

Relationship of AM to PM Noise in Selected RF Oscillators

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Abstract—We have studied the amplitude modulation (AM) and phase modulation (PM) noise in a number of 5 MHz and 100 MHz oscillators to provide a basis for developing models of the origin of AM noise. To adequately characterize the AM noise in high performance quartz oscillators, we found it necessary to use two-channel cross-correlation AM detection. In the quartz oscillators studied, the power spectral density (PSD) of the f^{-1} and f^0 regions of AM noise is closely related to that of the PM noise. The major difference between different oscillators of the same design depends on the flicker noise performance of the resonator. We therefore propose that the f^{-1} and f^0 regions of AM and PM noise arise from the same physical processes, probably originating in the sustaining amplifier.

I. INTRODUCTION

TO OUR KNOWLEDGE there has been little quantitative work analyzing the relationship between AM and PM noise in oscillators. Most of the literature assumes that the AM noise is much smaller than the PM noise. Leeson, Parker, and others have described models of PM noise in oscillators [1]–[3]. We are not aware, however, of any theory for the spectral dependence of the AM noise close to the carrier. We have therefore undertaken a detailed study of PM and AM noise in several quartz oscillators at 5 and 100 MHz to provide a basis for developing models of the origin of AM noise.

To adequately characterize the AM noise in high performance quartz oscillators, we found it necessary to use two-channel cross-correlation AM detection. The signal is split by a reactive power splitter into two signals that are each AM detected. The resulting baseband signals are then amplified and the power spectral density (PSD) of the cross spectrum between the two channels is computed. This technique provides a noise floor of order -190 dBc/Hz for AM noise at 50 kHz and -160 dBc/Hz at 10 Hz for signals of approximately +13 dBm.

An improved three-cornered-hat technique using cross-correlation makes PM noise measurements more quickly and accurately than the traditional three-cornered-hat measurement technique [4], [5]. In this approach the noise contribution of both the two references and the measurement systems average down to zero as $\frac{1}{\sqrt{N}}$ where N is the number of measurements taken. In our setup we were able to obtain a PM noise floor

of $1/2S_{\phi}(f) = -160$ dBc/Hz at 10 Hz and $1/2S_{\phi}(f) = -190$ dBc/Hz at 50 kHz.

In many of the quartz oscillators studied, the PSD of the f^{-1} and f^0 regions of AM noise is closely related to that of the PM noise over a substantial range in Fourier frequency. The major difference between different oscillators of the same design depends on the flicker performance of the resonator. AM and PM noise generally differ only in the region where the PM noise varies as f^{-3} (flicker FM noise) or in some cases f^{-4} (random walk FM noise). For example, we have found that the AM noise of 5 MHz oscillators is quite similar to the PM noise for Fourier frequencies above approximately 3 Hz. Likewise, the AM and PM noise of the 100 MHz quartz oscillators studied were quite similar for Fourier frequencies above approximately 300 Hz.

The relationship between the AM and PM noise can be exploited in many areas of analysis to assist in the design of new oscillators. It is tedious to determine the PM performance of a new oscillator design, especially one that exhibits performance superior to existing references. At the very minimum three oscillators of similar performance are required. (The requirements on the performance of the other oscillators can be relaxed considerably if the modified three-cornered-hat approach is used [4], [5].) On the other hand the AM performance of a single superior oscillator can be more easily assessed using the two-channel cross-correlation AM detection techniques presented in this paper. Therefore, it can be beneficial to use the readily obtained AM performance to evaluate the noise more quickly.

