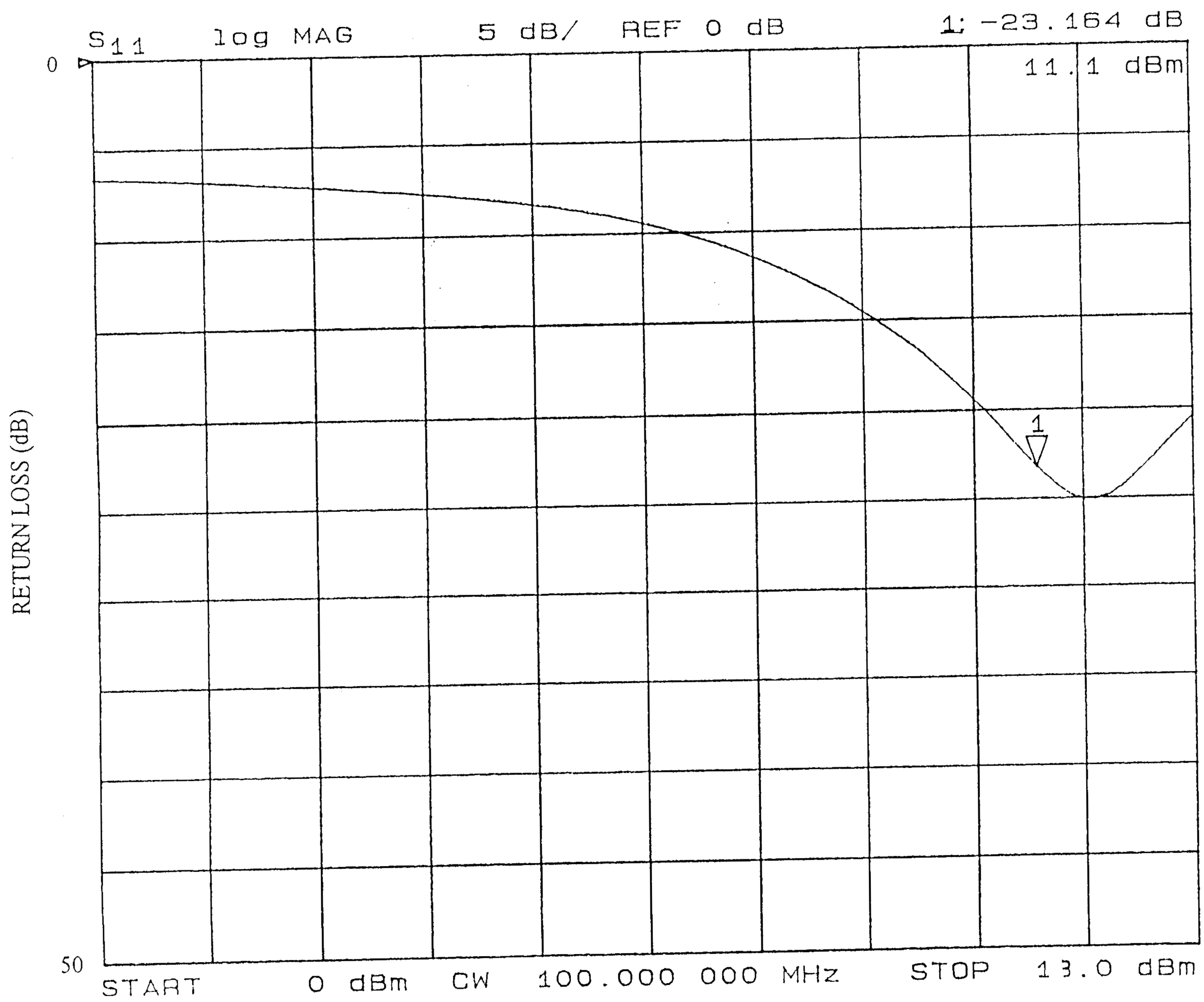
MIXER SENSITIVITY Kd VERSUS IF LOAD



MIXER A RETURN LOSS VERSUS RF POWER AT 100 MHZ AND
A LO OF 15 dBm



MIXER B RETURN LOSS VERSUS RF POWER AT 100 MHZ AND A LO OF 15 dBm



2. DETERMINATION OF AMPLIFIER GAIN VERSUS FOURIER OFFSET

G(f) DEPENDS ON

A. Intrinsic amplifier G(f)

B. Mixer output impedance

C. Signal power, impedance, and cable length through B.

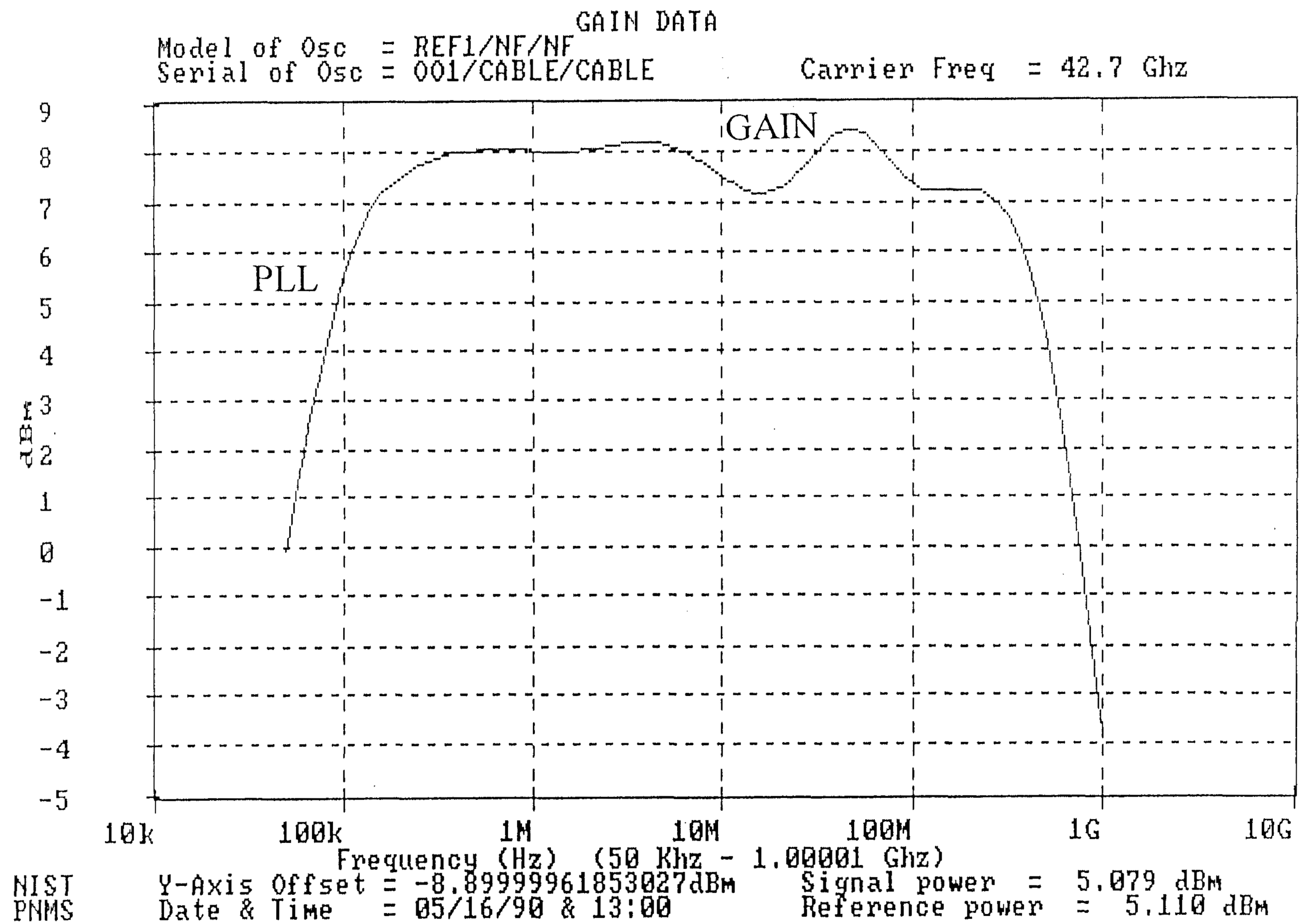
E. Reference power, impedance, and cable length through B.

ACCURACY OF DETERMINATION DEPENDS ON THE DEGREE ABOVE PARAMETERS HELD CONSTANT PLUS

A. Linearity and slewing rate of amplifier

CALIBRATION CONDITION MUST REPLICATE THE MEASUREMENT CONDITION AS CLOSELY AS POSSIBLE

PLL AND GAIN EFFECTS ON G(f) Kd



3. PLL EFFECTS (IF ANY)

ATTENUATION OF THE LOW FREQUENCY PHASE DEVIATION CAN BE REDUCED BY

A. Normal PLL loop. Results may be altered by additional filters in electronic frequency control (EFC) path

B. Signals that propagate through the power sources of the two oscillators

C. Signals that propagate through the air to pull the frequency of one or both signals

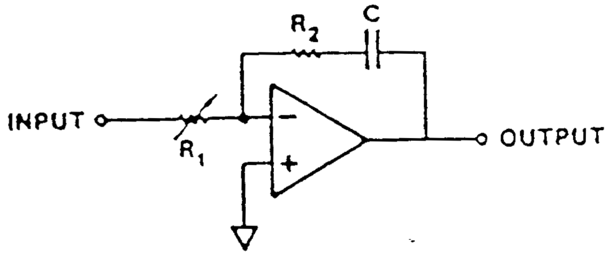
E. Signals that propagate through the measurement system (mixer) to pull the frequency

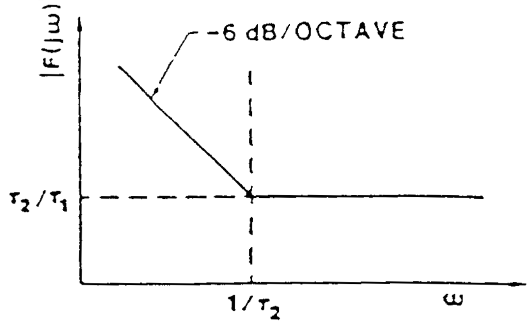
F. Injection lock feedback from the cavity discriminator or delay line discriminator

PLL EFFECTS SHOULD BE MEASURED IN SITU SINCE MANY EFFECTS IN THE EFC PATH ARE HIDDEN.

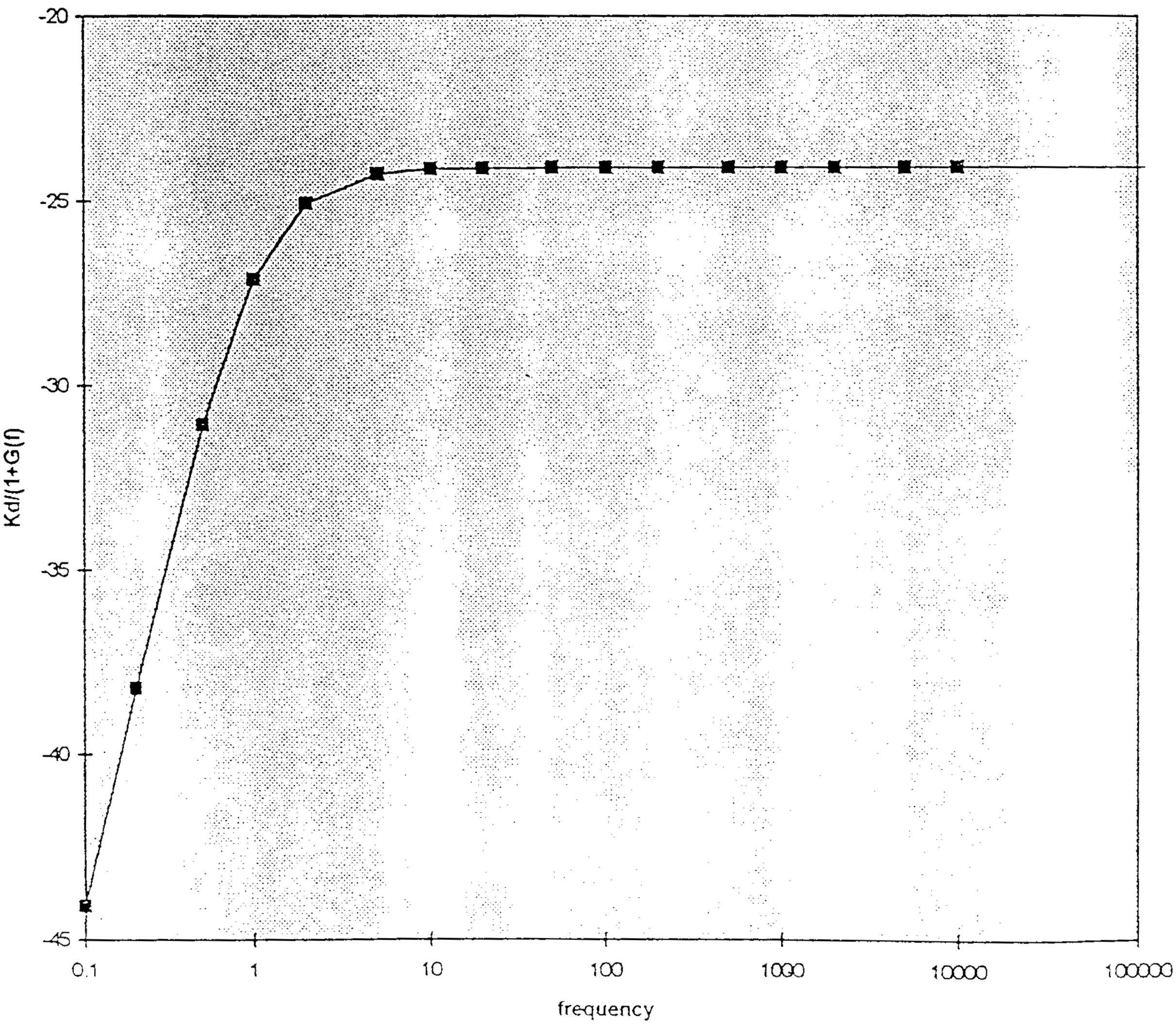
ERRORS IN PARAMETERS 1-3 ARE OFTEN CORRELATED

PLL RELATIONS

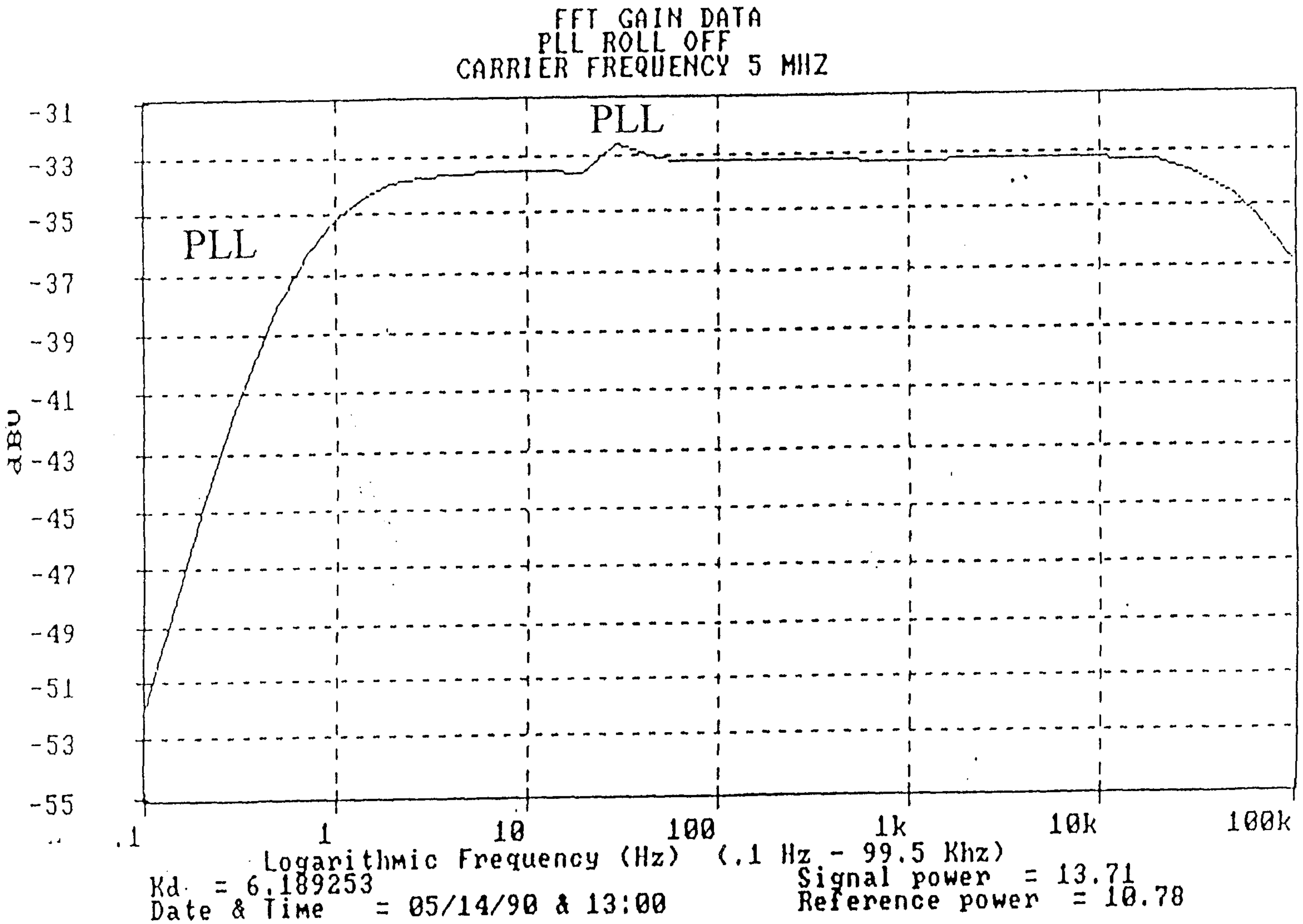




$$G(f)_{PLL} = C\frac{(1 + j\omega R_2 C)}{j\omega R_1 C} \qquad V_d = \frac{K_d(\Delta\phi_{test} - \Delta\phi_{ref})}{1 + G(f)_{PLL}}$$



PLL EFFECTS ON G(f) Kd



4. CONTRIBUTION OF AM NOISE

AM TO PM CONVERSION IS UNIVERSAL

A. Occurs via non-linear process

B. Typically -15 to -25 dB in double balanced mixers

C. Can reach - 3 dB in some amplifiers

D. Sets the noise floor in many measurements

MEASUREMENTS OF $S_\phi(f)$ @ MHz SYNTHESIZER VS OSCILLATOR

f (Hz)	$S_\phi(f)\|_{AB}$ (db/Hz)	$S_a(f)\|_{AB}$ (dB/Hz)	$S_a(f)\|_{A}$ $\beta^2 A$ (dB/Hz)	Measured Noise Floor dB rel Rad^2/Hz	Actual Noise Floor
32	-119.8	-126.0	≈ -151.0	-154.0	-160.0
100	-124.2	-127.0	≈ -152.0	-154.0	-165.0
1 K	-132.1	-132.0	≈ -157.0	-158.0	175.0
10 K	-137.3	-133.0	≈ -158.0	-158.0	175.0
100 K	-136.8	-133.0	≈ -158.0	-158.0	175.0

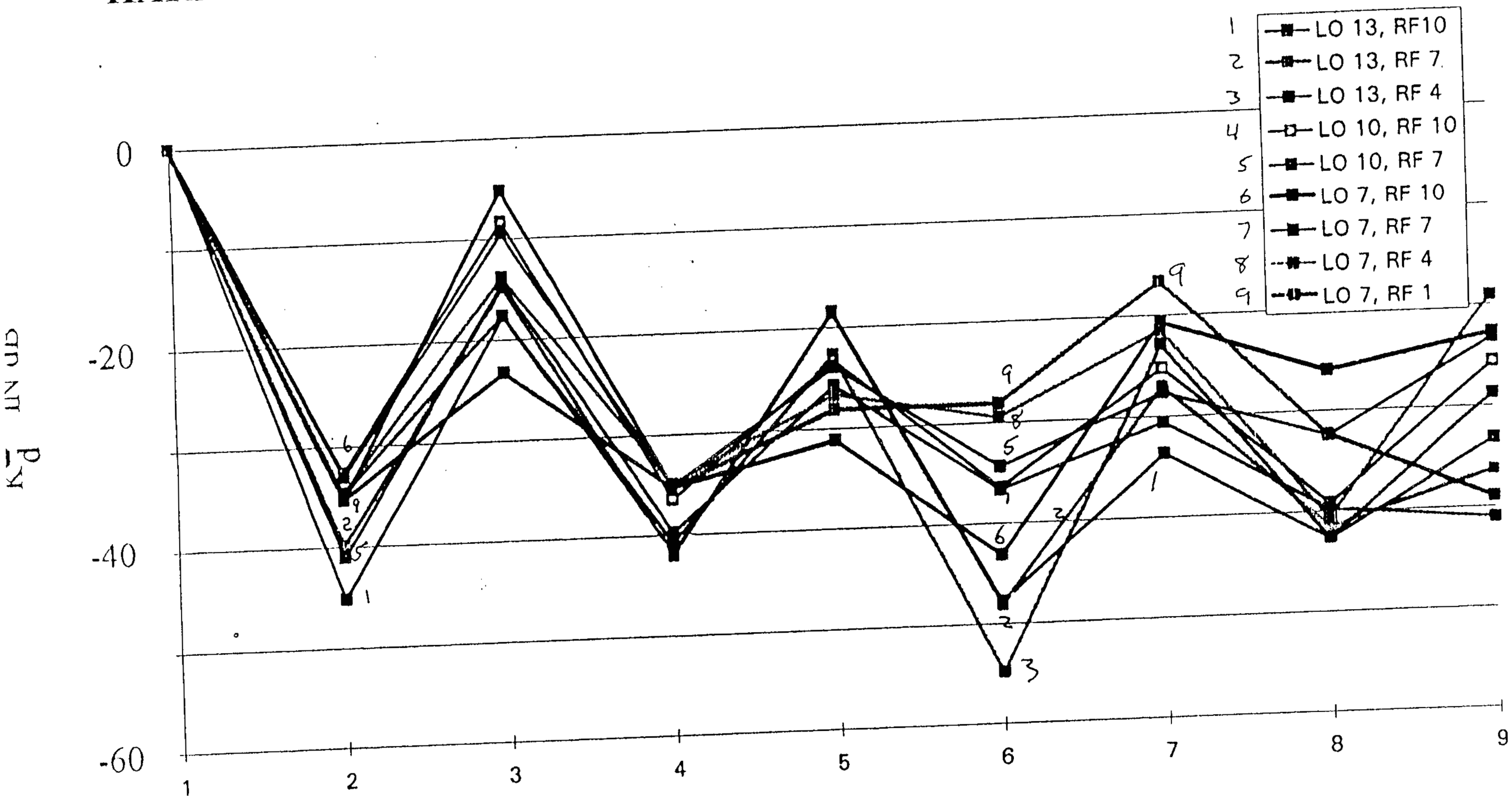
5. HARMONIC DISTORTION

A. Harmonics of signal and reference contribute to K and detected noise

B. PM noise on harmonics may not be same as fundamental

C. Sensitivity depends on power, impedance, harmonic number

HARMONIC SENSITIVITY OF MIXER VS RF AND LO POWER IN dB



TIME AND FREQUENCY DIVISION, NIST

PTTI 1994

6. CONTRIBUTION OF SYSTEM NOISE FLOOR

NOISE TERMS INCLUDED IN $\frac{PSD(V_n)}{K_d^2 G(f)^2}$

$$S_\phi(f) = \frac{[\Delta\phi_A(f) - \Delta\phi_B(f)]^2}{BW} + \frac{V_n(f)^2{}_{mixer}}{K_d^2 BW} + \frac{V_n(f)^2{}_{amp}}{K_d^2 BW}$$

$$+ \frac{V_n(f)^2{}_{SA}}{K_d^2 G(f)^2 BW} + S_{aA}(f)\beta_A^2 + S_{aB}(f)\beta_B^2$$

$$S_\phi(f)_{pair} = S_{\phi A}(f) + S_{\phi B}(f) + \frac{V_n(f)^2{}_{system}}{K_d^2 BW} + S_{aA}(f)\beta_A^2 + S_{aB}(f)\beta_B^2$$

