The Accuracy Evaluation of NIST-7

W. D. Lee, J. H. Shirley, J. P. Lowe and R. E. Drullinger

Abstract—**We have performed evaluations of the major systematic errors in NIST-7 with an overall uncertainty of less than a part in 10^{14}. The complete evaluation process has been separated into two parts. With a computer-controlled, digital servo system and some new measurement techniques, we now perform core evaluations (second-order Zeeman and Doppler shifts, cavity pulling and phase shift, line overlap and some electronic shifts) with an overall uncertainty of less than one part in 10^{14} in just a few days of measurements. The complete evaluation of all small and subtle effects in both the physics and electronics requires a few hundred days of data. But, these small effects are not variable at the 10^{-14} level and their infrequent evaluation does not detract from the operational accuracy of the standard.**

I. Introduction

The design goal for NIST-7 is for an overall accuracy of one part in 10^{14}. With the evaluation reported here, we have essentially attained that goal, see Table I. To insure that level of accuracy, each potential source of systematic error must be evaluated at a level of $3\text{-}5\times10^{-15}$. The short-term stability and atomic line Q of the standard strongly influence how we meet this requirement. The short-term stability is characterized by $\sigma_y(\tau) \approx 7\times10^{-13}\ \mathrm{s}^{-1/2}$ (see Fig. 1). This means that frequency measurements with a fractional uncertainty of a few parts in $10^{15}(1\sigma)$ can be made in about 10 hours. However, many measurements have to be made with operating parameters set to nonideal conditions. These measurements take more time. The atomic line Q is about 1.5×10^{8}. Hence, we must split the line with an uncertainty of less than a part in 10^{6}, a daunting task. Furthermore, the very broad velocity distribution found in an optically pumped standard leads to a relatively strong dependence of the frequency on microwave power and modulation parameters. To achieve confidence in measurements to such exacting accuracy we have tried to use two or more independent techniques to evaluate all the major sources of potential error.

The major sources of frequency bias in a thermal-beam, cesium frequency standard are: second-order Zeeman shift, second-order Doppler shift, end-to-end phase shift and possibly cavity pulling and line overlap shift. Because of their size, these effects might be considered variable on a level significant to the overall accuracy of the standard. Therefore, they should be evaluated rather frequently. For NIST-7, we have developed servo electronics that allow major portions of this evaluation to be automated. Additionally, we have developed a measurement technique which allows the very small effects of cavity pulling, line overlap shift and magnetic-

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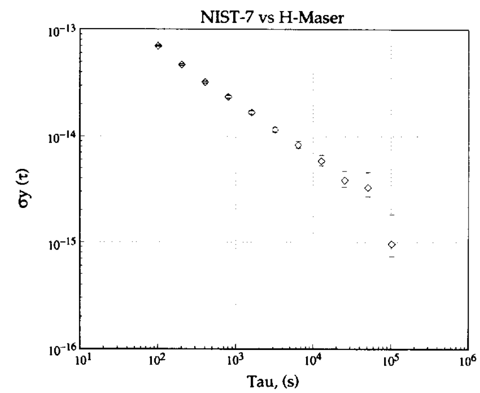

Fig. 1. Short-term stability of NIST-7. Error bars are 1σ.

TABLE I
Summary of Frequency Shifts and their Uncertainty in NIST-7; Units are Fractional Frequency

Effect	Frequency Bias ($\times 10^{15}$)	Uncertainty (1σ) ($\times 10^{15}$)
Second-Order Zeeman	$\approx$ 100 000†	≤ 1
Second-Order Doppler	$\approx$ 350†	3
End-to-End Phase Shift	733†	4
Distributed Phase Shift	——	< 1*
Cavity Pulling	2	< 1
Flourescence Light Shift	——	< 1*
Line Overlap	≤ 0.1	< 0.1
Electronics	——	< 3*

†These shifts vary with the operational parameters of C-field and microwave power.
*These are estimates based on our theoretical understanding of the system and preliminary measurements which have shown no shifts within a precision of 1-2 $\times$ 10^{-14}.

field inhomogeneity to be measured quickly with low precision measurements [1]. This relies on the fact that for these effects, the shift in the Rabi pedestal part of the line shape is large compared to that of the Ramsey fringe part of the line shape. Because of this leverage, the offset between the Ramsey line and its Rabi pedestal must be measured to only about 0.1 Hz to obtain corrections to the standard that are good to at least 1×10^{-15}. With the combination of the digital servo-system and this new measurement technique, we can evaluate the major systematic errors and check for any error that would show up in the frequency-vs-microwave power dependence to a combined, fractional uncertainty of 1×10^{-14} in just a few days.

The many other potential errors not investigated in the core evaluation just outlined are expected to be very small and not variable at the 10^{-14} level. The fact that their full evaluation requires several hundred days of data is not an impediment to frequent evaluations of the standard.

We report here the results of a core evaluation plus some preliminary results toward a complete evaluation of the smaller effects.