II. DESCRIPTION OF AM DETECTION CIRCUITS

Fig. 1 shows the single-sideband noise floor for a variety of available AM detectors at 5 MHz. All exhibit noise floors or resolution far above the phase noise of available oscillators at Fourier frequencies above 1 kHz and hence are unsuitable for measurements of the AM noise in these oscillators. Montress *et al.*, have used double-balanced mixers as phase detectors with good results at drive powers of +23 dBm [6]. Typically the oscillators with the best phase noise performance at 5 and 100 MHz have outputs of +13 dBm. Typical double-balanced mixers operated at this power level have relatively high levels of noise (see Fig. 2). The problem has been overcome by combining the approach of Montress *et al.* with a two-channel cross-correlation technique [4]–[8]. Fig. 3 shows the full AM measurement system schematic that was used for the cross-correlation AM noise detection.

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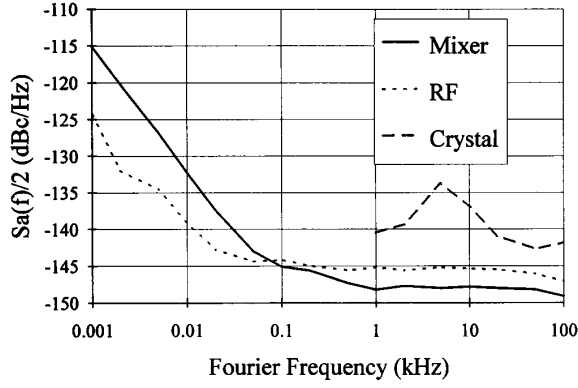


Fig. 1. AM noise floor of various detectors at 5 and 100 MHz at 13 dBm. The noise of a mixer is included for comparison. The mixer is a WJ M9HC, the RF detector is a WBE model A33, and the Crystal Detector is a HP 8472B.

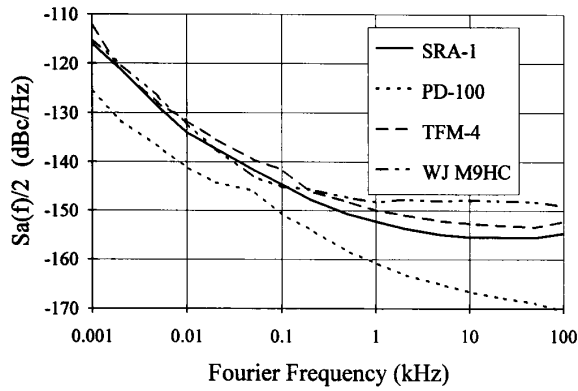


Fig. 2. AM noise floors of two mixer types at 5 and 100 MHz at 13 dBm. The PD-100 is a single-balanced mixer, the SRA-1 is a double-balanced mixer, the TFM-4 is a double-balanced mixer, and the WJ M9HC is a double-balanced mixer.

III. MEASUREMENT TECHNIQUES

At the output of the spectrum analyzer we obtain $S_a(f)/2$ for AM noise, or $S_\phi(f)/2$ for PM noise measurements. These are the single-sideband spectral densities of our signal. The practical definition for the power spectral density of amplitude fluctuations is given by

$$S_a(f) = \frac{[\Delta\varepsilon(f)]^2}{V_0^2} \frac{1}{\text{BW}} \quad (1)$$

where $\Delta\varepsilon(f)$ is the change in amplitude measured at Fourier separation f from the carrier, V_0 is the average carrier voltage and BW is the noise bandwidth of the spectrum analyzer. For the measurement system shown in Fig. 3, (1) becomes

$$S_a(f) = \frac{\text{PSD}(V_n)}{[k_a G(f)]^2} \quad (2)$$

where PSD denotes power spectral density, V_n is the noise voltage out, k_a is the sensitivity factor of the detector for converting $\Delta\varepsilon/V_0$ to a voltage, and $G(f)$ is the amplifier gain versus Fourier frequency.

At the output of the spectrum analyzer we obtain the power spectral density (PSD) for each of the two channels.