TO GET NOISE FLOOR SET A = B

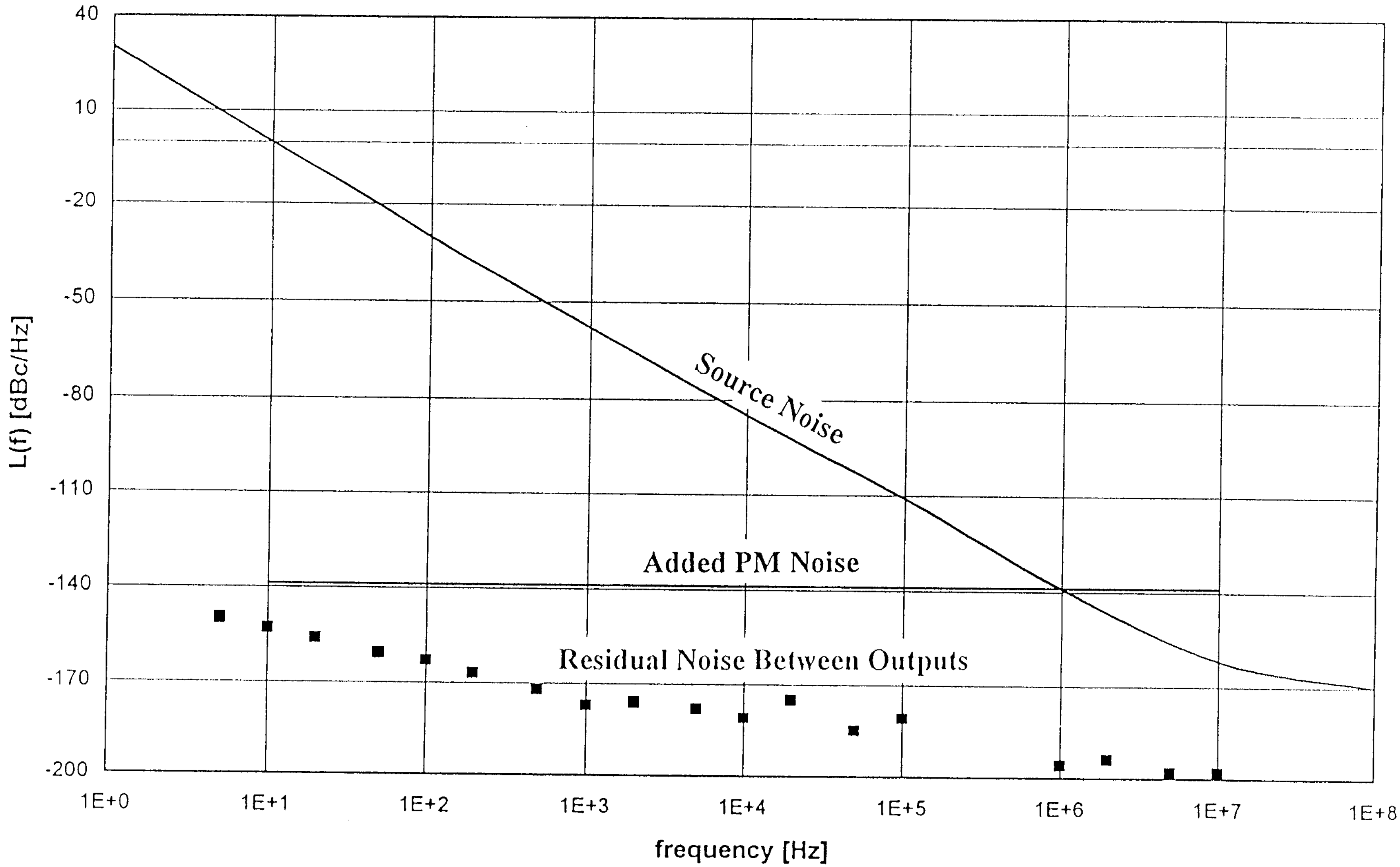
$$S_\phi(f)_{Noise\ Floor} = S_{\phi A}(2\pi f\tau_{delay})^2 + \frac{V_n(f)^2{}_{system}}{K_d{}^2 BW} + S_{aA}(f)\beta_A{}^2 + S_\phi(f)_{power\ splitter}$$

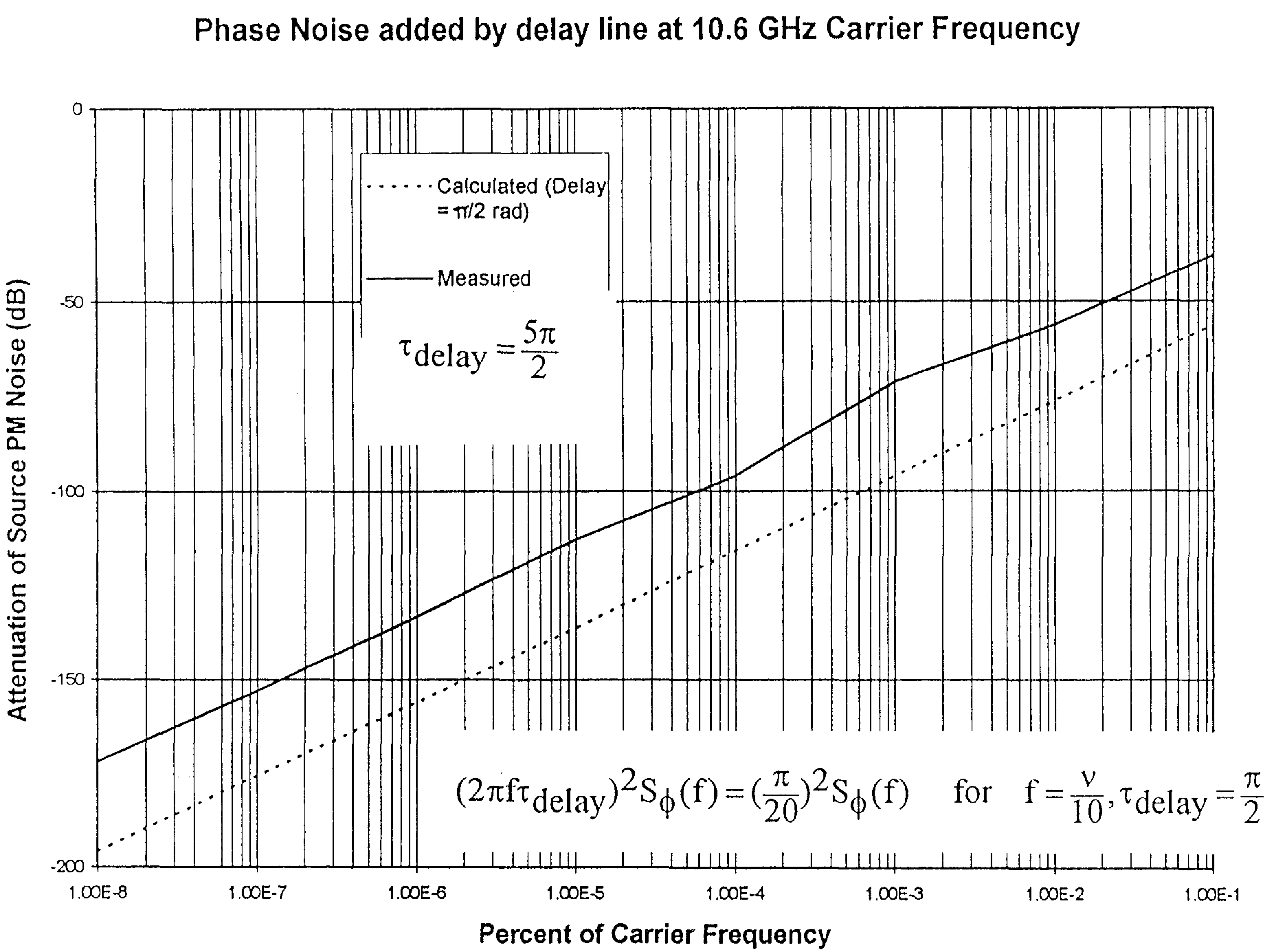
$$(2\pi f\tau_{delay})^2 S_\phi(f) = (\frac{\pi}{20})^2 S_\phi(f) \quad \text{for} \quad f = \frac{\nu}{10}, \tau_{delay} = \frac{\pi}{2}$$

TO CALCULATE INDIVIDUAL PM NOISE FOR AN OSCILLATOR

$$S_\phi(f)_{AB} + S_\phi(f)_{AC} - S_\phi(f)_{BC} = 2S_{\phi A}(f) + \frac{V_n^2}{K_d{}^2 BW} + 2S_{aA}(f)\beta_A{}^2$$

L(f) vs Frequency of NIST PM/AM standard at 10.6 GHz




Phase Noise added by delay line at 10.6 GHz Carrier Frequency
Calculated (Delay = π/2 rad)
Measured
$\tau_{delay} = \frac{5\pi}{2}$
$(2\pi f \tau_{delay})^2 S_\phi(f) = (\frac{\pi}{20})^2 S_\phi(f)$ for $f = \frac{\nu}{10}, \tau_{delay} = \frac{\pi}{2}$
Attenuation of Source PM Noise (dB)
0
-50
-100
-150
-200
1.00E-8
1.00E-7
1.00E-6
1.00E-5
1.00E-4
1.00E-3
1.00E-2
1.00E-1
Percent of Carrier Frequency

7. CONTRIBUTION OF REFERENCE AM AND PM NOISE

NOISE TERMS INCLUDED IN $\frac{PSD(V_n)}{K_d^2 G(f)^2}$

$$S_\phi(f) = \frac{[\Delta\phi_A(f) - \Delta\phi_B(f)]^2}{BW} + \frac{V_n(f)^2{}_{mixer}}{K_d^2 BW} + \frac{V_n(f)^2{}_{amp}}{K_d^2 BW}$$

$$+ \frac{V_n(f)^2{}_{SA}}{K_d^2 G(f)^2 BW} + S_{aA}(f)\beta_A^2 + S_{aB}(f)\beta_B^2$$

$$S_\phi(f)_{pair} = S_{\phi A}(f) + S_{\phi B}(f) + \frac{V_n(f)^2{}_{system}}{K_d^2 BW} + S_{aA}(f)\beta_A^2 + S_{aB}(f)\beta_B^2$$

8. STATISTICAL CONFIDENCE OF THE DATA

Table 1. Approximate : 68% confidence Intervals for FFT Spectral Estimates $N > 10$

power law		window	
noise type	uniform	Hanning	flattened peak
f^{0}	$1.02/\sqrt{N}$	$0.98/\sqrt{N}$	$0.98/\sqrt{N}$
f^{-2}	$1.02/\sqrt{N}$	$1.04/\sqrt{N}$	$1.04/\sqrt{N}$
f^{-3}	unusable	$1.04/\sqrt{N}$	$1.04/\sqrt{N}$
f^{-4}	unusable	$1.04/\sqrt{N}$	$1.04/\sqrt{N}$

$$S = S_m\left(1 \pm \frac{B}{\sqrt{N}}\right)$$

STATISTICAL UNCENTAINTY OF FFT SPECTRAL DENSITY MEAUREMENTS

$S_m(f) = S(f)\,[1 \pm k/N^{1/2}]$

$k = 1 \rightarrow 68\%$, $k = 1.9 \rightarrow 95\%$ CONFIDENCE $\quad N \geq 10$

N = number of samples averaged

Number of Samples	k = 1 (approx. 68%) $S_m = S[1\pm\delta]$, $S_m\,{}^{-\gamma}_{+\beta}$ dB			k = 1.9 (approx. 95%) $S_m = S[1\pm\delta]$, $S_m\,{}^{-\gamma}_{+\beta}$ dB		
	δ	γ	β	δ	γ	β
4	0.54	-2 ,	+3.3	2.5	-3 ,	+6
6	0.42	-1.5,	+2.3	1.4	-2.5,	+5
10	0.32	-1.2,	+1.7	0.61	-2.1,	+4
30	0.18	-0.72,	+ .86	0.35	-1.3,	+1.8
100	0.1	-0.41,	+0.46	0.19	-0.76,	+0.92
200	0.058	-0.24,	+0.25	0.14	-0.46,	+0.51
1000	0.032	-0.13,	+0.13	0.06	-0.26,	+0.28
3000	0.018	-0.08,	+0.08	0.035	-0.15,	+0.15
10000	0.01	-0.04,	+0.04	0.019	-0.08,	+0.08

D. B. Percival and A.T. Walden,"Spectral Analysis for Physical Application," Cambridge Univ. Press, 1993.

B. N. Taylor and C. E. Kuyatt, NIST Technical Note TN1297, 1993.

STATISTICAL UNCENTAINTY OF SWEPT RF SPECTRAL DENSITY MEAUREMENTS

$S_m(f) \; = \; S(f)\,[1 \pm k\,(VIDEO_{BW}/NRES_{BW})^{1/2}]$

$k = 1 \rightarrow 68\%$, $k \approx 1.9 \rightarrow 95\%$ CONFIDENCE $\quad N \geq 10$

$VIDEO_{BW}$ = video bandwidth

N = number of sweeps averaged

RES_{BW} = resolution bandwidth $\leq f/10$

$\frac{NRES_{BW}}{VIDEO_{BW}}$	k = 1 (approx. 68%) $S_m = S[1\pm\delta]$, $S_m\,{}^{-\gamma}_{+\beta}$ dB			k = 1.9 (approx. 95%) $S_m = S[1\pm\delta]$, $S_m\,{}^{-\gamma}_{+\beta}$ dB		
	δ	γ	β	δ	γ	β
4	0.54	-2 ,	+3.3	2.5	-3 ,	+6
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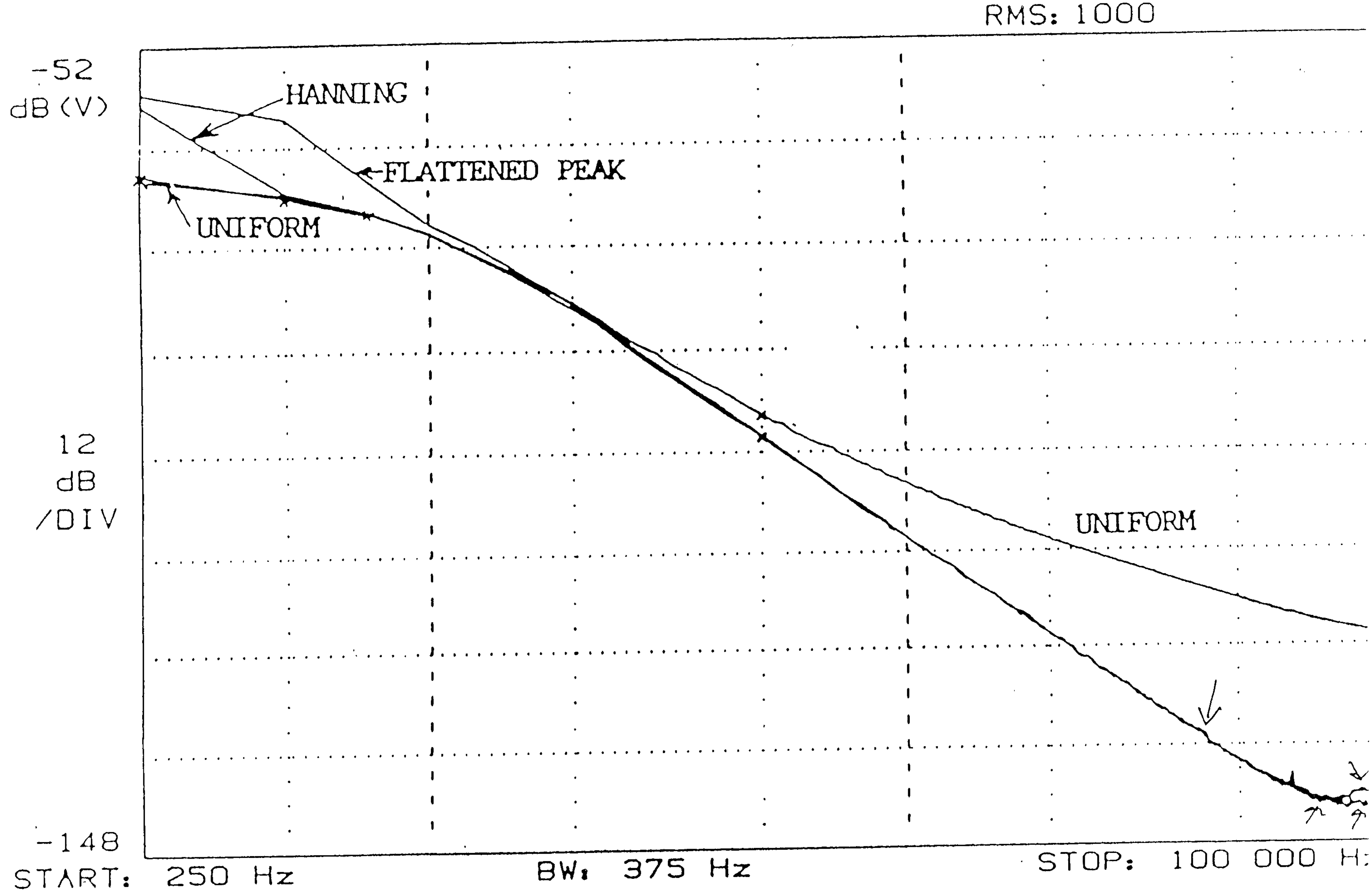
9. LINEARITY OF SPECTRUM ANALYZER

A. Accuracy of wide dynamic range

B. Digitizing errors

C. Need to segment spectrum with filters

DYNAMIC RANGE AND DIGITIZING ERRORS



10. ACCURACY OF THE PSD FUNCTION

DEPENDS ON

A. Signal type

Use flat top window for bright lines

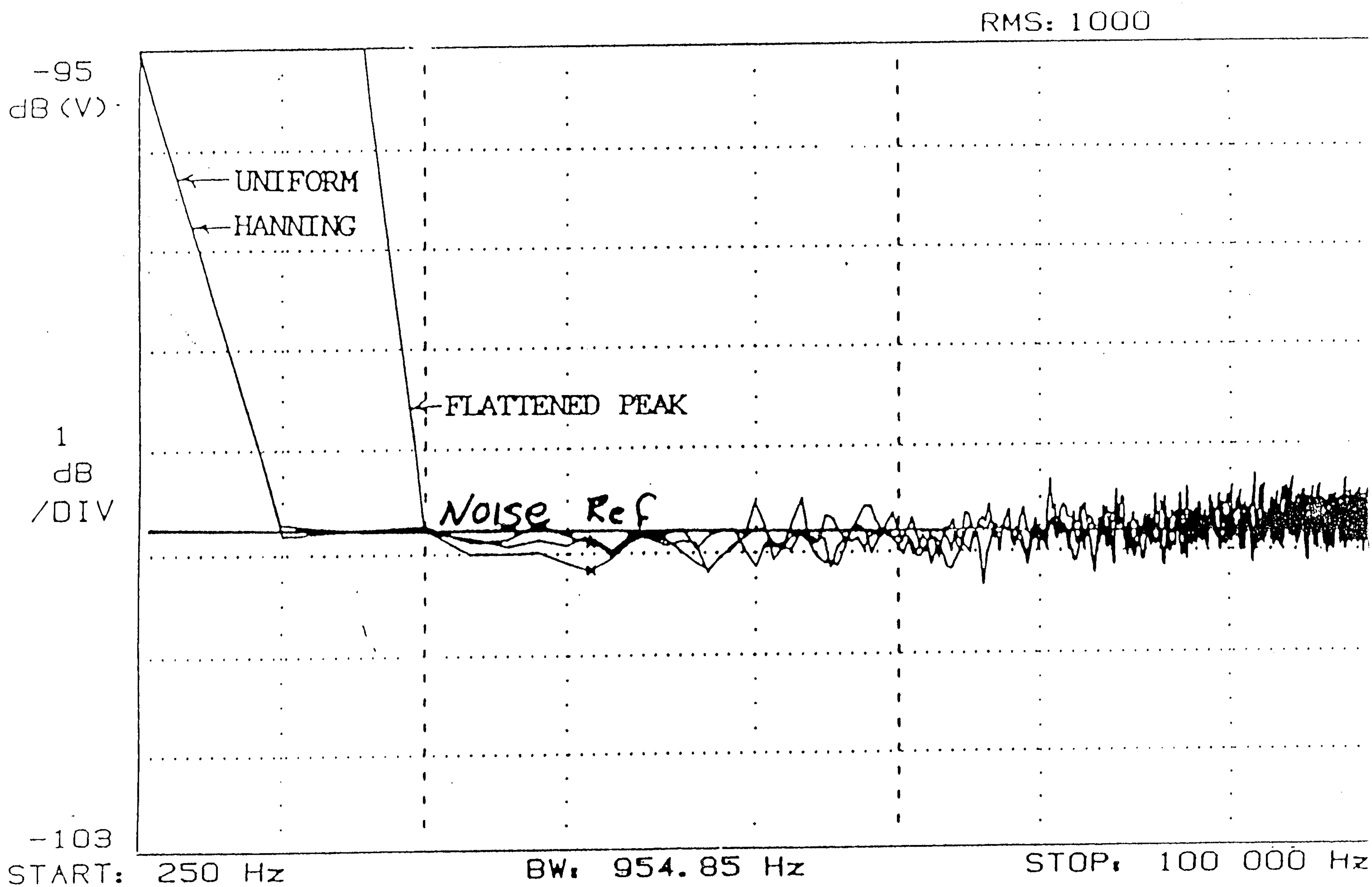
Use Hanning window for noise

B. Window function and Fourier frequency (leakage)

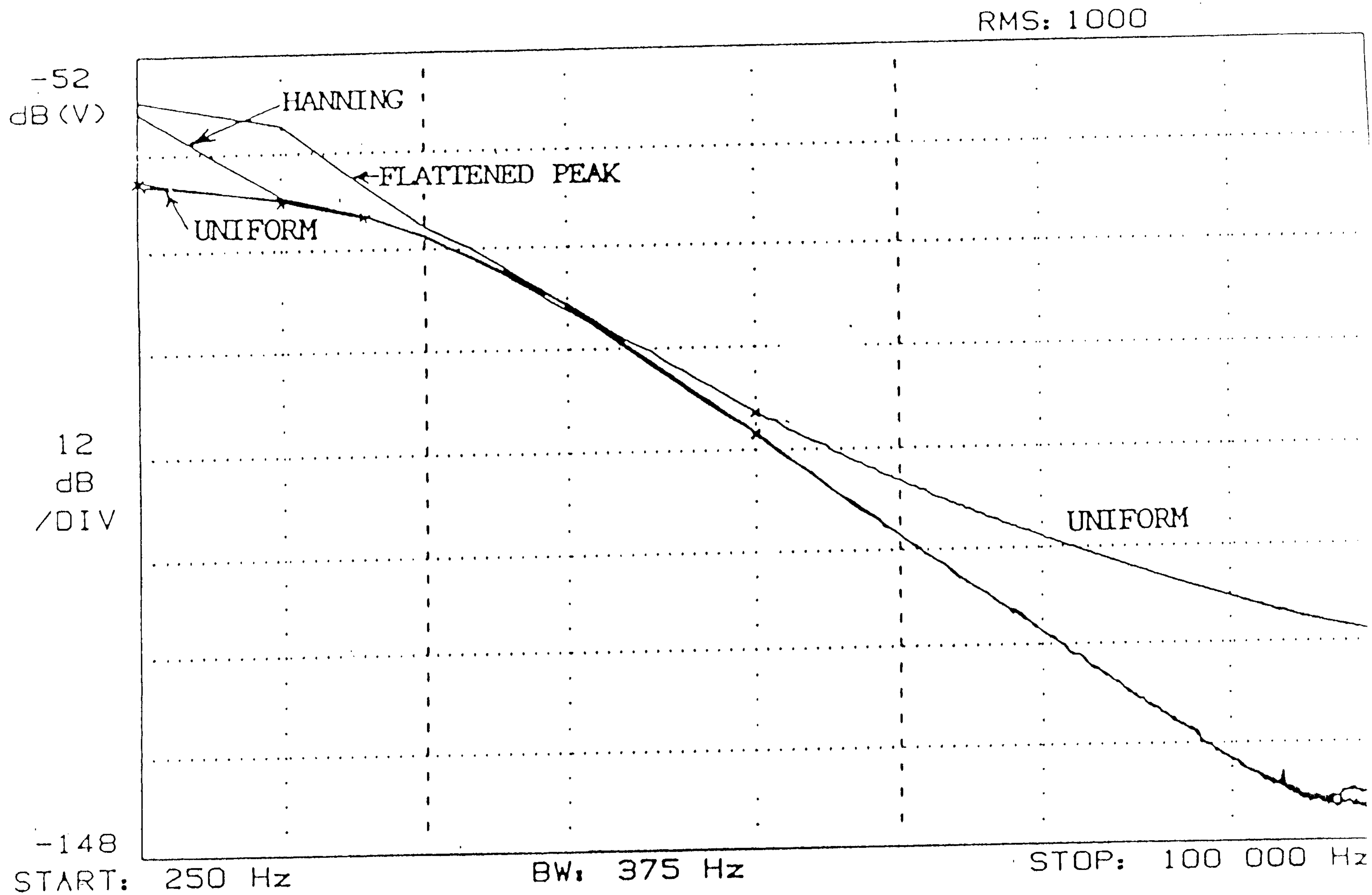
f should be less than span/23 for Flat top window