II. Apparatus

The atomic beam tube has been described elsewhere [2]–[4]. Briefly, it has a Ramsey cavity 1.55 m long and an atomic beam 3 mm in diameter. An axial C-field is employed for field uniformity and control of the Rabi pedestal shape. The cavity ends are designed so that the Poynting vector vanishes at the center of the atomic beam window [5], thus minimizing distributed-cavity phase shift effects. We operate the standard at a temperature of 40°C.

The laser system uses a single, extended-cavity, grating-tuned, diode-laser which is frequency referenced to the $F = 4 \rightarrow F' = 5$ saturated absorption feature in an evacuated, cesium cell. The optical pumping beam ($F = 4 \rightarrow F' = 3$) is synthesized from this laser with an acousto-optic modulator.

We use a number of different frequency-control servo systems which have been described elsewhere [6]–[8]. The pure analog system uses 49 Hz, sine-wave modulation followed by square-wave demodulation and a second-order integrator loop. Its microwave synthesis is via direct multiplication from a crystal at 5.00688 MHz. The advantages of this system are its relatively high modulation frequency, direct 5 MHz output for use in the time scale, and well understood sources of systematic error. The pure digital system uses several different microwave synthesis schemes that all involve the addition of a 10–12 MHz offset near the top of the multiplication chain. This frequency comes from a computer controlled "direct digital synthesizer" (DDS). The entire system is frequency referenced to an active hydrogen maser and the output is in the form of a table of offset values sent to the DDS. This system uses slow, square-wave frequency modulation ($\approx$2 Hz) with blanking during the signal transients. Its advantage is its extreme frequency agility that allows us to interrogate a number of features in the information-rich cesium spectrum. We have also combined the best aspects of both the analog and digital servo-systems in a hybrid system [6]. This combination of servo systems has allowed us to automate large parts of the evaluation process. It also gives us cross checks on some of the electronic offsets.

III. Results

A. Magnetic Field Effects

In both our digital and hybrid servo systems, we operate the C-field under closed-loop servo control. This low-duty-cycle (5–10%) servo maintains the first-order Zeeman frequency constant to within 10 mHz of a preselected value. By itself, this would contribute no more than about 10^{-16} fractional error to the standard. However, since what we measure is the average field and what is needed is the average squared value of the field, the uncertainty in the correction for the second-order Zeeman shift is completely dominated by the magnetic field inhomogeneity. Field measurements made during assembly [4] showed a fractional field variation at the position of one Ramsey cavity of 5×10^{-4} relative to the mean field. This corresponds to a shift of less than 2×10^{-15}. We know that the field inhomogeneity arises from the poor fit of one end cap on the shield that surrounds the C-field solenoid. This is confirmed by the fact that the measured inhomogeneity did not change when the current in the C-field coil was reversed. Measurements of the offset of the field dependent Ramsey lines from the centers of their corresponding Rabi pedestals confirms the size of the inhomogeneity shift [1]. The associated uncertainty in the clock transition is less than 1×10^{-15}.

B. Second-Order Doppler Effect

The second-order Doppler shift is of the order of 3×10^{-13}. To achieve our accuracy goal requires a measurement of the effective, ensemble-averaged velocity with an uncertainty of $\leq$1%. We have used both a Ramsey lineshape inversion technique [9] and a pulsed optical pumping technique [10] to make these measurements. The second-order Doppler correction was computed for several microwave power levels using the two methods. While the corrections varied by nearly 2 parts in 10^{13} over the 7.5 dB power range, the corrections computed from the two methods differed by no more than 2×10^{-15} and we take this as the uncertainty.

To maintain stability of the second-order Doppler shift to this level requires control of the microwave field experienced by the atoms to about 0.1 dB. To this end, we have installed a power level servo. At present, this uses a precision power splitter and power meter which is known to be *stable* to 0.02 dB. The computer searches for the optimum power level by fitting the observed signal on resonance to a function of the measured power. The Ramsey inversion program used to measure the velocity profile also returns a measure of the absolute power. The power value returned by the Ramsey inversion program and that determined by our power level servo never differ by more that 0.05 dB.

C. Cavity-Related Errors

The end-to-end phase shift is measured in a conventional beam reversal experiment. The fractional frequency shift on beam reversal is 1.466×10^{-12} with an uncertainty of 4×10^{-15} arising purely from the statistics of the frequency measurements.

The distributed-cavity phase-shift is expected to be vanishingly small in this machine due to the use of the "ring" cavities to terminate the Ramsey structure [5]. Theoretical analysis of this cavity shows that the phase shift is quadratic in the beam retrace error and a 1 mm displacement would only lead to an error of 2.5×10^{-14}. The ovens are aligned during construction to $\pm$0.15 mm and are not movable. The actual atomic beam position is defined primarily by the beam passing holes in the Ramsey cavity. Hence, we believe the retrace error to be small compared to 0.1 mm and the distributed-cavity phase-shift to be much smaller than a part in 10^{14}. We have a system of beam masks near each end of the Ramsey cavity to investigate the effects of forced atomic beam displacement. However, blocking half of the beam at each end of the cavity reduces the signal flux by a factor of 4 and increases the measurement time for a given precision by the same factor. Preliminary investigations with these masks show no frequency shifts at the level of a few parts in 10^{14}.

