Rabi Pedestal Shifts as a Diagnostic Tool in Primary Frequency Standards

Jon H. Shirley, W. D. Lee, G. D. Rovera and R. E. Drullinger

***Abstract*—Some of the systematic errors in primary, atomic-beam frequency standards are much larger for the Rabi pedestal part of the lineshape than for the Ramsey fringe part. We have used a digital servo system to measure the frequency offset between the Rabi pedestal and the Ramsey fringe for all seven Zeeman components of the cesium hyperfine transition. The dependence of these shifts on magnetic field, modulation amplitude, and microwave power enables us to separate three distinct causes: Rabi pulling, cavity pulling, and magnetic field inhomogeneity. From the measured pedestal shifts, we infer the corresponding shifts for the clock transition with uncertainties of the order of one part in 10^{15} or less.**

I. INTRODUCTION

IN 1949, Norman Ramsey introduced the concept of separated oscillating fields for exciting radio-frequency or microwave transitions [1]. This technique produces a very narrow spectral feature ("Ramsey fringe") superimposed on a broader resonance ("Rabi pedestal") as shown in Fig. 1. This technique is now widely used for precise spectroscopic measurements, especially those found in atomic standards of frequency.

In this paper, we present some experimental measurements of shifts between the Rabi pedestal and Ramsey fringe. Analysis of the dependence of the shifts on magnetic field, microwave power, and Zeeman line is applied to identify three distinct causes. The pedestal shifts are then used to estimate the corresponding very small shifts of the Ramsey fringe itself.

II. EXPERIMENT

The experiments were done on the primary frequency standard NIST-7 using a digital servo system. A computer program drives a frequency synthesizer with slow square-wave frequency modulation. The modulation amplitude can be chosen so the system locks either to the center of the Ramsey fringe or to the center of the Rabi pedestal. When locked to the pedestal there is an enormous decrease in stability as shown by the Allan variance plot in Fig. 2. The ratio of the Allan variances for the Ramsey versus Rabi line on the clock transition is roughly 200 to 1, the approximate ratio of the slopes of the lineshapes where the height is measured. Despite the poorer stability, the center frequencies of the Rabi pedestals can be adequately measured in just a few minutes.

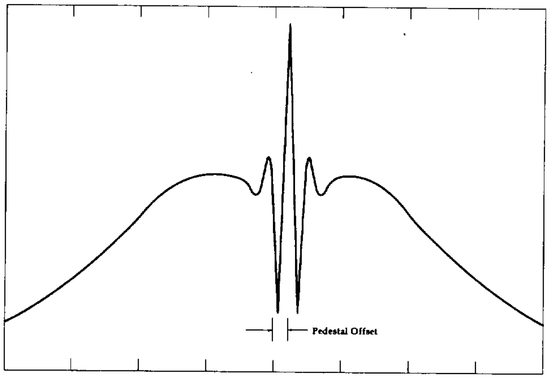

Fig. 1. Illustrative lineshape showing a Ramsey fringe on an offset Rabi pedestal. For NIST-7 both the Ramsey fringe width and the pedestal offset are too small to show on the same scale with the Rabi pedestal.

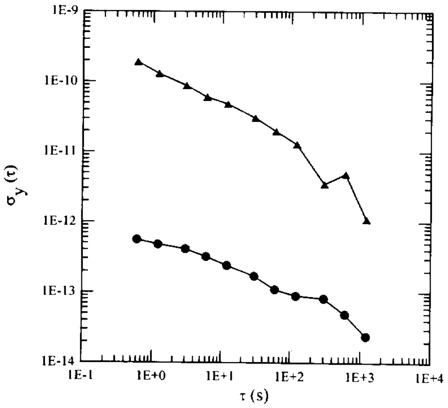

Fig. 2. Allan variance, $\sigma_y(\tau)$, for the Ramsey fringe (dots) and Rabi pedestal (triangles) of the $m = 0$ clock transition.