f should be less than span/75 for Flat top window

PSD OF f^0 NOISE



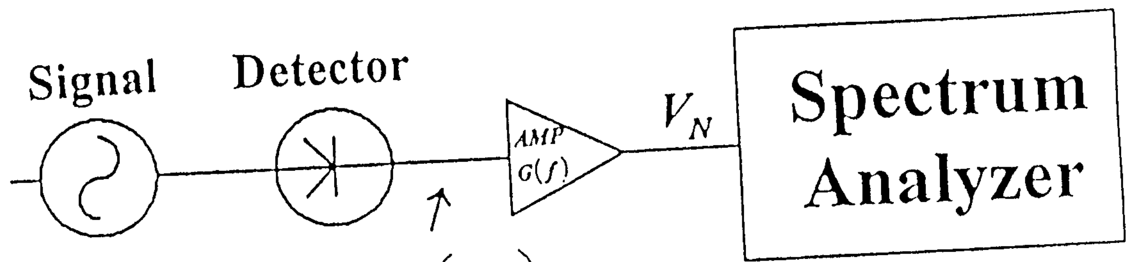
PSD OF f^4 NOISE



APPROXIMATE BIASES IN FFT SPECTRAL DENSITY ESTIMATORS

	Noise Type f^0			Noise Type f^{-4}		
Channel #	Flat Top	Hanning	Uniform	Flat Top	Hanning	Uniform
1	20.1 dB	19.6 dB	19.6 dB	10.0 dB	8.6 dB	Not Useable
2	16.7	Small	Small	9.1	0.4	
3	7.22	↓	↓	4.0	0.4	
4	Small			1.2	Small	
5	↓			1.1	↓	
6				1.1		
7				1.0		
8				0.8		
9				0.6		
10				0.6		
11				0.5		
12				0.4		
13				0.4		
14				Small		
15				↓		

Simple AM Measurement



$$V(t) = k_a \delta\left(\frac{\varepsilon}{V_0}\right)$$

$\frac{PSD\,V_N}{[k_a G(f)]^2}$ measures $S_a(f)$ of the Signal plus the system noise.

It is difficult to seperate the system noise from a signal with low AM noise.

ERROR MODEL FOR AM MEASUREMENTS

1. DETERMINATION OF K
2. DETERMINATION OF AMPLIFIER G(f)
3. CONTRIBUTION OF SYSTEM NOISE FLOOR
4. STATISTICAL CONFIDENCE OF DATA
5. LINEARITY OF SPECTRUM ANALYZERS
6. ACCURACY OF PSD FUNCTION

1. DETERMINATION OF Ka

DETECTOR SENSITIVITY DEPENDS ON

A. Carrier frequency

B. Signal power and impedance

C. Detector termination both ports

D. Cable lengths

E. Fourier frequency

Sensitivity to Fourier frequency is often difficult to measure due to bandwidth of most AM modulators

CALIBRATION CONDITION MUST REPLICATE THE MEASUREMENT CONDITION AS CLOSELY AS POSSIBLE

2. DETERMINATION OF AMPLIFIER G(f)

Depends on

A. Detector output impedance

B. Signal power, impedance, and cable length through A

C. Fourier frequency

CALIBRATION CONDITION MUST REPLICATE THE MEASUREMENT CONDITION AS CLOSELY AS POSSIBLE

3. CONTRIBUTION OF AM SYSTEM NOISE FLOOR

A. Noise floor difficult to measure in single channel systems

B. Cross-correlation can be used to determine noise floor (part III)

CALIBRATION CONDITION MUST REPLICATE THE MEASUREMENT CONDITION AS CLOSELY AS POSSIBLE

NOISE MODEL OF AMPLIFIERS

AM and PM similar 1/f + thermal

NOISE MODEL OF OSCILLATORS

PM complicated-see examples

PM typically includes $1/f^3$ + thermal

AM depends on circuit and degree of limiting

AM sometimes 1/f + attenuated thermal

NOISE MODEL OF PM MEASUREMENT SYSTEMS

1/f + thermal for two oscillator type

$1/f^3$ + thermal for single oscillator type

NOISE MODELS OF AM DETECTORS

1/f + thermal

MODEL FOR PM IN AMPLIFIERS

PM AM

$$S_\phi(f) = \left[\frac{\alpha_E}{f} + \frac{2kTFG}{P}\right] \Rightarrow S_a(F)$$

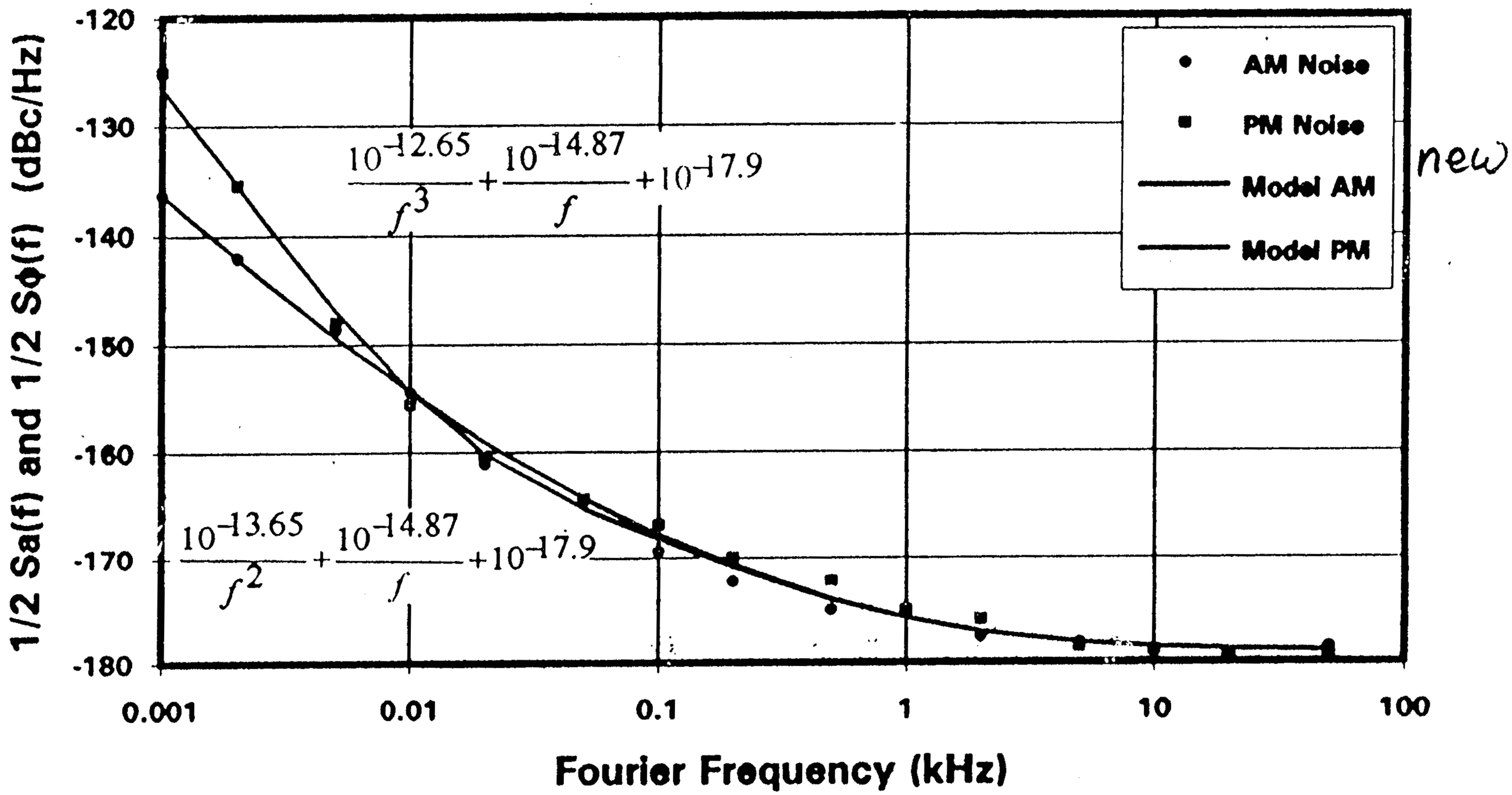
NEW

LEESON'S MODEL FOR PM IN OSCILLATORS

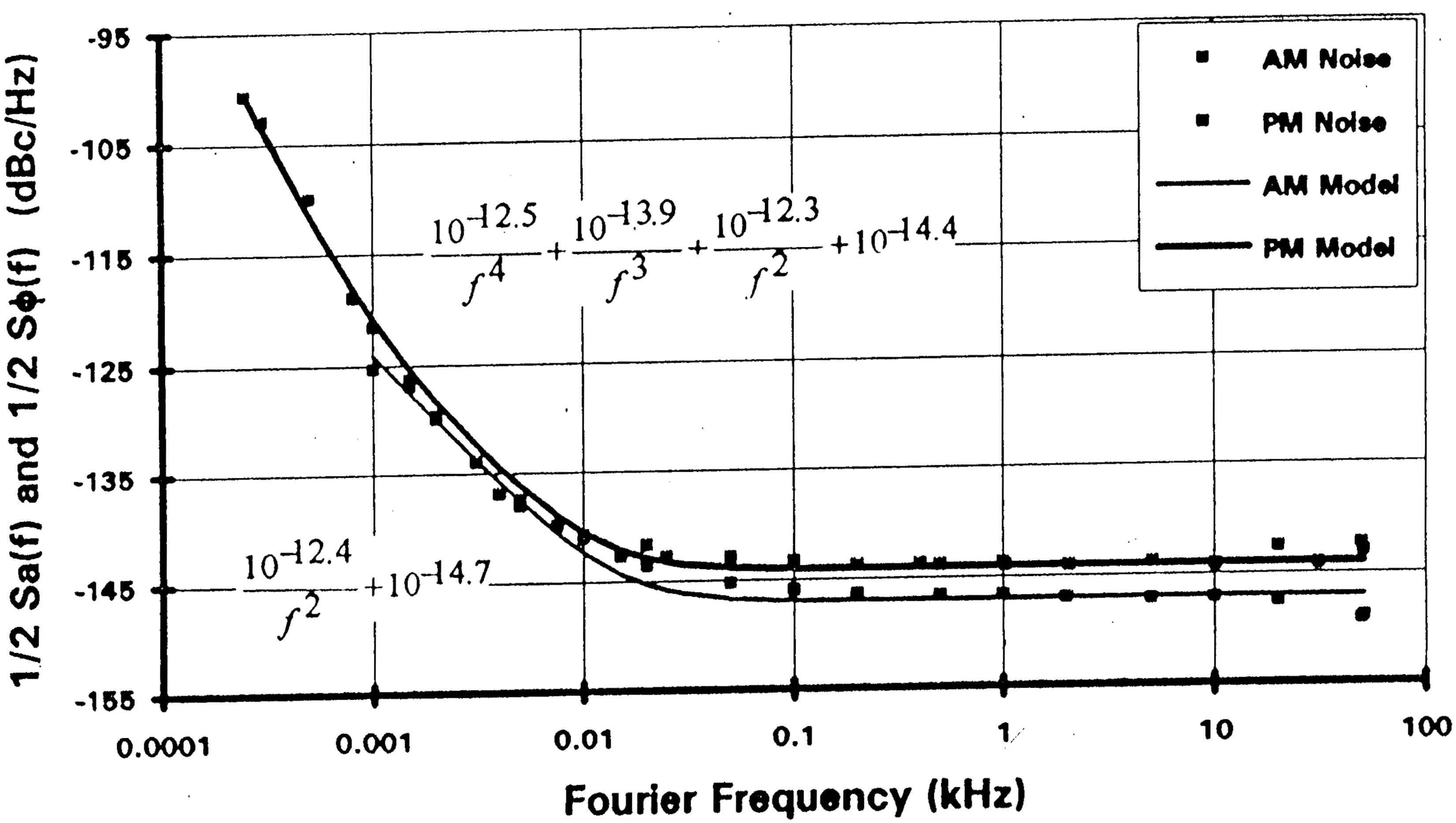
$$S_\phi(f) = \left(\frac{\upsilon_o}{2Q_L}\right)^2 \frac{1}{f^2}\left[\frac{\alpha_E}{f} + \frac{2kTFG}{P}\right] + \left[\frac{\alpha_E}{f} + \frac{2kTFG}{P}\right] + \left(\frac{\upsilon_0}{2Q_L}\right)^2 \frac{1}{f^2}\left[\frac{\alpha_R}{f}\right]$$

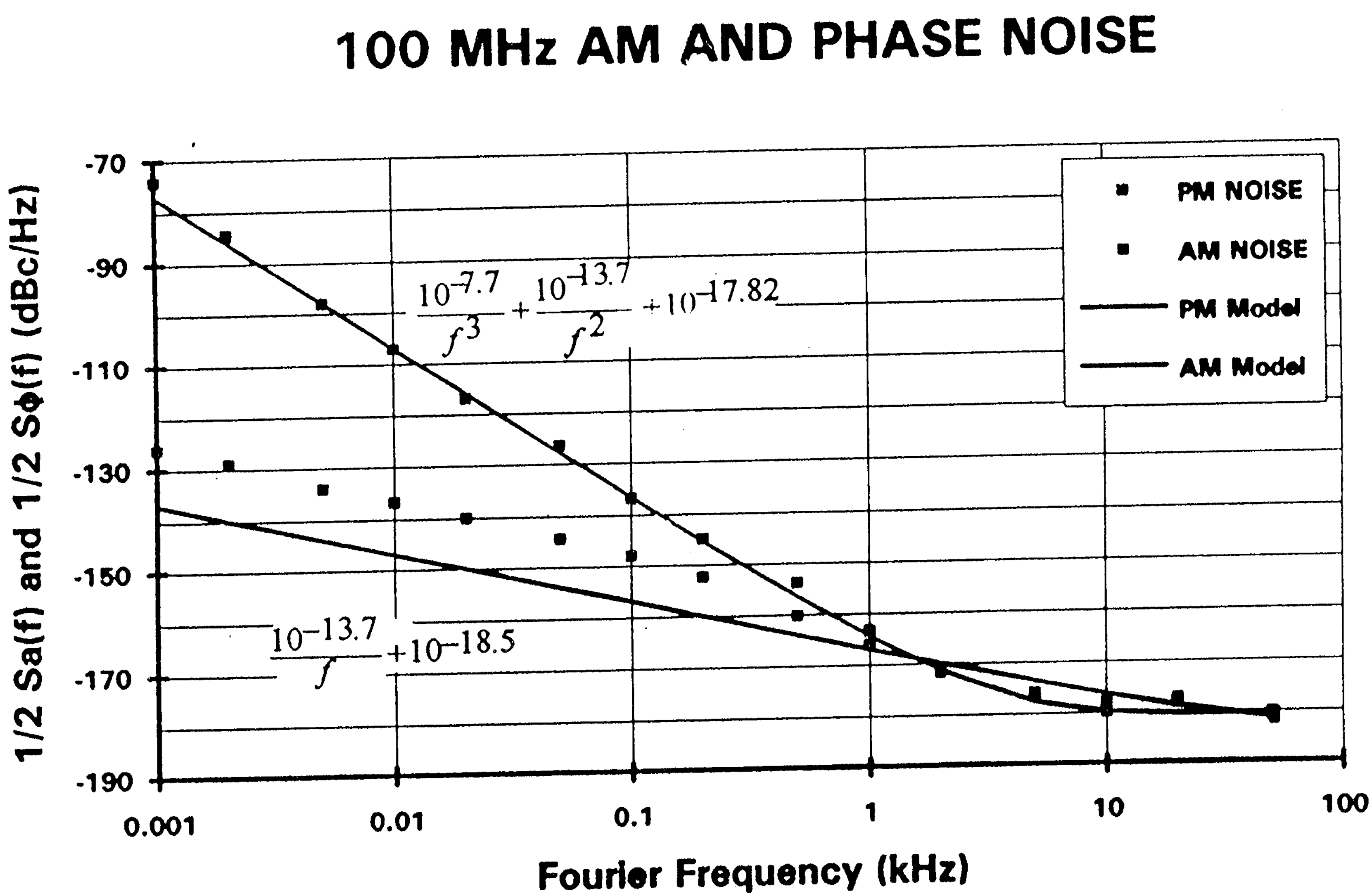
	Amplifier		Resonator
$BW = \upsilon_0/2Q_L$	f<BW	f>BW	f<BW