Cavity pulling has been investigated by a number of techniques; the most sensitive of which is the measurement of the offset of the Ramsey lines from the center of their respective Rabi pedestals [1]. The shift near optimum power is 2×10^{-15} with an uncertainty of less than 10^{-15}.

D. Line Overlap Shift

Because of the symmetry of the spectrum, as well as the very smooth Rabi line wings produced in this standard by the use of an H-plane cavity, Rabi pulling is extremely small. The offset of the Ramsey structures from their corresponding Rabi lines shows that Rabi pulling is less than 10^{-16} for all C-field values greater than 4 μT [1].

E. AC Stark Shift

The AC stark shift in the standard caused by the black body radiation at 40°C (the operating temperature of the beam tube) is calculated to be 20×10^{-14} [11]. It is too small to be measured by actual temperature change in the standard but must be accounted for in the evaluation. The uncertainty in this value is less than a part in 10^{15}.

AC stark shift is also possible from the near resonant light that is a byproduct of the optical pumping process (fluorescence). Theoretical analysis of this effect in the geometry of NIST-7 shows it to be negligible at the 10^{-15} level [12]. A complete, experimental evaluation involves a host of optical parameters: pumping transition and polarization; beam angles, diameters and power. We have made a preliminary search and have seen no effect at the few parts in 10^{14} level.

F. Microwave Leakage

This insidious effect arises from microwave radiation in the region outside the Ramsey cavity. It has the same effect as end-to-end phase shift and would be accounted for in a beam reversal if it were stable in both phase and amplitude at every point in space and time. Leakage from microwave structures outside the beam-tube that finds its way into the standard is not stable in phase and amplitude because it travels over uncontrolled and varying pathways. It contributes to both frequency errors and long-term instability in the standard. We have used a heterodyne detector to find the sources of this external leakage and a radiator to find the points where the radiation enters the standard [13]. We have reduced both the leakage and the coupling into the standard by about 50 dB from a level where these effects produce shifts of several parts in 10^{14}. Hence, leakage from outside the standard causes shifts of much less than a part in 10^{15}. Leakage from the microwave structures within the beam-tube, if present, should be stable and accounted for in the beam-reversal evaluation of end-to-end cavity phase-shift if nothing moves inside the standard during the evaluation.

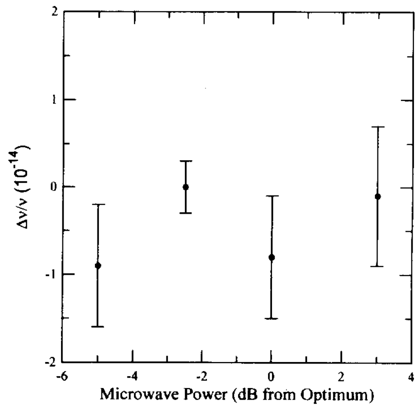

Fig. 2. Residual frequency shift as a function of microwave power after the effect of second-order Doppler shift has been removed and data from both beam directions is averaged. Error bars are 1σ.

G. Electronics

The analog servo is susceptible to errors from modulation distortion and integrator offsets. The digital servo can have subtle errors from switching transients, round off errors and aliasing. Both are susceptible to RF spectral purity and phase noise. Rather than use the frequency of the standard as a diagnostic tool to study these effects, a painfully slow process, we have developed electronic tests which are both quick and sensitive. We have developed models for and experimentally verified the sensitivity of the standard to all mixtures of FM, AM, and PM spectral sidebands [14]. We have also used heterodyne measurements to study the phase stability of the synthesized 9.2 GHz with time, temperature and following frequency switching in all of the synthesizer schemes. As an example of the sensitivity of these tests, we have found (and fixed) phase perturbations of a few hundred microradians caused by synchronous operations of the microprocessor in the commercial DDS.

The final verification of these tests lies with the standard itself. The frequency-versus-microwave power measurements outlined below are a check for most types of error. The full search through modulation parameters and switching schemes may take quite a while.

H. Frequency vs. RF Power

Most frequency biasing errors in a cesium beam frequency standard are microwave power dependent. It is customary to measure the frequency shift versus RF power in each beam direction as a check for any unexpected errors not otherwise accounted for in the evaluation. We have made these measurements and find nothing other than the expected dependence on second-order Doppler shift and end-to-end cavity phase shift (see Fig. 2). We take the lack of any residual shift with power to mean that all significant shifts have been evaluated.

IV. Summary

We can now routinely evaluate the sources of major frequency bias in NIST-7 with an overall uncertainty of less than one part in 10^{14} with just a few days of measurements. Complete evaluation of all small and subtle effects takes much longer but such effects are stable at the 10^{-14} level and their infrequent evaluation does not detract from the accurate operation of the standard.

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