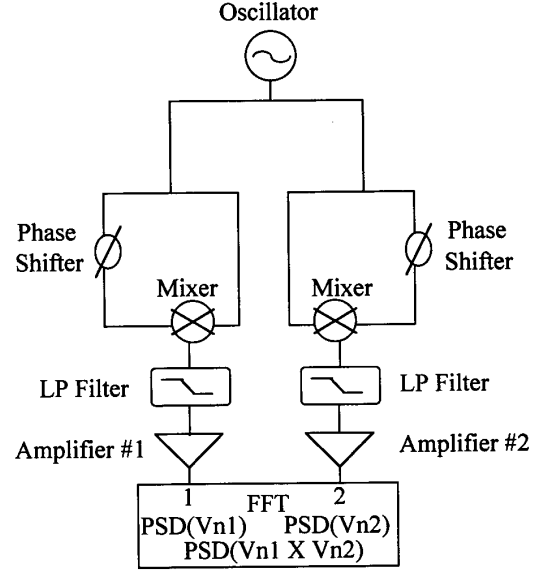


Fig. 3. AM cross-correlation measurement system.

This provides us with the noise from the source and the measurement noise in each respective channel [see (3) and (4)]. By using cross-correlation we can obtain the AM noise of the source as shown in (5). The uncorrelated noise between the channels averages away as the reciprocal of the square root of the number of averages.

$$\text{PSD}(V_{n1}) = \alpha(\text{AM}_{\text{Source}} + \text{System}_1), \quad (3)$$

$$\text{PSD}(V_{n2}) = \alpha(\text{AM}_{\text{Source}} + \text{System}_2), \quad (4)$$

$$\text{PSD}(V_{n1} \times V_{n2}) = \alpha \left[\text{AM}_{\text{Source}} \times \left(+ \frac{\text{System}_1 + \text{System}_2}{N^{1/2}} \right) \right], \quad (5)$$

$$\text{PSD}(V_{n1}) - \text{PSD}(V_{n1} \times V_{n2}) = \alpha(\text{System}_1), \quad (6)$$

$$\text{PSD}(V_{n2}) - \text{PSD}(V_{n1} \times V_{n2}) = \alpha(\text{System}_2), \quad (7)$$

where the proportionality constant $\alpha = [k_a G(f)]^2$. Subtracting the source noise from (3) and (4), respectively, we are able to obtain the noise in each channel [see (6) and (7)].

IV. AM AND PM NOISE FOR SELECTED OSCILLATORS

AM and PM noise measurements on various oscillators are described below. The mathematical models for their relationships are presented in the following section.

Fig. 4 shows the AM and PM noise of a 5 MHz oscillator that uses an AT-cut resonator driven at low power. The AM noise was measured using the techniques described in the previous section. Calibration of the PM noise was done using a three-cornered-hat technique outlined in [8]. The PM noise in the 0.3 to 3 Hz range shows flicker frequency (f^{-3}) as expected with white PM noise at higher Fourier frequencies.

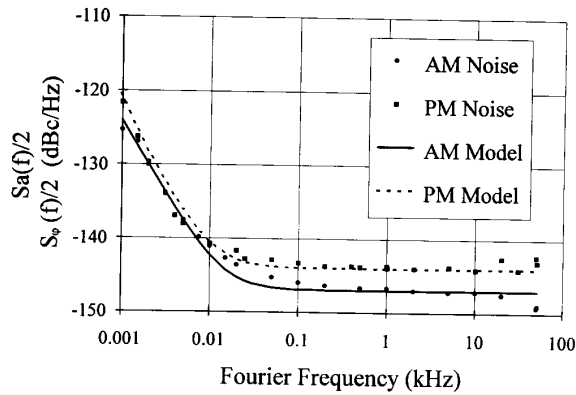


Fig. 4. AM and PM noise with models for AT-cut 5 MHz oscillator.