5 MHz AM and PM Noise



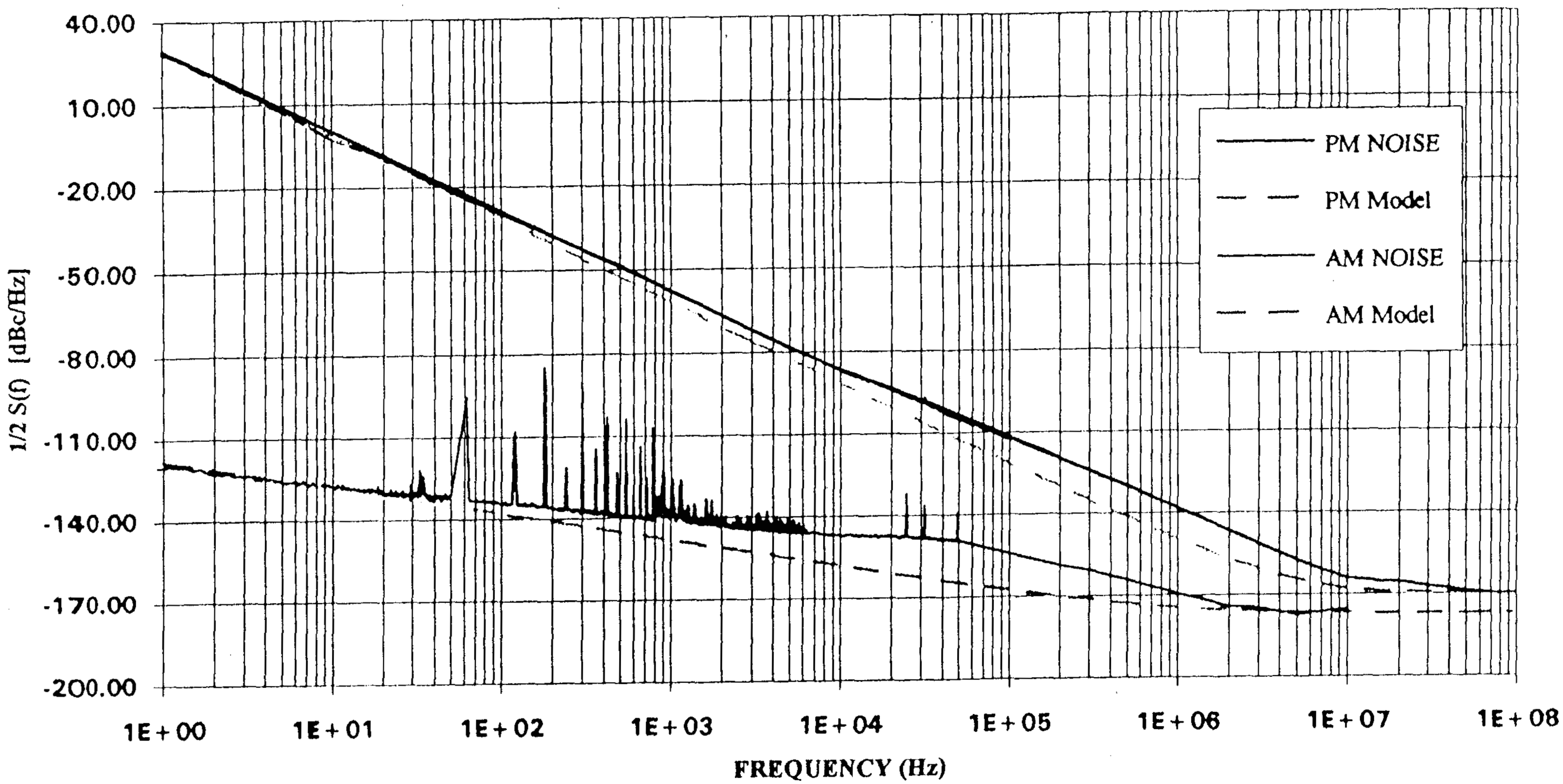
5 MHz AM and Phase Noise





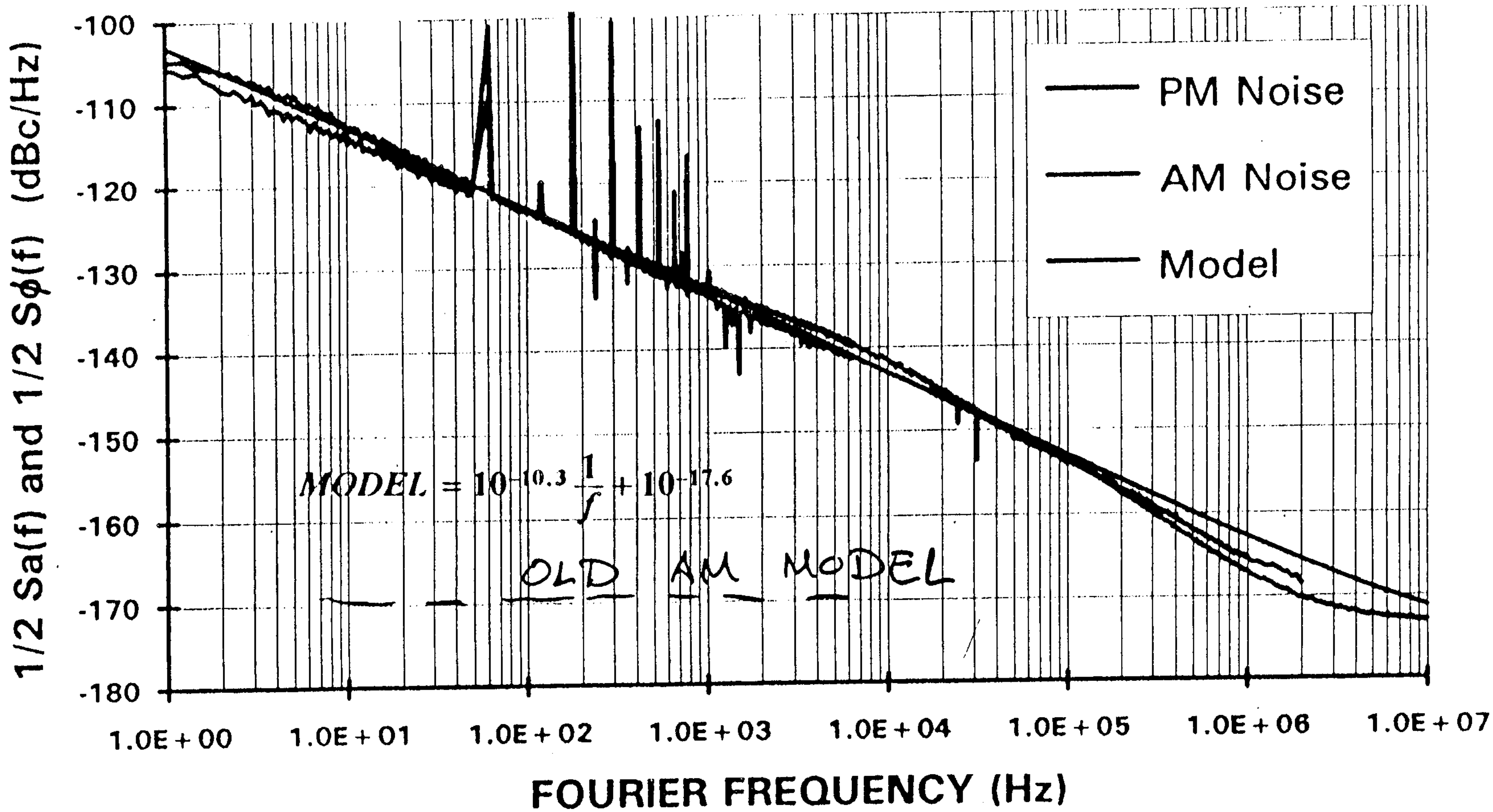
100 MHz AM AND PHASE NOISE
1/2 Sa(f) and 1/2 Sϕ(f) (dBc/Hz)
-70
-90
-110
-130
-150
-170
-190
0.001
0.01
0.1
1
10
100
Fourier Frequency (kHz)
PM NOISE
AM NOISE
PM Model
AM Model
$\frac{10^{-7.7}}{f^3}+\frac{10^{-13.7}}{f^2}+10^{-17.82}$
$\frac{10^{-13.7}}{f}+10^{-18.5}$

PM AND AM NOISE IN DRO 247



AM AND PM NOISE IN AMPLIFIER #1

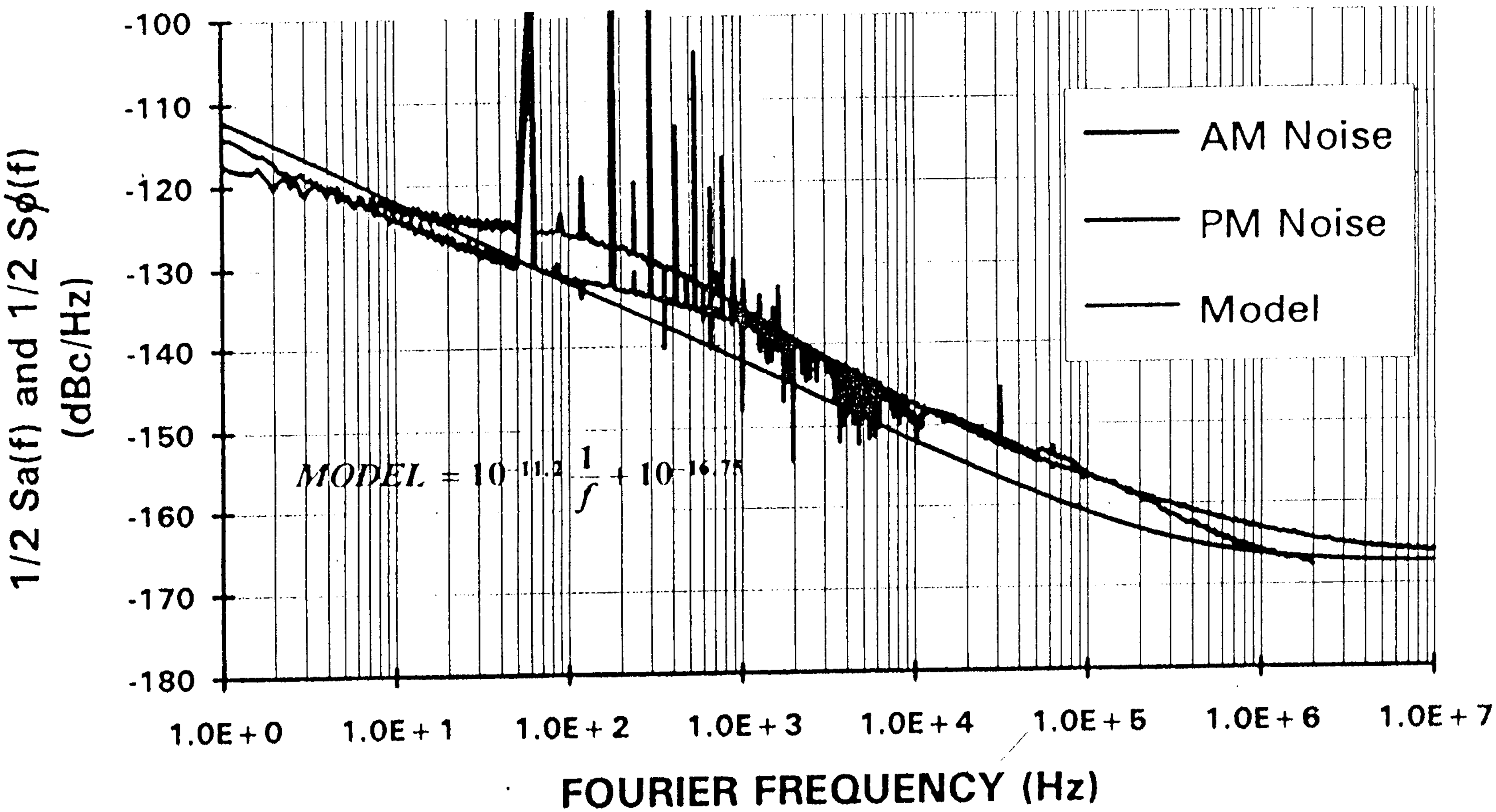
10 G 117



$$\frac{1}{2}S_\phi(f) = \frac{1}{2}S_a(f) = \alpha_E \frac{1}{f} + \frac{2kTFG(f)}{P_o}$$

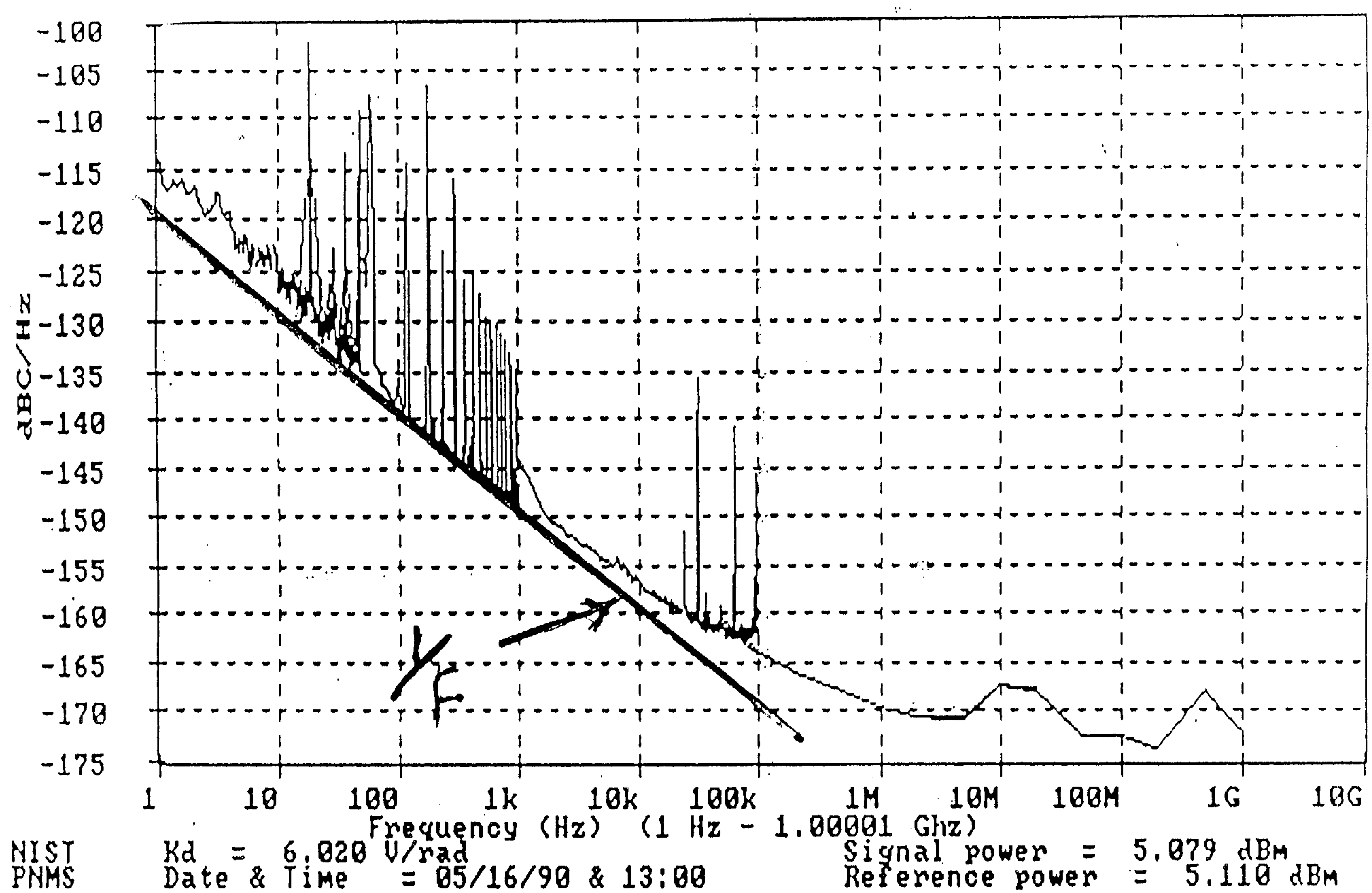
AM AND PM NOISE IN AMPLIFIER #2

10 GHz

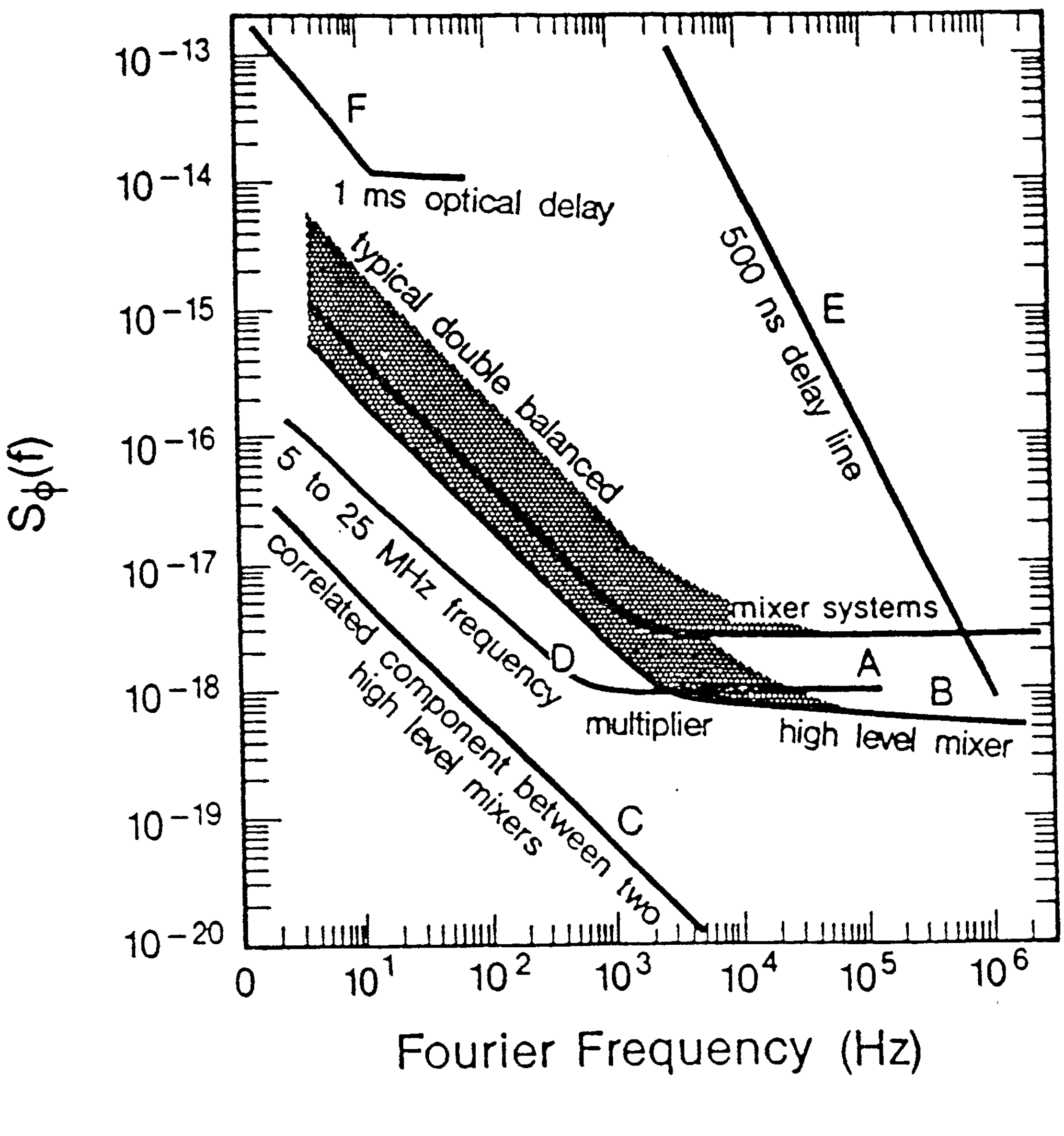


$$\frac{1}{2} S_\phi(f) = \frac{1}{2} S_a(f) = \alpha_E \frac{1}{f} + \frac{2kTFG(f)}{P_o}$$

NOISE FLOOR OF WIDE-BAND NIST PM MEASUREMENT SYSTEM AT 42 GHz



Comparison of Noise Floor for Different Techniques



PHASE NOISE RELATIONSHIPS

$$S_\phi(f) = \mathcal{L}(\nu_o - f) + \mathcal{L}(\nu_o + f)$$

$$dBC/Hz = 10 \log \mathcal{L}(f)$$

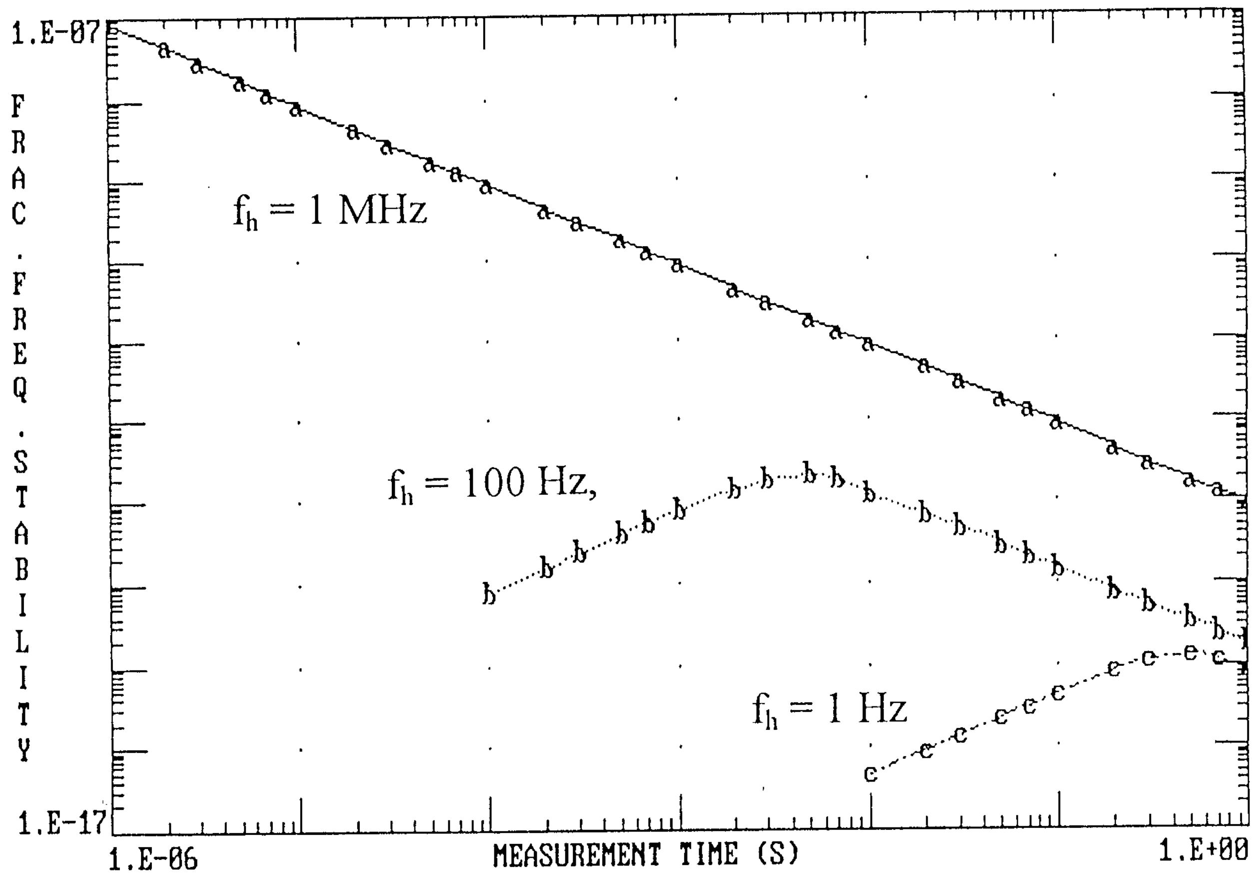
$$S_\phi(f) = \frac{\nu_o^2}{f^2} S_y(f) \quad rad^2/Hz \quad 0<f<\infty$$

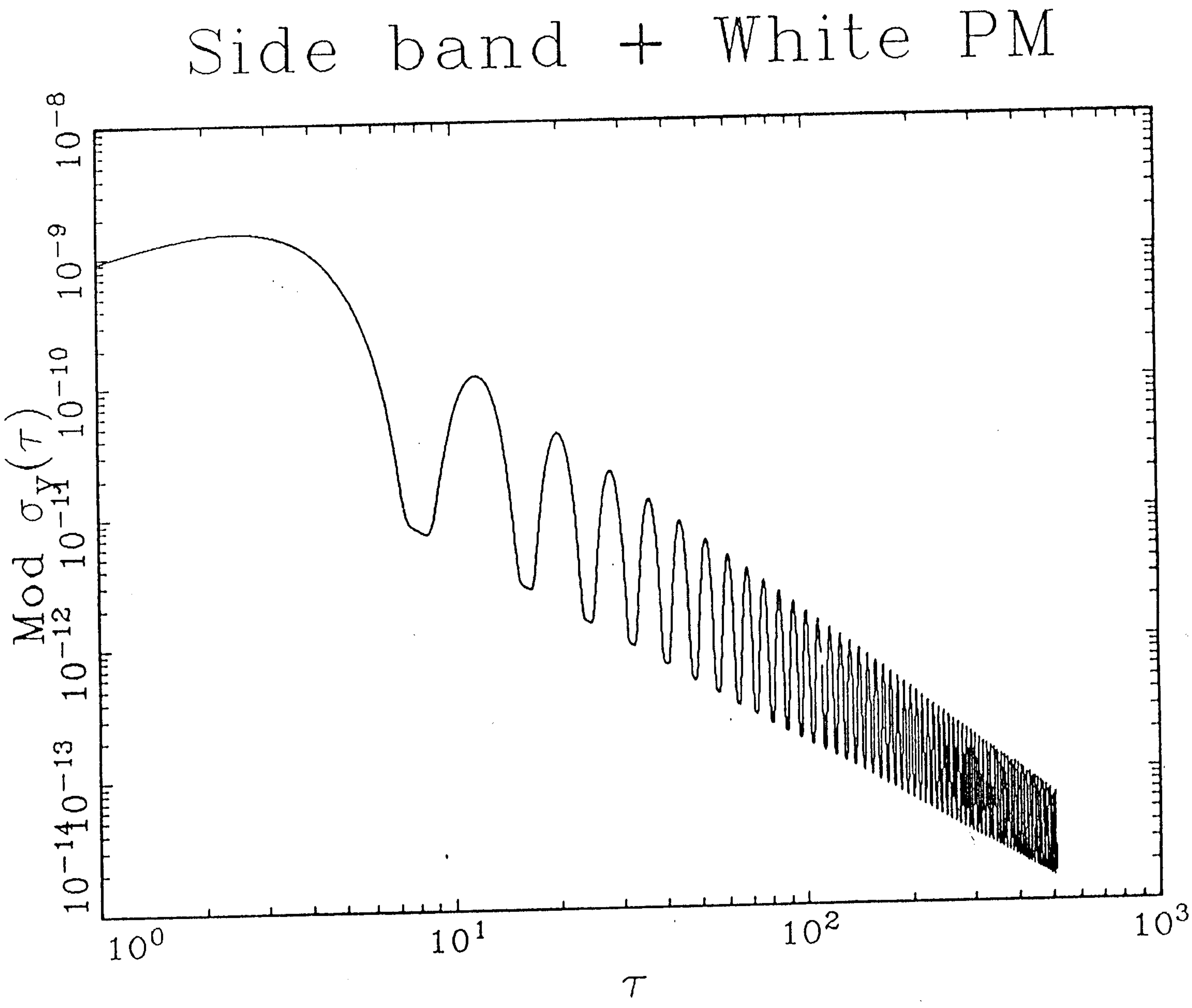
$$\sigma_y^2(\tau) = 2\int_0^\infty df\, S_y(f) \frac{\sin^4(\pi f \tau)}{(\pi f \tau)^2}$$

$$Mod\ \sigma_y(n\tau_o) = \left(\frac{2}{n^2(\pi n \tau_o)^2} \int_0^{f_h} S_y(f) \frac{\sin^6(\pi f n \tau_o)}{f^2 \sin^2(\pi f \tau_o)}\, df \right)^{1/2}$$