The measurements reveal that the Rabi pedestal for the clock transition is shifted about 2 Hz from the position of the Ramsey fringe (see Fig. 4.). Much larger shifts are seen for the field-dependent transitions. Fig. 3 illustrates how the pedestal shifts for the 7 $\Delta m = 0$ Zeeman lines vary as a function of m. The straight line fits the shifts for $m = 0$, $+1$, and -1 well, but deviates for higher absolute values of m.

III. THEORY

We have gone through a list of systematic errors for primary cesium frequency standards to consider whether they affect the

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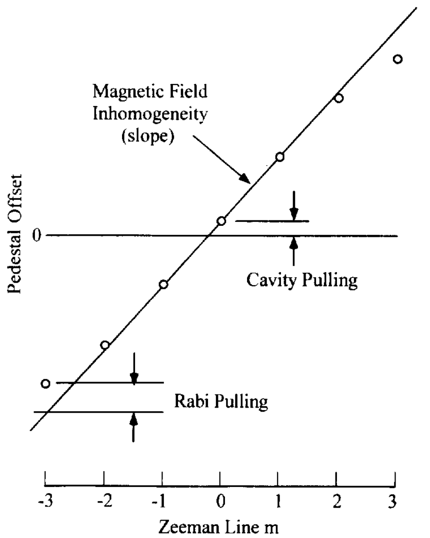

Fig. 3. Pedestal shifts for 7 Zeeman lines as a function of m showing how the three pulling effects are identified. For clarity, the effects are not shown in correct relative scale.

centers of the Rabi pedestals and Ramsey fringes in the same way. The following errors should be the same for both the pedestal and the fringe:

- Quadratic Zeeman
- Second-order Doppler
- Black-body.

The following errors affect the Rabi pedestal much more strongly than they affect the Ramsey fringe:

- Magnetic field inhomogeneity
- Rabi pulling
- Cavity pulling
- Asymmetric microwave spectrum
- Light shift (for optically pumped standards).

The following errors affect only the Ramsey fringe:

- End-to-end cavity phase difference
- Distributed cavity phase

Since the latter errors are typically no bigger than 10^{-12}, they cannot explain the experimental observations. We therefore confine our attention to the second group of errors.

A. Magnetic Field Inhomogeneity

The Rabi pedestal is made up of the probability that a transition occurs in the first excitation region and not in the second, plus the probability that a transition occurs in the second excitation region and not in the first. Its Zeeman shift therefore corresponds to the average of the magnetic fields in the two excitation regions. The Ramsey fringe has a Zeeman shift corresponding to the average field in the drift region. Since these fields can differ when inhomogeneity is present, we expect the Rabi pedestal to be shifted from the center of the Ramsey fringe. For the field-dependent transitions a 0.1% inhomogeneity in a field producing a 40 kHz Zeeman separation will produce a pedestal shift of 40 Hz, which can be easily measured. This shift should be proportional to the m value of the transition observed, but independent of microwave power or modulation amplitude.

However, for the clock transition, which has only a quadratic Zeeman shift, a 0.1% inhomogeneity will result in a shift of the pedestal of only 2×10^{-13} when the Zeeman separation is 40 kHz. This is too small to measure.

The fact that the Ramsey fringe sits on top of a background that does not have the same center induces a shift in the center of the Ramsey fringe. This shift is smaller than the shift of the pedestal by the ratio of the slopes where the two center frequencies are measured, or roughly $\ell/2L$ where ℓ is the length of the excitation region of the cavity and L is the distance between excitation regions. Thus a 2×10^{-13} shift of the clock transition pedestal will translate into less than a 2×10^{-15} shift of the Ramsey fringe. But since the inhomogeneity can be measured to a few percent, the clock frequency can be corrected for this shift with an uncertainty less than 10^{-16}.

The 40 Hz pedestal shift for the $m = 1$ line also shifts the corresponding Ramsey fringe, by about 0.7 Hz. If the frequency of the $m = 1$ line is used to measure the magnetic field and no correction is made for this inhomogeneity shift, we use an erroneous value to compute the quadratic Zeeman shift for the clock transition. This error translates into an error in the clock frequency of about 2×10^{-15}, but with the opposite sign of the error found in the previous paragraph. The two errors nearly cancel, leaving a net error of order 10^{-16}. Even this net error can be estimated from the measured inhomogeneity.