CONVERSION OF $S_\varphi(f)$ TO $\sigma_y(\tau)$ FOR

$$S_\phi(f) = \frac{4X10^{-16}}{f} + 1X10^{-17} \text{ AT 100 MHz}$$





Side band + White PM
Mod $\sigma_y(\tau)$
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{0}
10^{1}
10^{2}
10^{3}
τ

STATE-OF-THE-ART MEASUREMENT TECHNIQUES FOR PM AND AM NOISE

Craig Nelson

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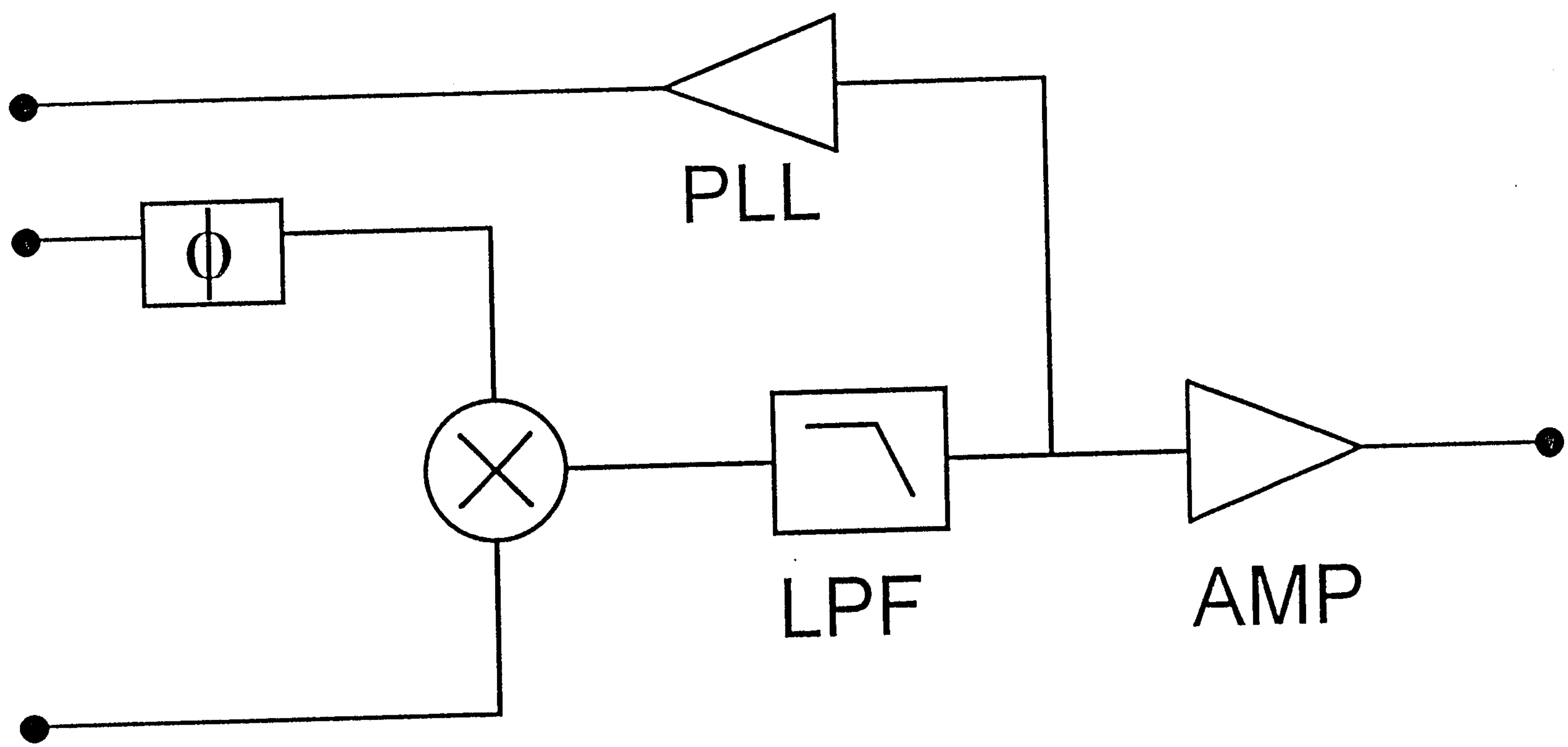
State-of-the-art measurement techniques for PM and AM noise

- Ultra wideband measurements (Fourier frequencies 0.1Hz to 1 GHz)
- Integral PM and AM noise standards
- Ultra low-noise PM and AM measurement systems ($S(f) \leq -190$ dBc/Hz)

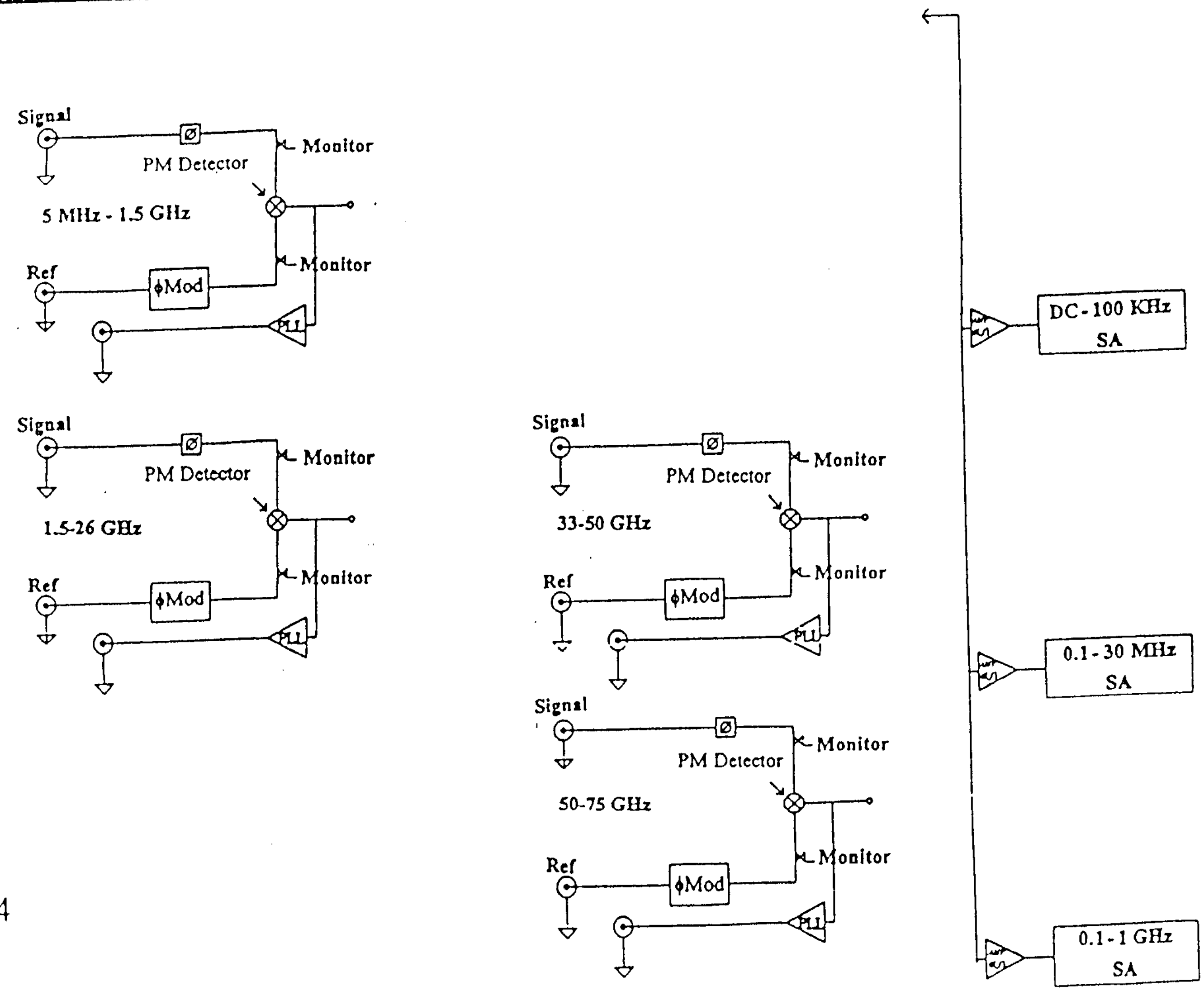
NIST PM/AM noise measurement system

- Separates PM from AM noise
- Measures carrier frequencies from 5 MHz to 75 GHz
- Extends Fourier analysis to 1GHz
- Measurement accuracy: 0.3 - 3 dB
- Calibrates most PM/AM measurement error models

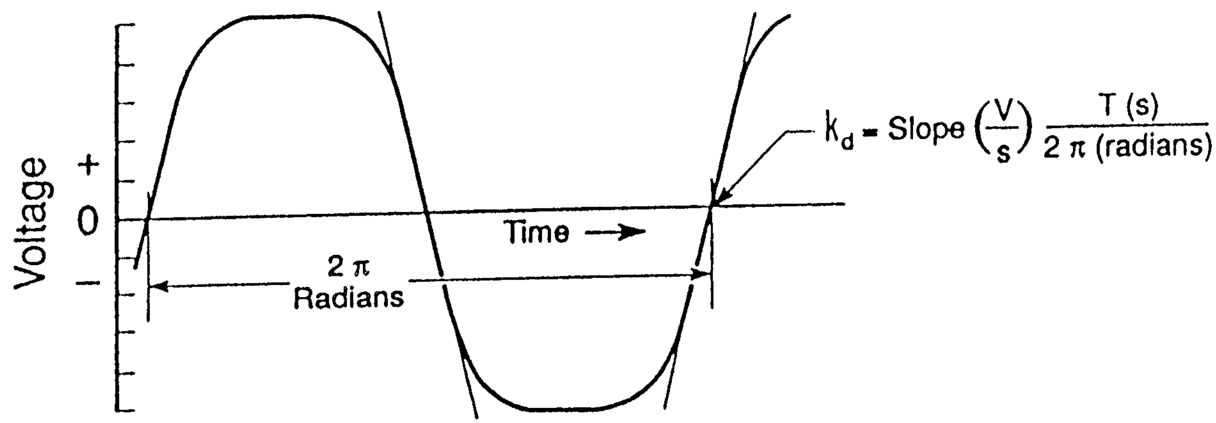
Basic phase noise measurement



NIST wideband measurement system



Determination of Kd

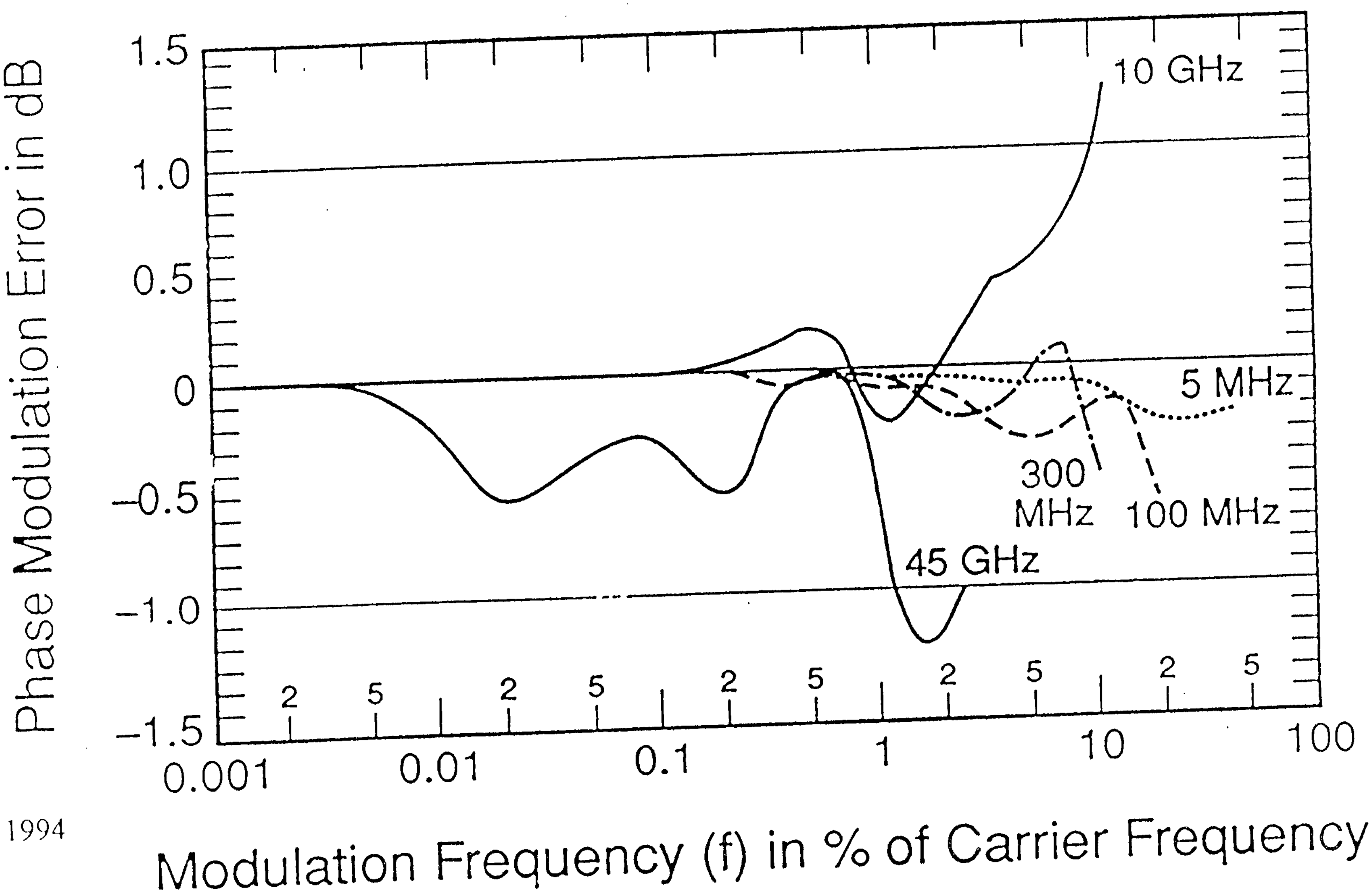


- Determines Gain at single frequency
- Does not calibrate pll effects

NIST modulator

- Can be adjusted for pure PM or AM modulation
- Extremely flat frequency response
- Calibrates Kd(f) with system locked

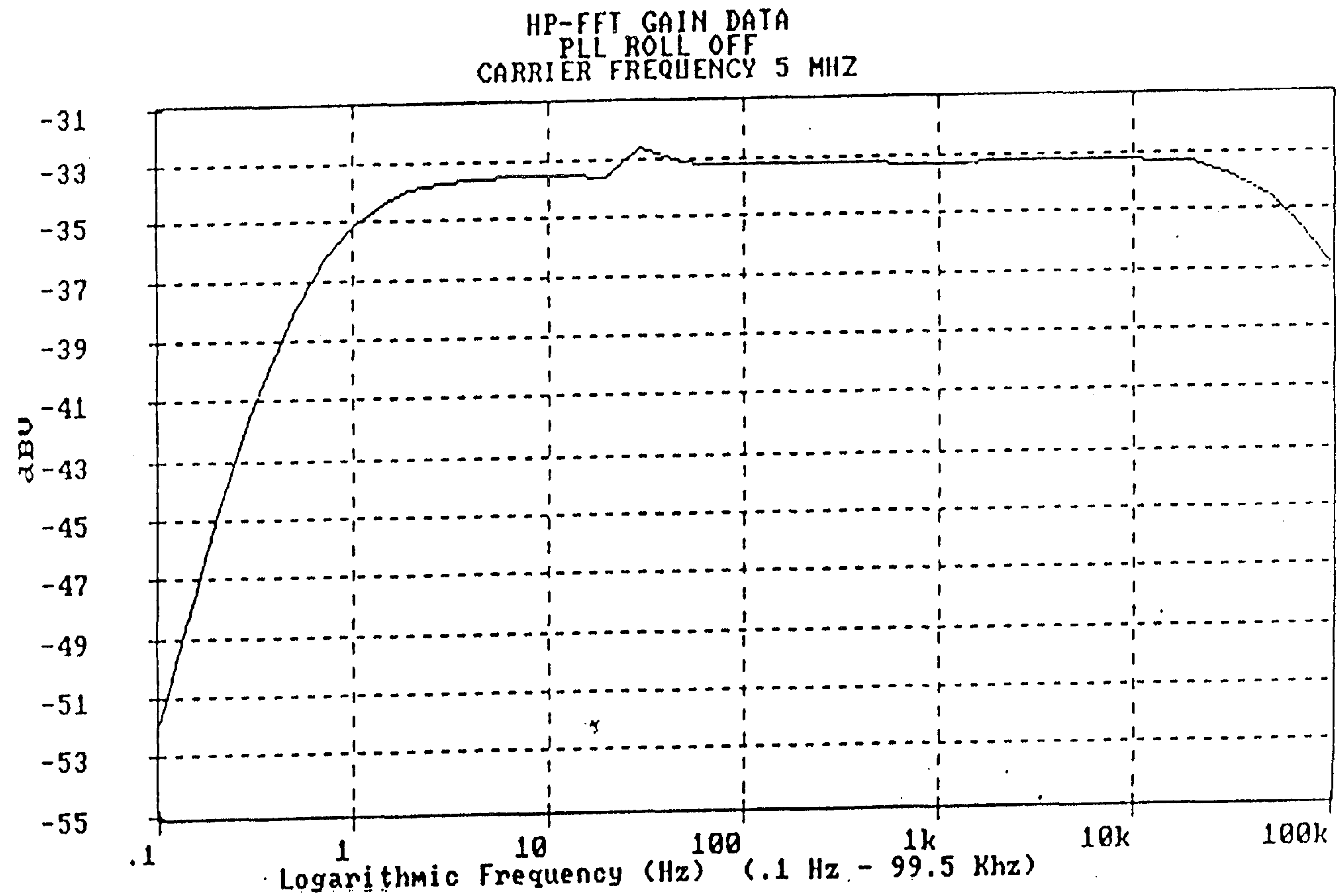
Errors in the NIST modulator



Tips for measuring gain Vs Fourier frequency

- Measure power spectrum not PSD
- Use flattop windows for FFT
- Only small number of averages required
- Use zero span width on spectrum analyzers
- 3-5 points per decade
- create gain curve with cubic spline

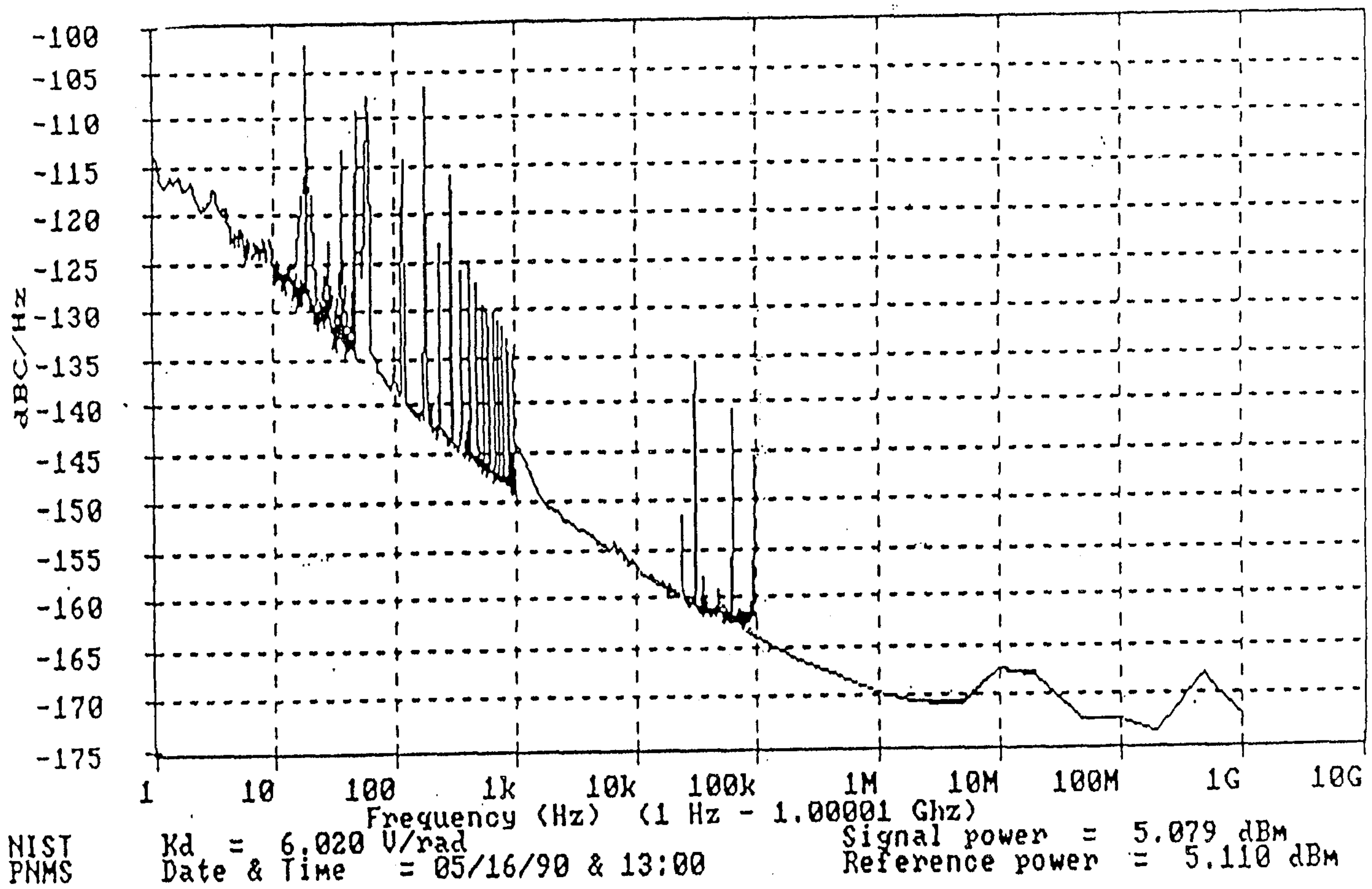
Sample gain curve at X-band



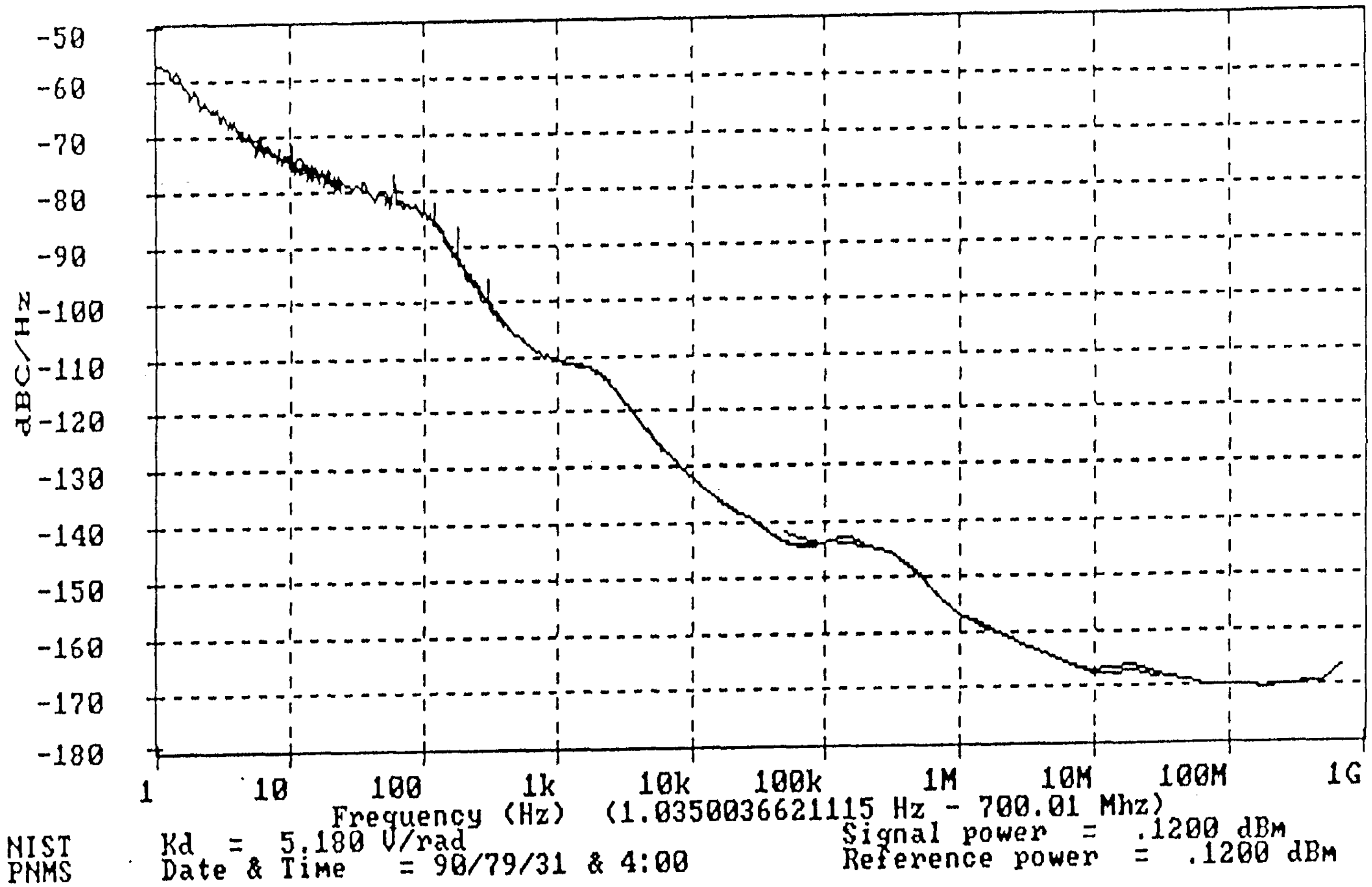
Tips for measuring noise

- Use PSD on FFT
- Using Hanning window
- Confidence interval depends on number of averages
- Confidence interval depends also on resolution and video bandwidth for swept analyzers
- Keep level of system noise floor in mind

Noise floor of NIST system at 42 GHz



Phase noise of X-band synthesizer



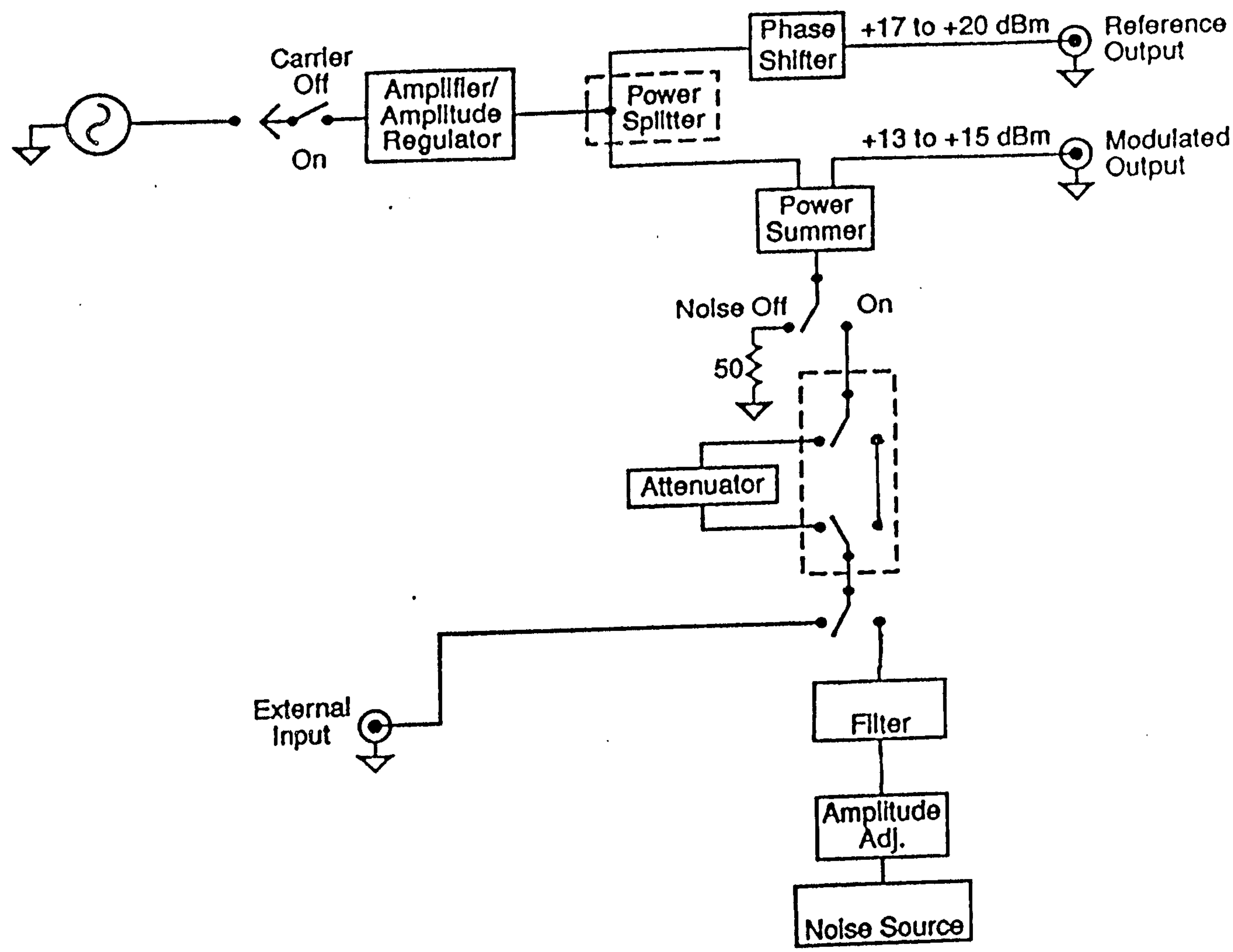
Current performance of NIST phase noise measurement system

Noise Floor	Accuracy
5 to 1500 MHz	
-140 dBc/Hz at 1Hz	±0.5 dB 1Hz to 32 MHz
-173 dBc/Hz Floor	±1 dB 32 MHz to 150 MHz
1.5 to 26 GHz	
-135 dBc/Hz at 1 Hz	± 1dB 1Hz to 500 MHz
-170 dBc/Hz Floor	±2 dB 500 MHz to 1GHz

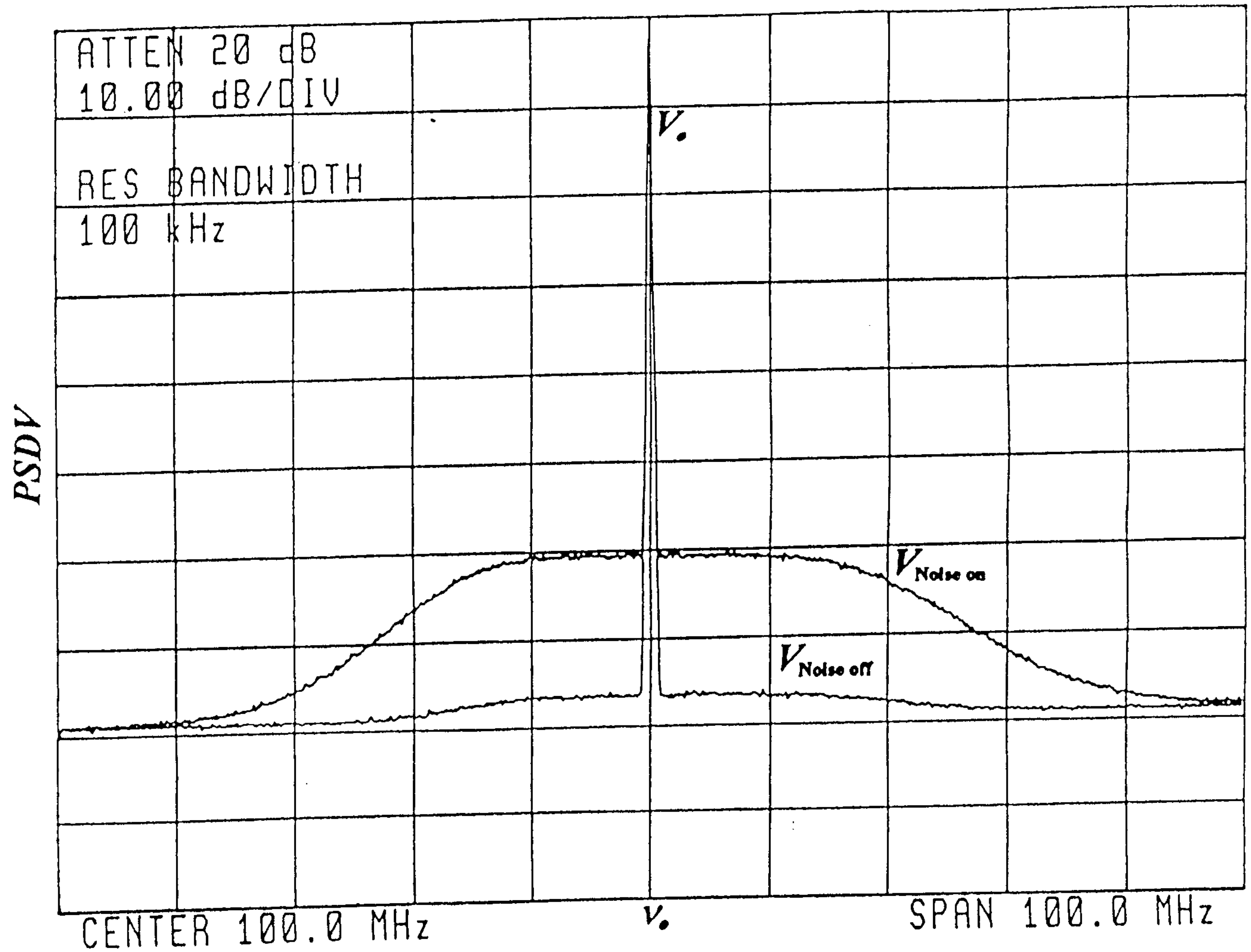
Integral PM and AM noise standards

- Low noise signal source
- Two outputs with extremely low differential AM and PM noise
- Calibrated noise source
- Greatly simplifies AM and PM measurements

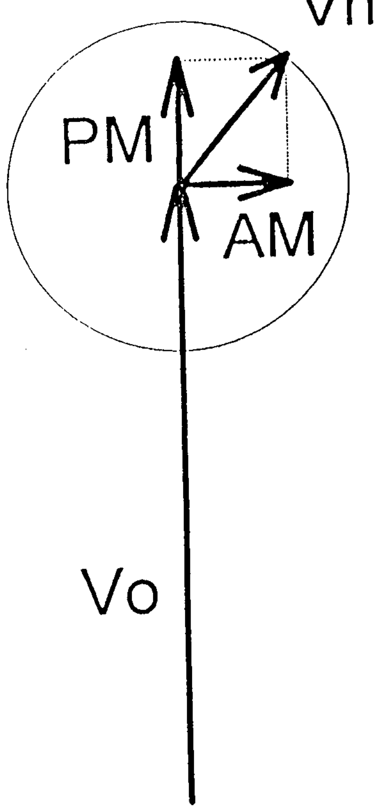
Block diagram of NIST PM/AM noise standard



Addition of noise to carrier



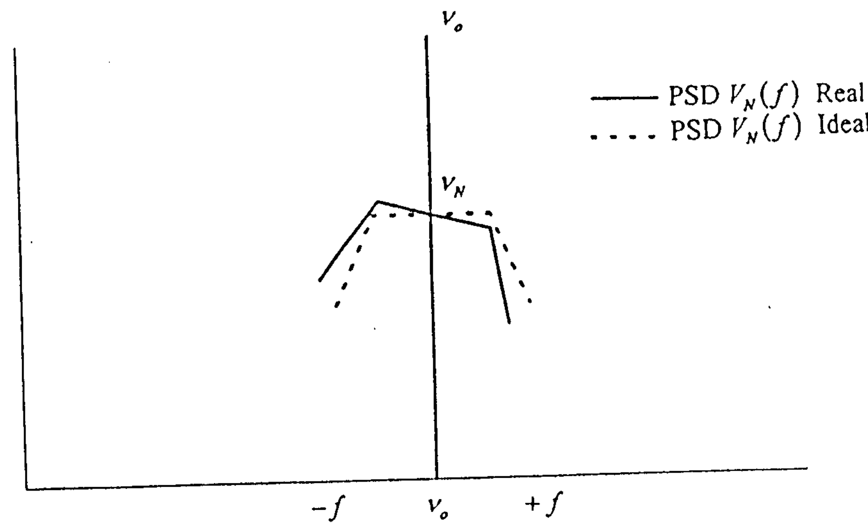
Added noise appears as equal amounts of AM and PM



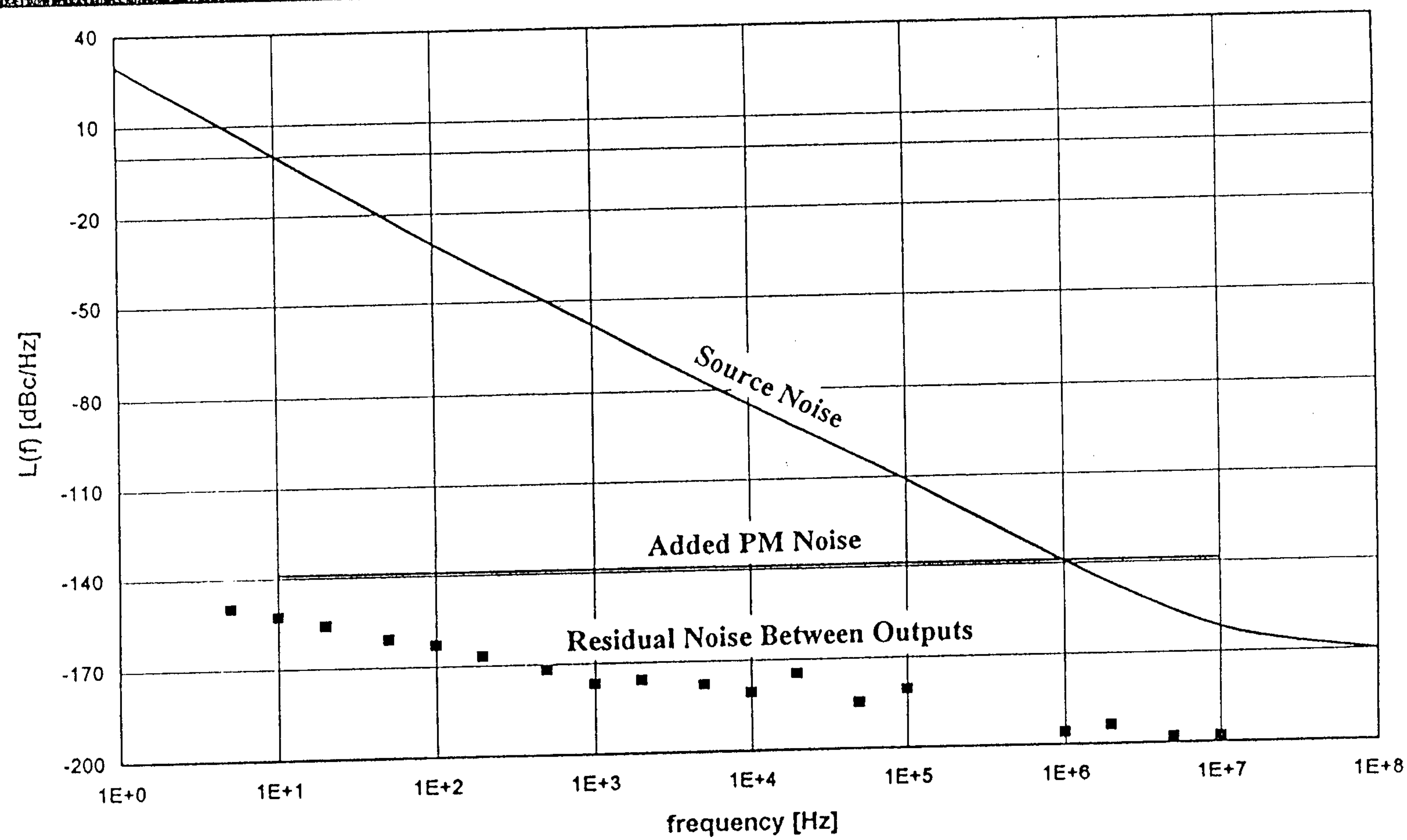
$$S_a(f) = S_\phi(f) = \frac{PSDV_n(\nu_0 - f) + PSDV_n(\nu_0 + f)}{2V_0^2}$$

While

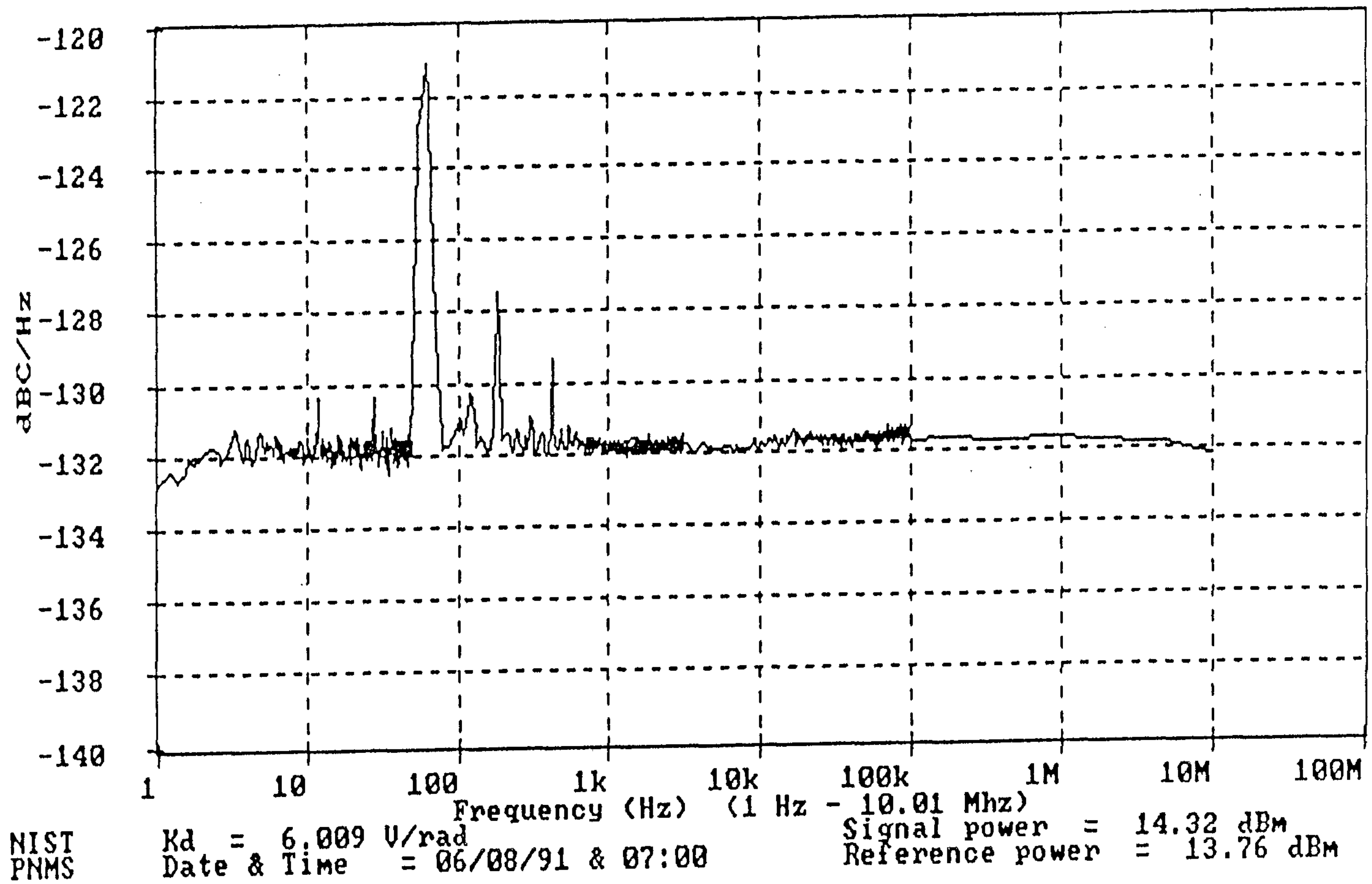
$$\int_0^\infty S_\phi(f) << 0.1$$



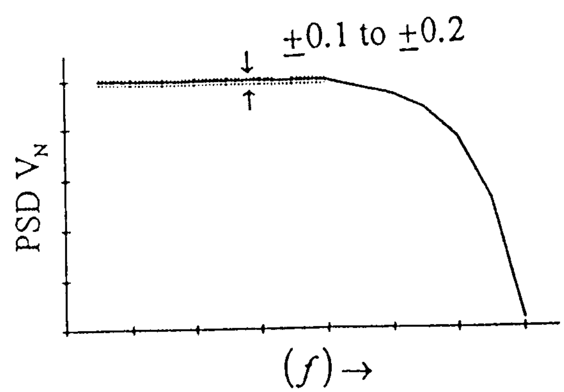
Phase noise of NIST X-band PM/AM standard