B. Cavity Pulling

Cavity pulling arises because, in general, the transition probability depends on microwave amplitude, and the microwave amplitude depends on frequency when the cavity is mistuned [2]. A generic formula for the cavity pulling shift when observed with slow square-wave modulation is

$$\lambda_c = -\frac{\partial \bar{P}/\partial b}{\partial \bar{P}/\partial \lambda} \times \frac{db}{d\lambda} \times \omega_m \tag{1}$$

where λ is the detuning and $\bar{P}$ is the velocity-averaged Rabi pedestal lineshape evaluated at the detuning of the modulation amplitude ω_m. The Rabi frequency, b, is proportional to the microwave field amplitude. The derivative $db/d\lambda$ depends only on the linewidth and mistuning of the cavity.

For a cavity mistuning large compared to the separation of Zeeman lines, $db/d\lambda$, and hence the cavity pulling shift, should be nearly independent of m. However, the shift has a strong dependence on microwave power, as shown in Fig. 4, and on modulation amplitude.

Equation (1) can also be used to compute cavity pulling of the Ramsey fringe by using the Ramsey lineshape for $\bar{P}$. Since $db/d\lambda$ is the same for both the Rabi and Ramsey shifts, the ratio of their shifts depends only on the lineshape parameters. The derivatives $\partial \bar{P}/\partial b$ differ by at most a factor 2, so the shift ratio becomes roughly the ratio of the modulation amplitudes divided by the ratio of the detuning slopes at the modulation amplitudes, or the order of $(\ell/L)^2$. For NIST-7 this ratio is on the order of 3×10^4. Hence an observed 2 Hz pedestal shift due

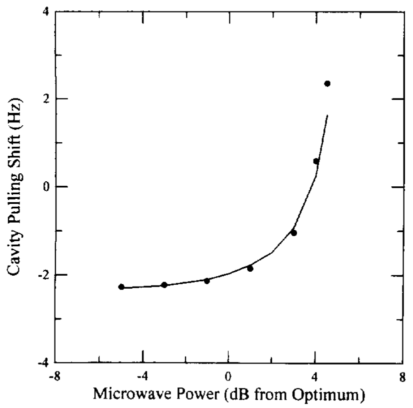

Fig. 4. Power dependence of the cavity pulling shift of the Rabi pedestal; dots are experimental points while solid line is theory with cavity tuning as only adjustable parameter.

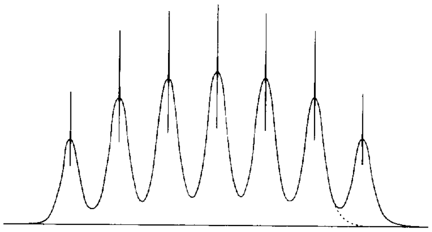

Fig. 5. Zeeman spectrum with overlapping Rabi pedestals showing the origin of Rabi pulling. The separation of the lines is 23 kHz.

to cavity pulling corresponds to a relative shift of the Ramsey line of 6×10^{-15}. Measurement of the pedestal shift permits computation of the Ramsey line shift with an uncertainty less than 1×10^{-15}.

C. Rabi Pulling

Rabi pulling is the result of the overlap of the wings of adjacent transitions in the Zeeman spectrum [3] (see Fig. 5). It is strongest for the outlying $m = 2$ and $m = 3$ transitions where the strength of adjacent transitions varies greatly because of the change in matrix elements. Rabi pulling shifts will increase linearly with microwave power and also increase with modulation amplitude. They decrease rapidly with increasing magnetic field.

For the clock transition, Rabi pulling cancels unless asymmetry exists in the spectrum. Asymmetry could arise from the optical pumping (not possible if pumping light is linearly polarized) or from a change in microwave amplitude due to cavity mistuning or changes in the source. Measurement of the Rabi pulling shift of the $m = 3$ line plus an estimate of asymmetry permits one to place limits on the possible Rabi pulling shift of the $m = 0$ pedestal.