Added PM noise at 100 MHz



Stability of noise standard

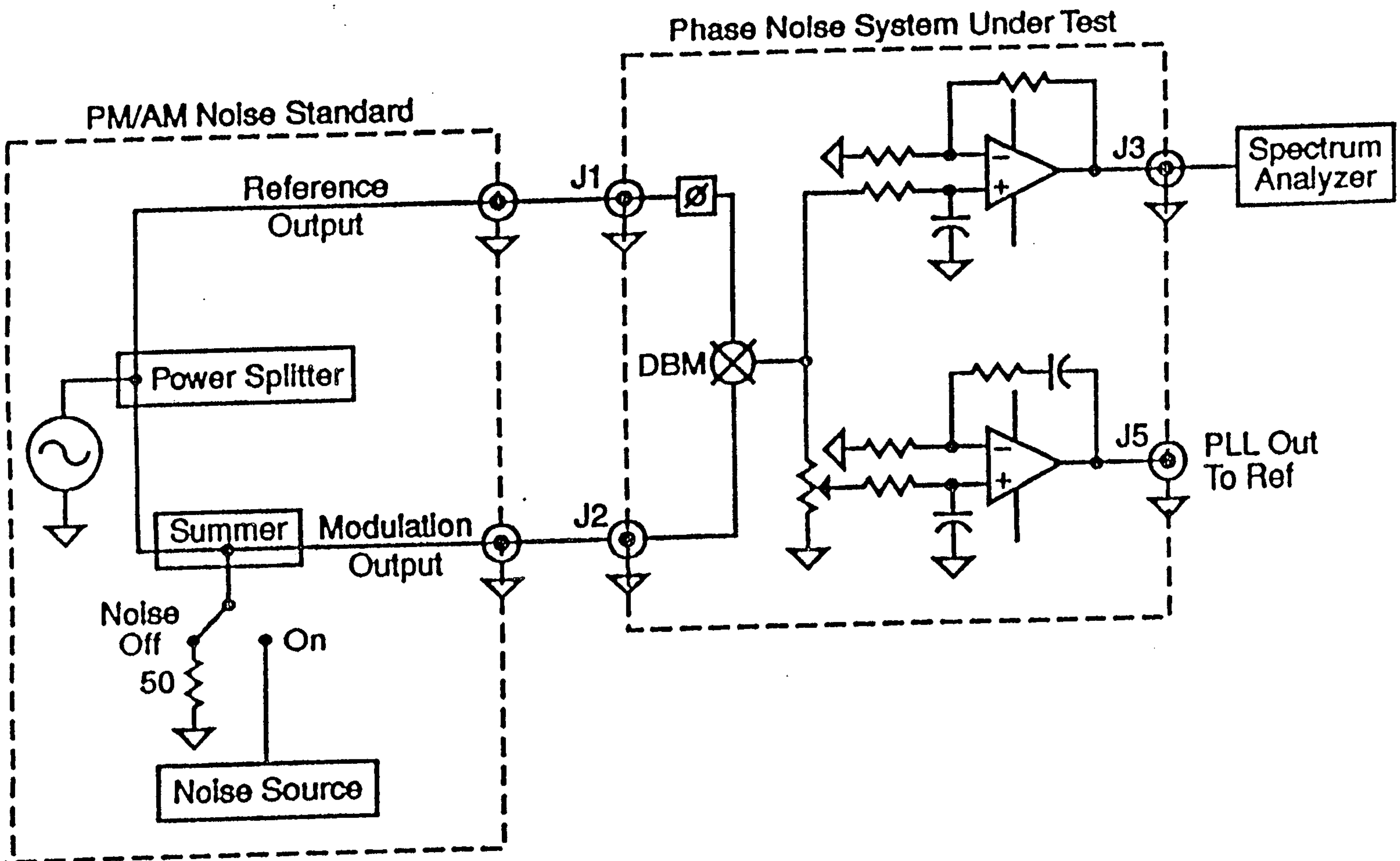


$$\frac{dS_\phi(f)}{dTemp} < 0.02\, dB/^\circ C$$

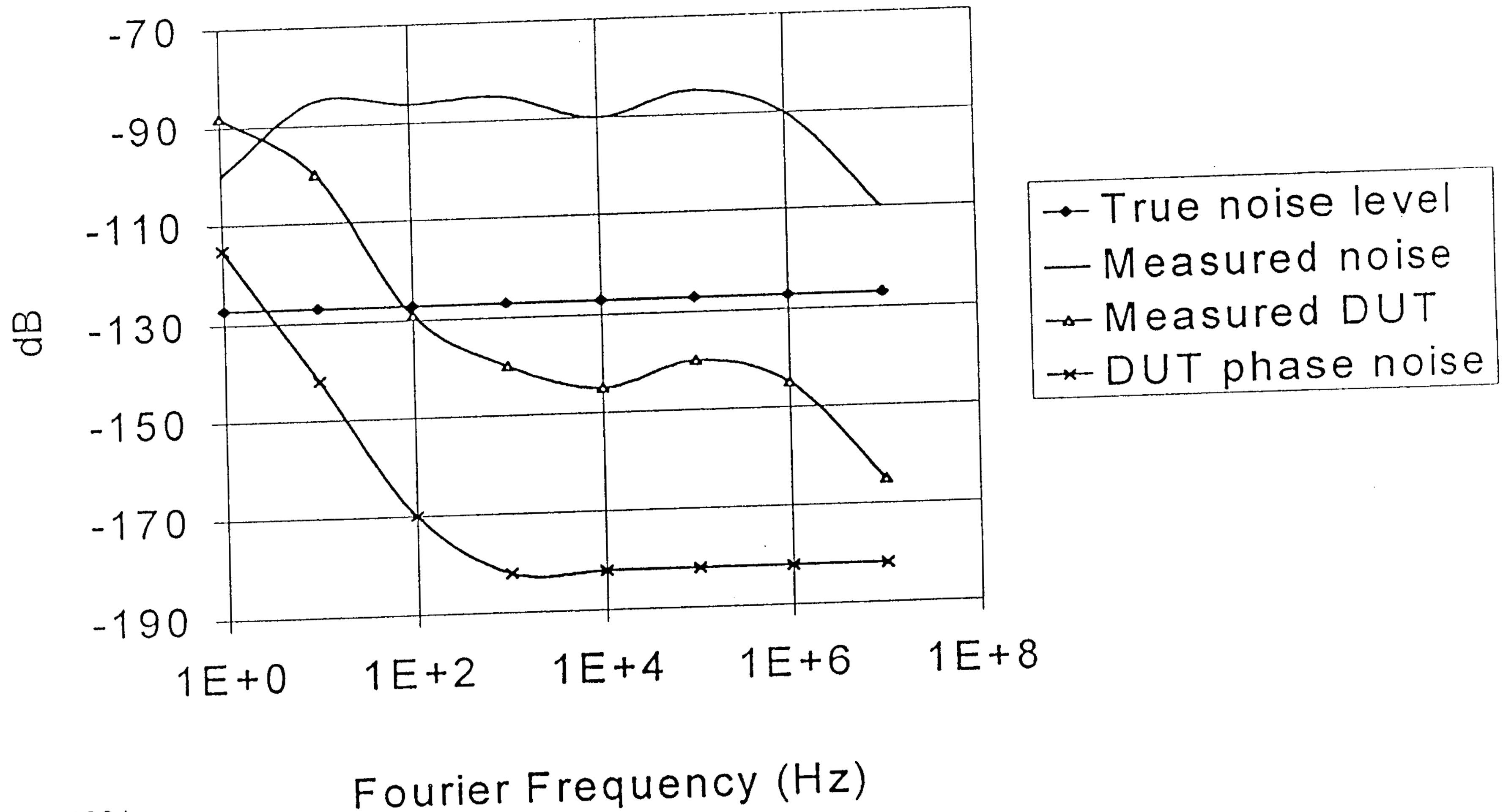
$$\frac{dS_\phi(f)}{dtime} < 0.2\, dB/year$$

$$accuracy/_{calib.} \pm 0.15 dB$$

System noise floor for PM



Use of noise calibration level



Calibration of noise floor and system accuracy

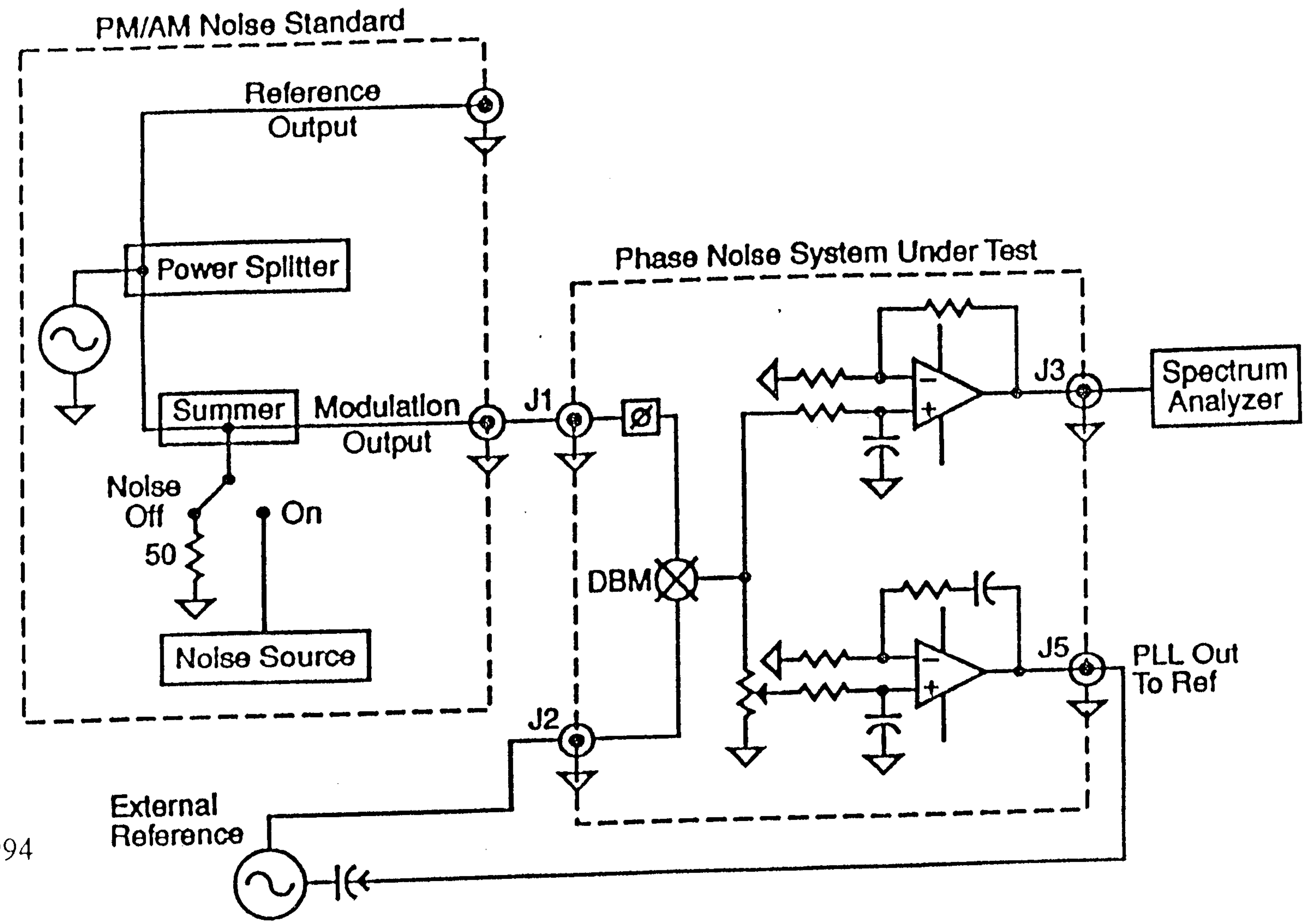
MAXIMUM RESIDUAL NOISE BETWEEN CHANNELS dBc/Hz

SOURCE FREQUENCY	1 Hz	10 Hz	100 Hz	1 kHz	10 kHz	100 kHz	1 MHz	10 MHz
5 MHz	-162	-172	-182	-190	-194	-191		
10 MHz	-161	-176	-183	-191	-197	-194		
100 MHz	-152	-162	-172	-182	-193	-193	-194	
10.6 GHz		-153	-163	-173	-181	-181	-196	-198

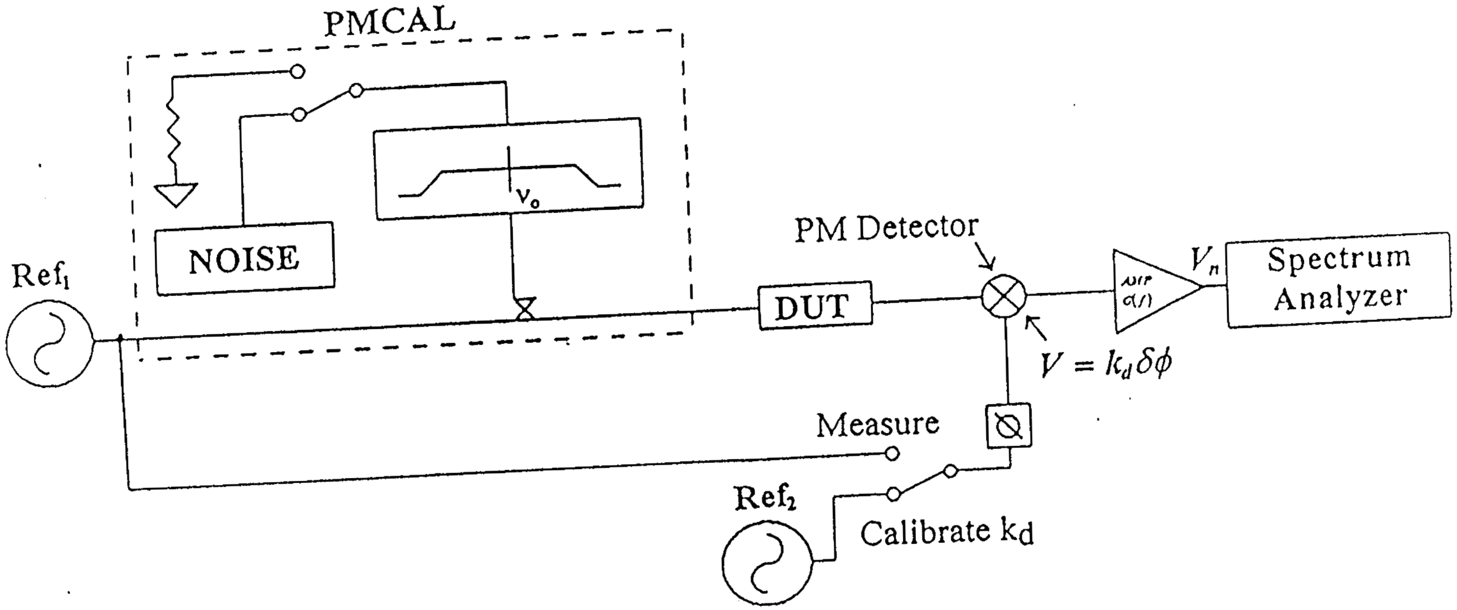
DIFFERENTIAL PM/AM NOISE LEVEL ± 0.2 dBc/Hz

SOURCE FREQUENCY	1 Hz	10 Hz	100 Hz	1 kHz	10 kHz	100 kHz	1 MHz	10 MHz
5 MHz	-127.3	-127.3	-127.3	-127.3	-127.3	-127.3		
10 MHz	-128.4	-128.4	-128.4	-128.4	-128.4	-128.4	-128.4	
100 MHz	-129.5	-129.5	-129.5	-129.5	-129.5	-129.5	-129.5	-129.8
10.6 GHz	-138.9	-138.9	-138.9	-138.9	-138.9	-138.9	-138.9	-138.9

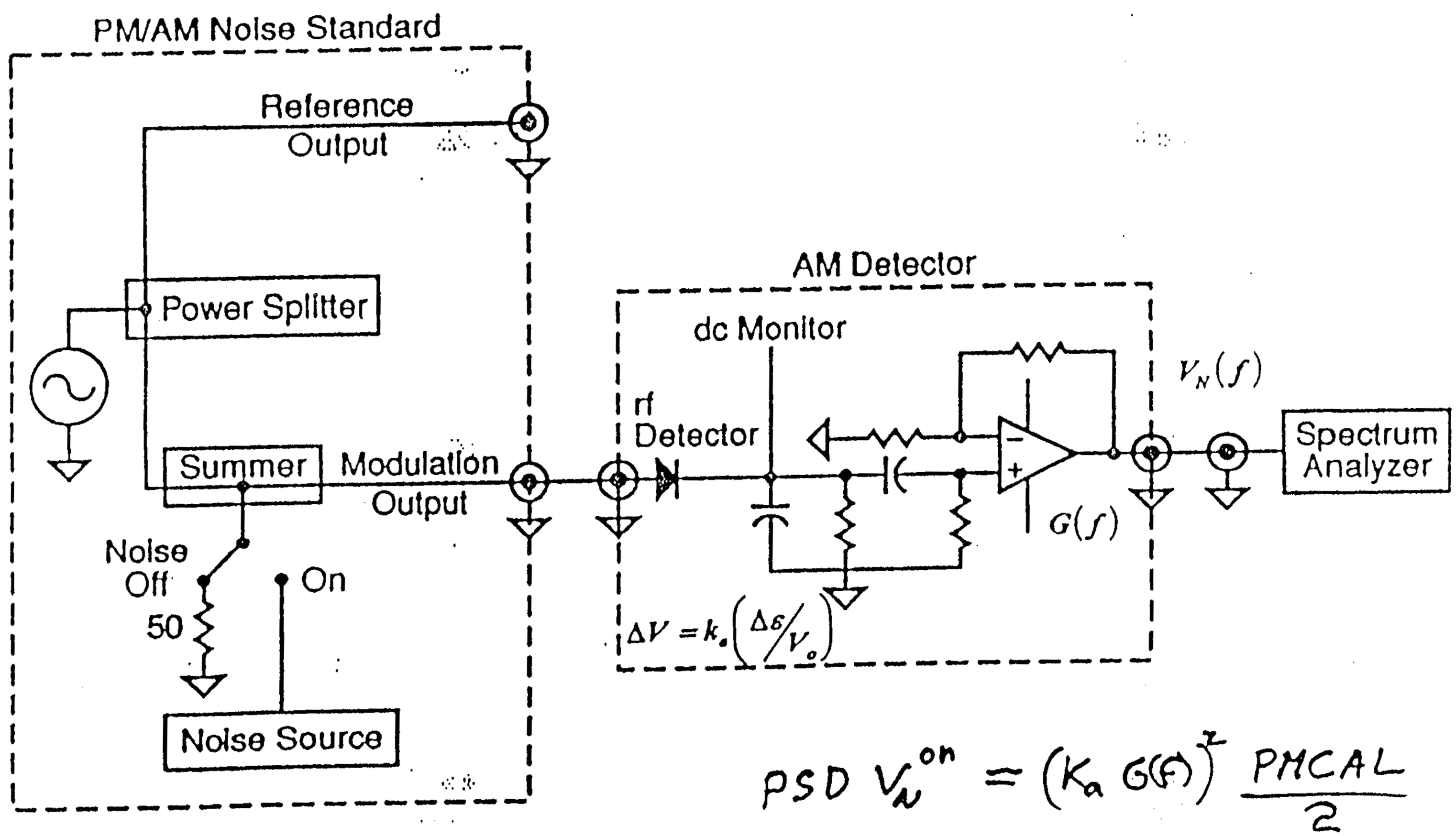
Measurement of an external oscillator



Measurement of other devices



Calibration of a simple AM measurement

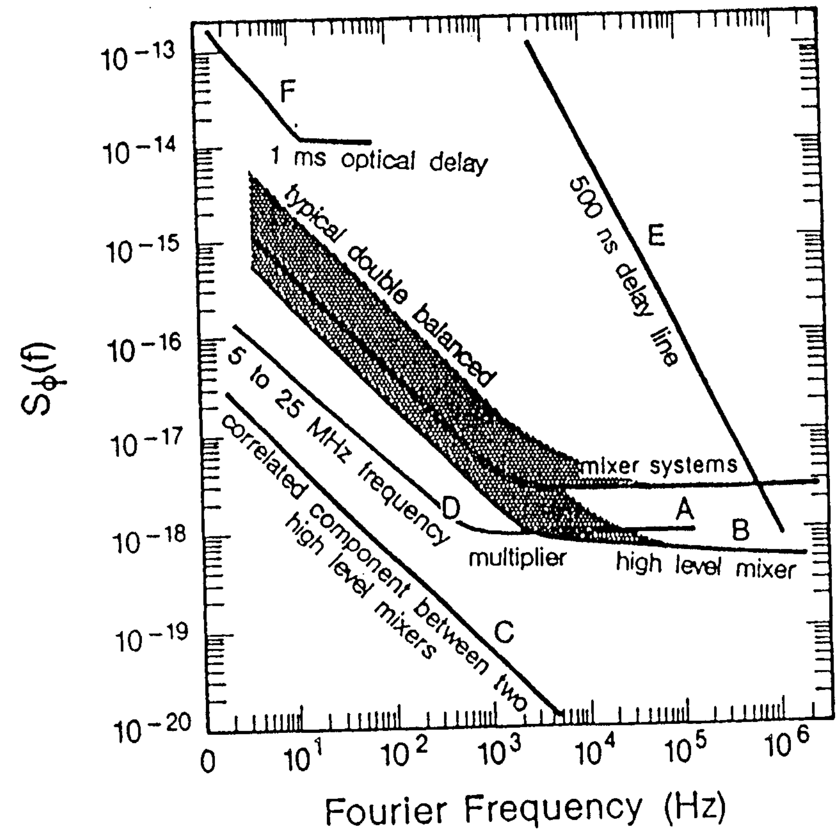


$$PSD\ V_N^{on} = (K_a\, G(F))^2\ \frac{PMCAL}{2}$$

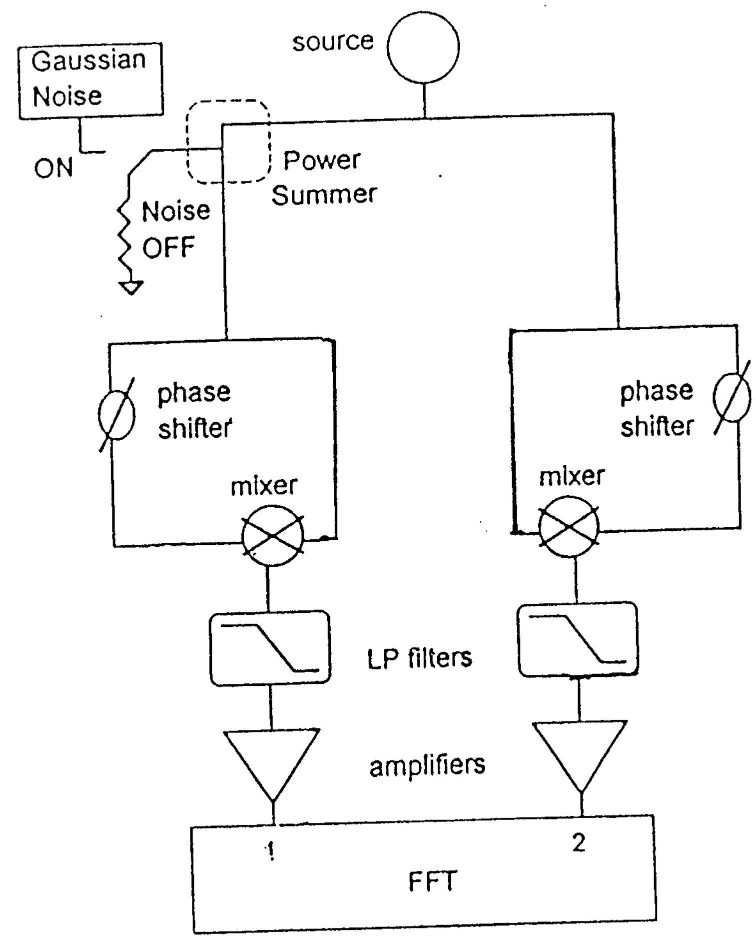
Ultra low PM and AM measurement systems

- Cross-Correlation

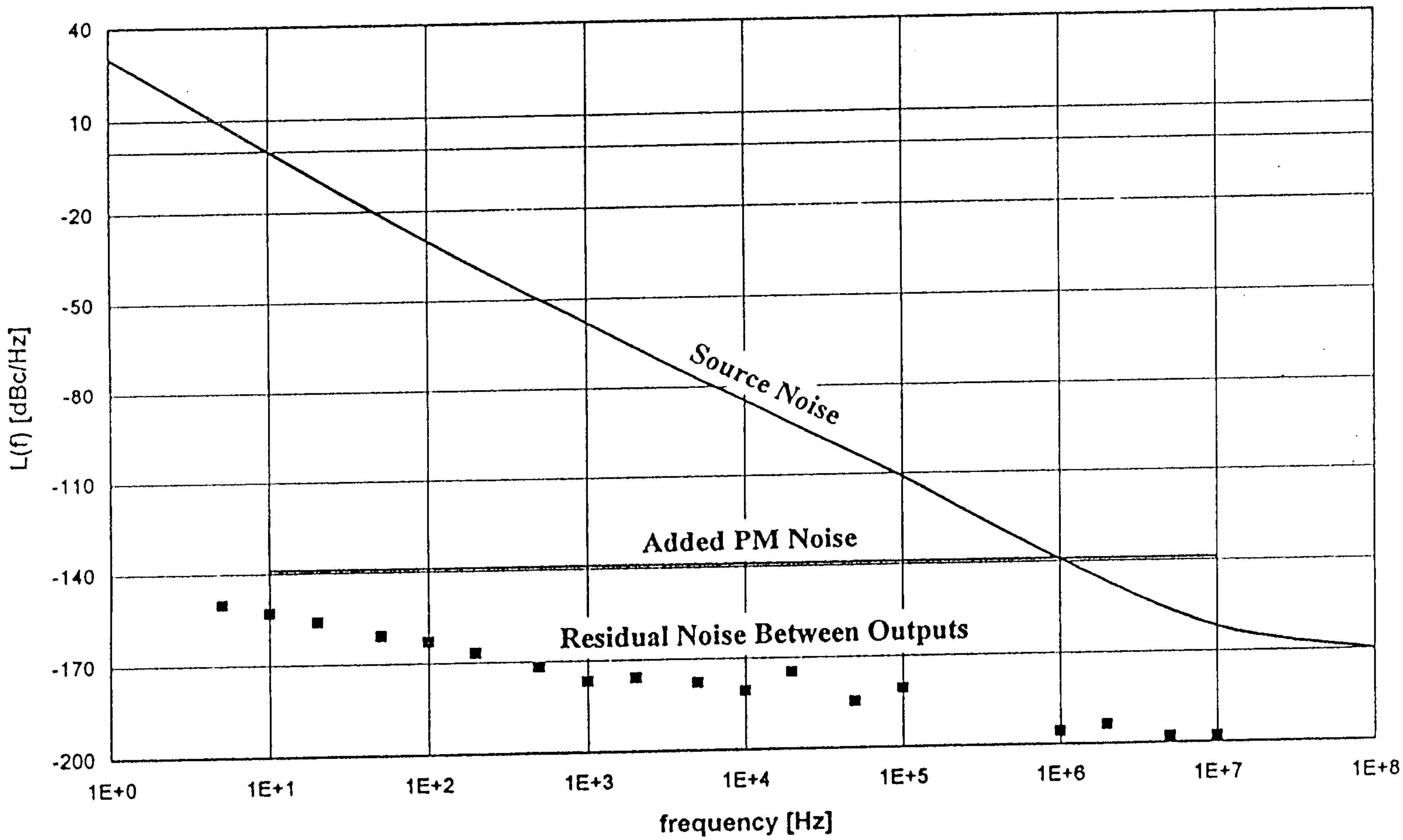
Comparison of Noise Floor for Different Techniques



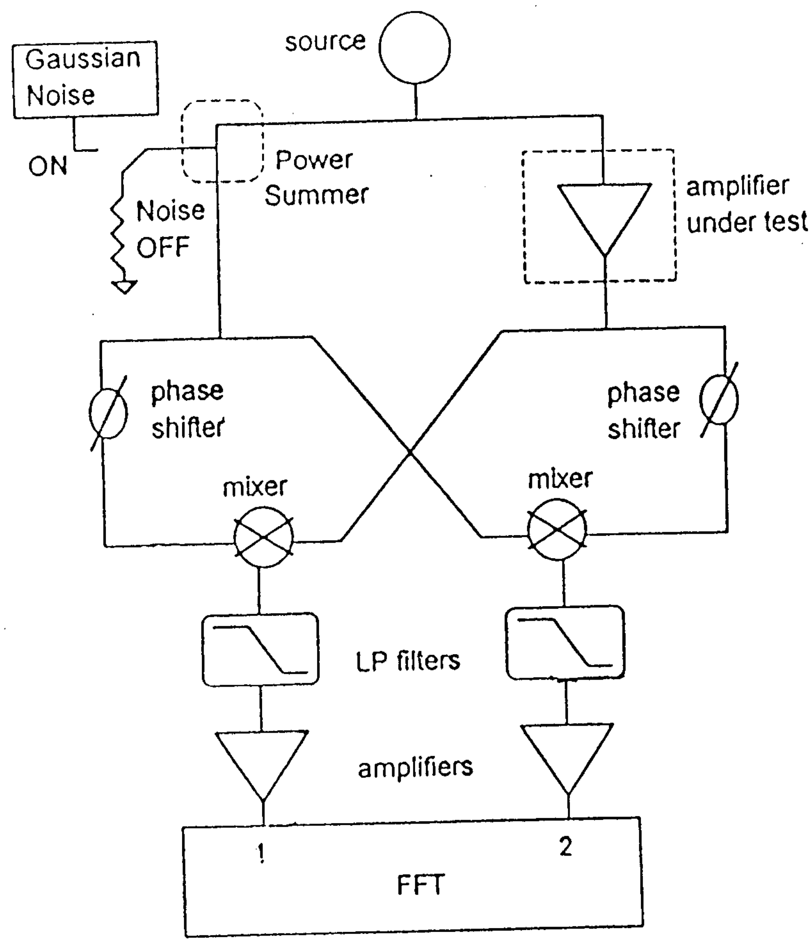
Cross-correlation PM noise floor measurement



Residual noise between channels of NIST phase noise standard



Cross-correlation PM noise system for amplifier measurements

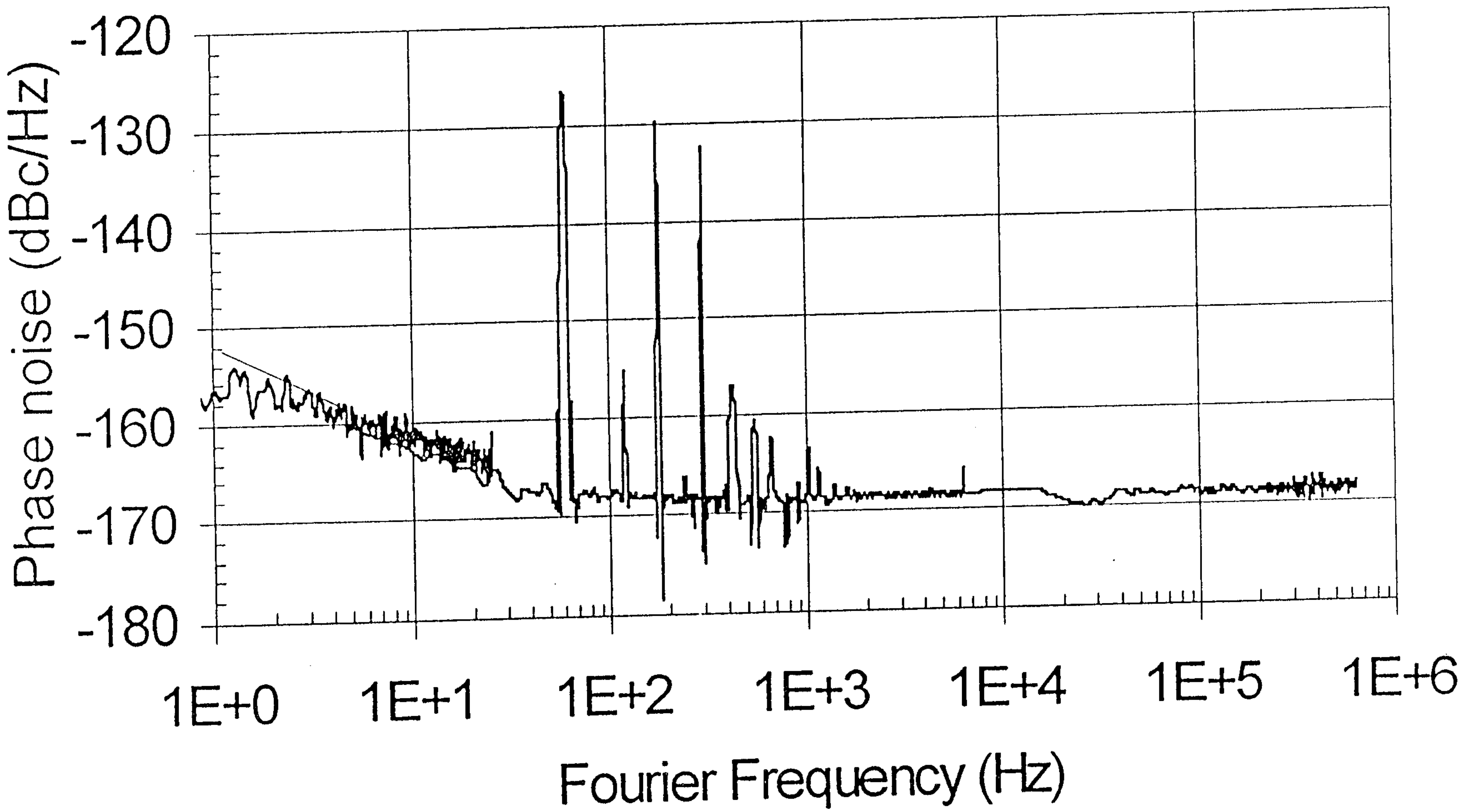


$PSD1 = PM_{amplifier} + NOISE_{channel1}$

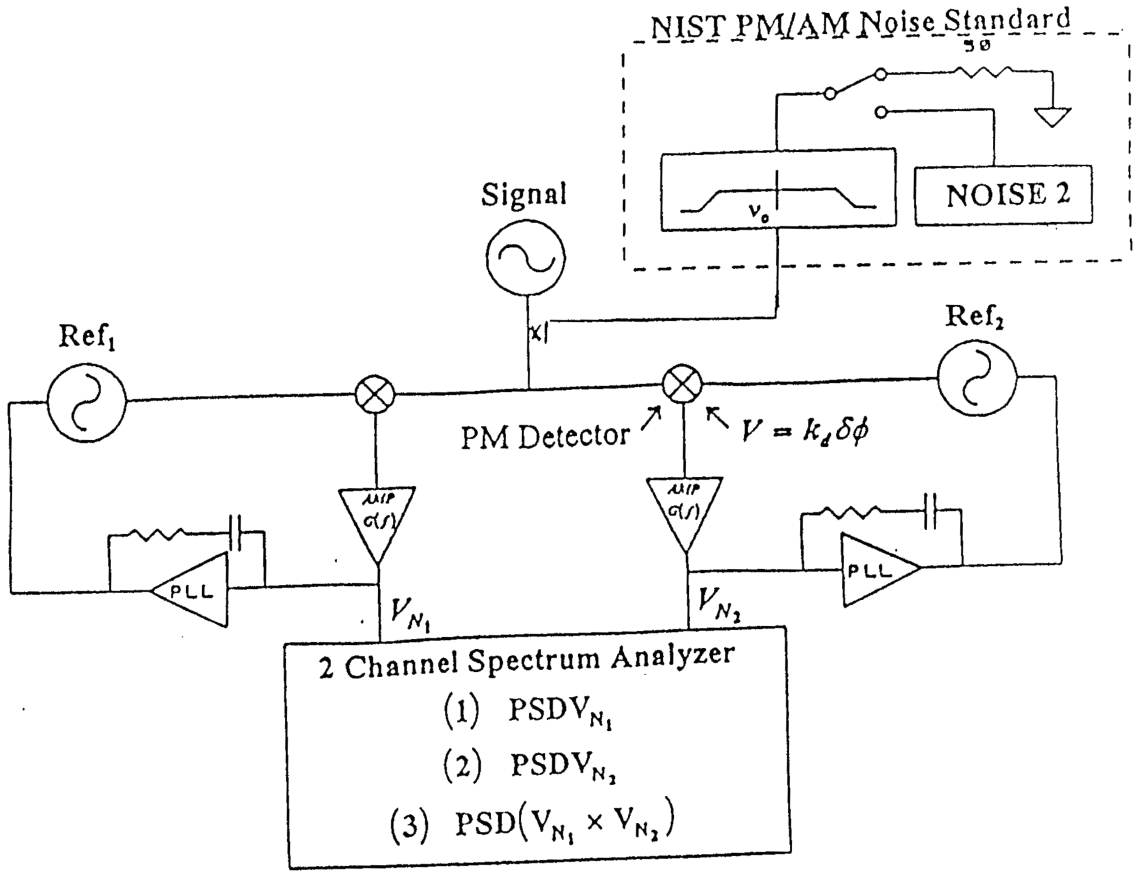
$PSD2 = PM_{amplifier} + NOISE_{channel2}$

$PSD(CROSS) = PM_{amplifier}$

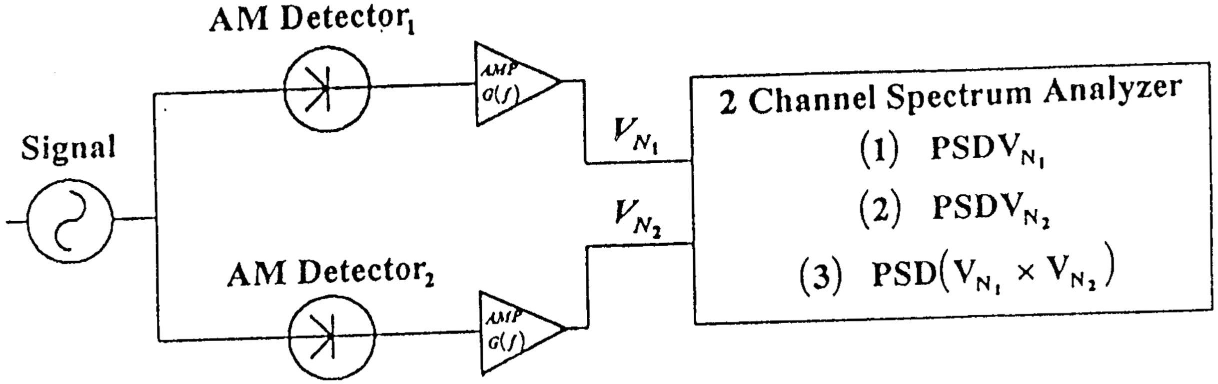
Ultra-low noise amplifier measurement



Cross-correlation oscillator measurements



Cross-correlation AM measurements

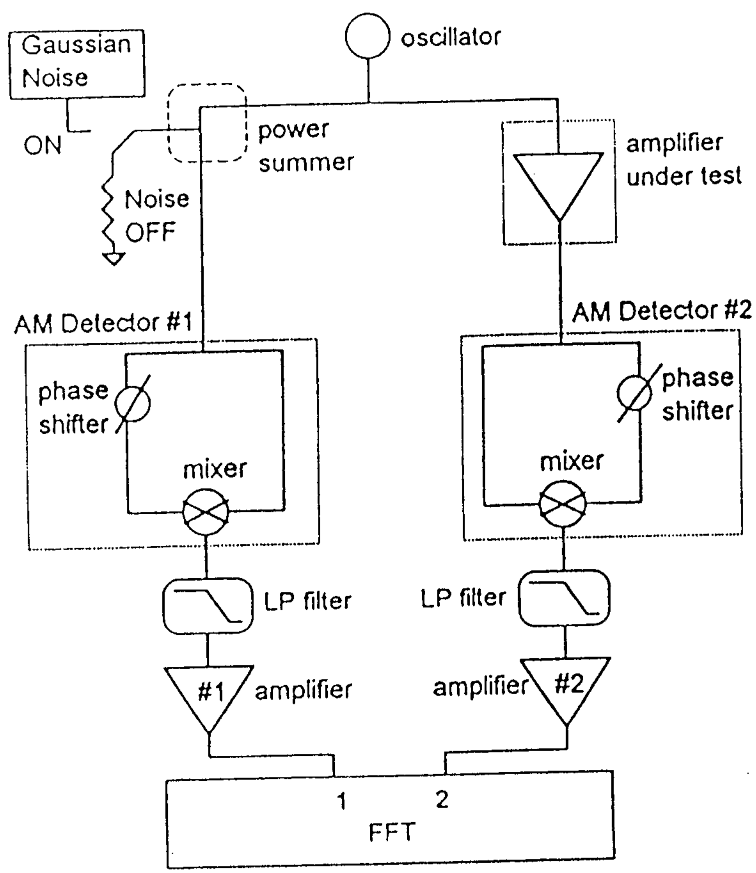


(1) $\frac{PSDV_{N_1}}{[k_a G(f)]^2}$ **measures $S_a(f)$ of the signal plus System$_1$ noise.**

(2) $\frac{PSDV_{N_2}}{[k_a G(f)]^2}$ **measures $S_a(f)$ of the signal plus System$_2$ noise.**

(3) $\frac{PSD(V_{N_1} \times V_{N_2})}{[k_a G(f)]^2}$ **measures $S_a(f)$ of only the signal since, System$_1$ noise is uncorrelated with System$_2$ noise.**

Cross-corelation AM amplifier measurements



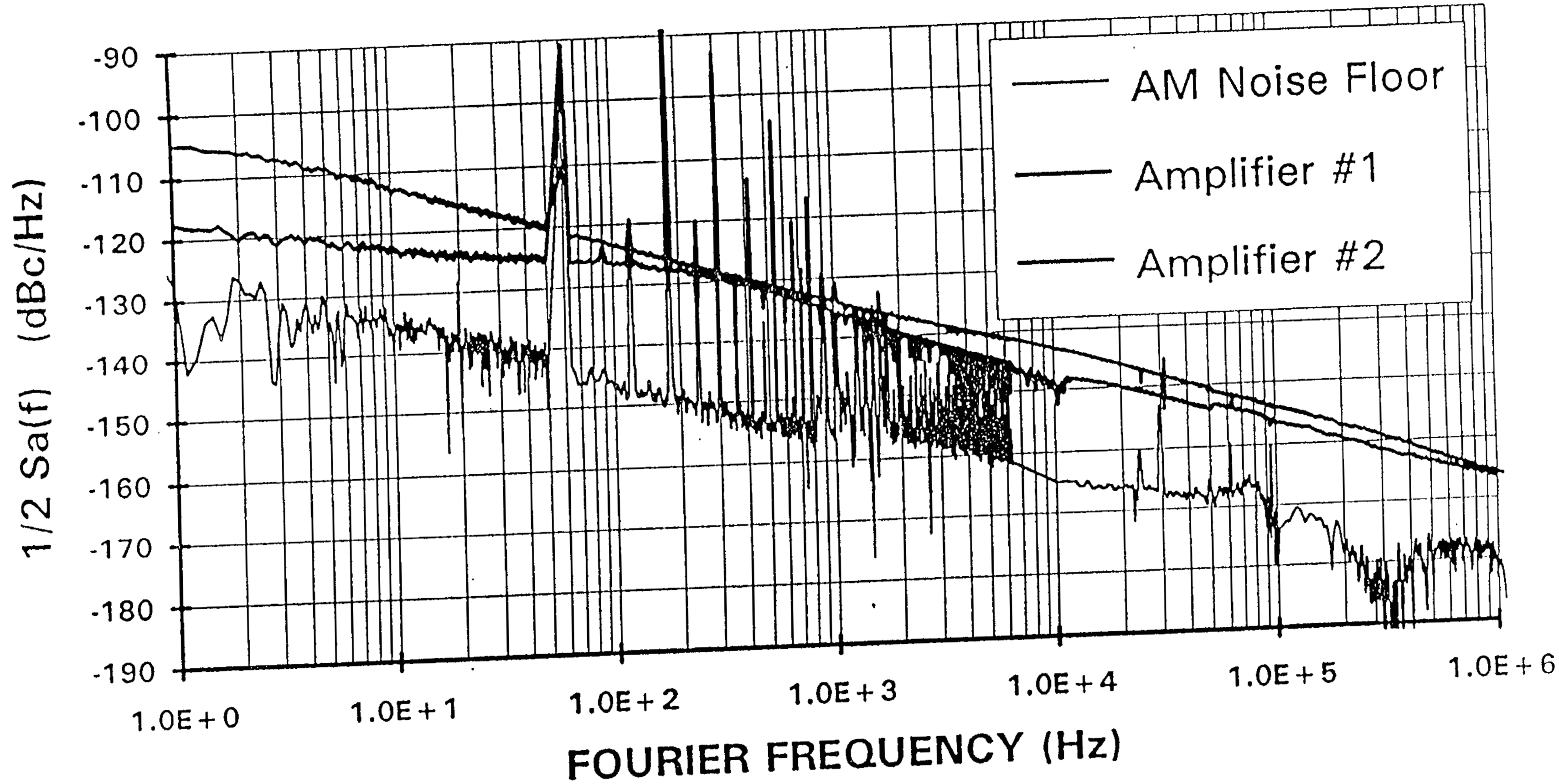
$PSD1 = NOISE_{channel\ 1} + AM_{source}$

$PSD2 = NOISE_{channel\ 2} + AM_{source} + AM_{amplifier}$

$PSD1 - PSD2 = AM_{amplifier} + NOISE_{channel\ 1} - NOISE_{channel\ 2}$

$PSD(CROSS) = AM_{source}$

Noise floor of AM measurement system



Conclusions

- To increase Fourier range a modulation technique (PM or FM) can be used.
- Using an added noise source greatly simplifies PM and AM measurements as well as decreases measurement times.
- For ultra-low noise floors cross-correlation techniques must me used.