Rabi pulling of the Ramsey fringe is smaller than that for the pedestal both because of the smaller modulation amplitude and because of the steeper slope. In addition, since the pedestal wing is curved, the average slope of the wing is smaller under the Ramsey fringe than across the width of the pedestal. Hence, we expect the Rabi pulling shift of the Ramsey fringe to be smaller than that of the pedestal by more than $(\ell/2L)^2$, 3×10^{-5} for NIST-7.

TABLE I
FREQUENCIES OF RAMSEY FRINGES AND RABI PEDESTALS

m	Ramsey Freq.	Rabi Freq.	Difference
0	1.361	-1.0	- 2.3
1	39 514.00	39 487.3	-26.7
-1	- 39 511.44	-39 489.5	22.0
2	79 026.51	78 975.4	-51.1
-2	- 79 024.45	-78 978.3	46.2
3	118 538.90	118 463.1	-75.8
-3	-118 537.70	-118 466.9	70.8

TABLE II
ANALYSIS OF RABI PEDESTAL SHIFTS

|m|	Mean Shift	Difference	|m| =1 Difference times |m|
0	-2.3		
1	-2.4	-48.7	-48.7
2	-2.5	-97.3	-97.2
3	-2.5	-146.6	-145.8

IV. ANALYSIS OF DATA

Table I shows a sample set of frequency measurements of both the Ramsey fringe and the Rabi pedestal for all 7 Zeeman lines. All frequencies are in Hz relative to 9 192 631 770 Hz. Each line was measured for 3000 s. The uncertainties are about 1 in the last figure. The microwave power was set 2.5 dB below optimum.

The observed pedestal shifts (difference column) are the sum of shifts due to all three causes. To separate the causes we recall that inhomogeneity shifts are linear in m while cavity pulling is insensitive to m. Accordingly we tabulate the mean and difference of the pedestal shifts for $+m$ and $-m$ transitions as shown in Table II.

We interpret the mean shift as due to cavity pulling. This interpretation was strengthened by measuring the power and modulation dependence of the mean shift and finding agreement with theoretical predictions based on numerical evaluation of (1); (Fig. 4). We interpret the difference in shifts as due to magnetic field inhomogeneity. These differences showed very little dependence on microwave power or modulation amplitude.

The last column of Table II shows the difference in pedestal shifts for $m = +1$ and $m = -1$ multiplied by m. Its close agreement with the difference column verifies the predicted m dependence for inhomogeneity shifts. The 0.8 Hz discrepancy for $|m| = 3$ is interpreted as a 0.4 Hz Rabi pulling shift for each of the outer lines. Multiplying this shift by a 1.2% asymmetry between the wings of the $m = +1$ and $m = -1$ lines (the amount inferred from the cavity pulling measurements) we obtain an estimate of the Rabi pulling shift of the clock transition pedestal less than 0.01 Hz. The corresponding shift for the Ramsey fringe will be less than one part in 10^{17}. Much larger Rabi pulling shifts have been seen at smaller magnetic fields.

V. SUMMARY

From the NIST-7 data in Tables I and II (taken at the indicated Zeeman frequency and an RF power level 2.5 dB below optimum) we have identified the following Rabi pedestal shifts: a shift of 24 Hz times the m value of the line due to magnetic field inhomogeneity, a shift of each Zeeman line by about 2 Hz due to cavity pulling, and a shift of 0.4 Hz for the two outermost Zeeman transitions due to Rabi pulling. From these observed shifts we can infer the following frequency errors for the standard.

Magnetic field inhomogeneity: A shift of 2×10^{-15}. But this shift is nearly canceled if the Ramsey frequency of a field-dependent transition is not corrected for the inhomogeneity shift and then used to compute the quadratic Zeeman shift.

Cavity pulling: A shift of 6×10^{-15} with a strong dependence on microwave power.

Rabi pulling: A shift of less than 1×10^{-16} at the magnetic field used for the measurements.

We emphasize that because of the large ratio between Rabi pedestal shifts and Ramsey fringe shifts and the use of measurements on field-dependent lines, these small error limits were obtained with frequency measurements no better than 1×10^{-11}.

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