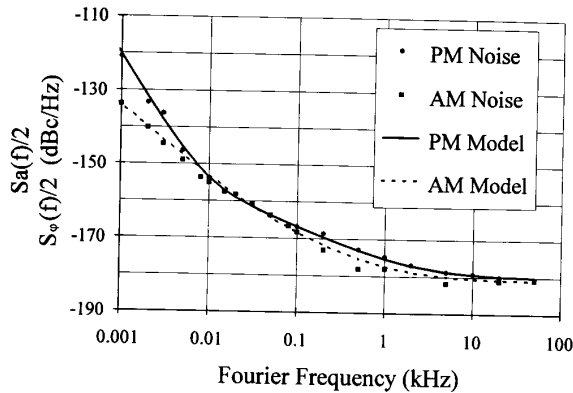


Fig. 5. AM and PM noise with models for SC-cut 5 MHz oscillator.

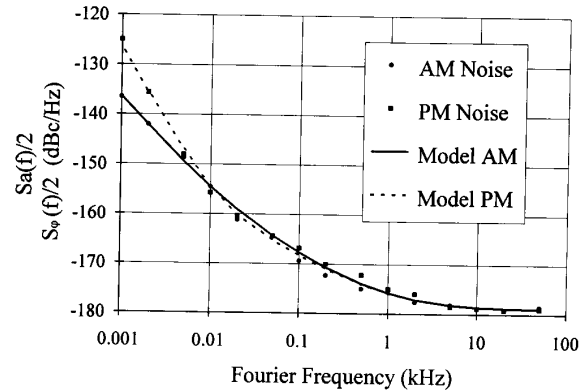


Fig. 6. AM and PM noise with models for SC-cut 5 MHz oscillator.

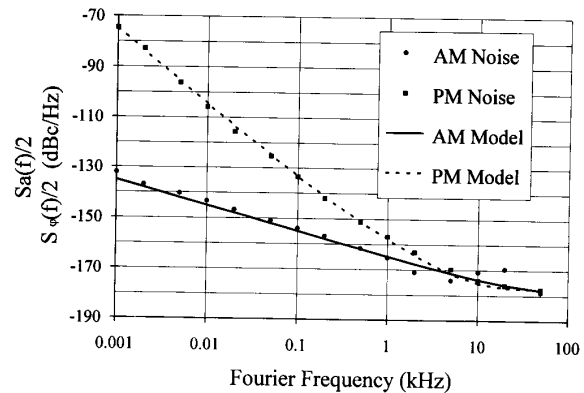


Fig. 7. AM and PM noise with models for SC-cut 100 MHz oscillator.

The AM noise shows f^{-2} (white frequency) behavior as does the PM noise until 30 Hz and continues to follow the white PM noise to higher Fourier frequencies. The models for the AM and PM noise are both similar for the f^{-2} and f^0 components of the noise.

Fig. 5 shows AM and PM noise of a 5 MHz high performance oscillator that uses an SC-cut resonator driven at relatively high power. The AM and PM noise were measured with the techniques described above. The PM noise exhibits flicker FM, flicker PM, and white PM noise. The AM noise tracks the PM noise from approximately 10 Hz to at least 50 kHz. The models for the AM and PM noise are both similar for f^{-1} and f^0 components of the noise.

Fig. 6 shows AM and PM noise of another 5 MHz high performance oscillator of the same SC-cut design as Fig. 5. The PM noise was measured using a cross-correlation technique described by Walls [5]. Again the PM noise exhibits flicker FM, flicker PM, and white PM noise. The AM noise close to the carrier has a white frequency (f^{-2}) component but follows the PM noise in both the flicker PM and white PM noise characteristics at higher frequencies.

Fig. 7 shows AM and PM noise of a high performance 100 MHz oscillator. The AM noise was measured with the technique previously described. A three-cornered-hat technique was used to obtain both the AM and PM noise. The PM noise

fits the classic model described by Parker with flicker FM, white frequency, and white PM noise [10]. The AM noise falls as f^{-1} and joins with the PM noise at higher frequencies.

Fig. 8 shows AM and PM noise for another high performance 100 MHz oscillator. The AM noise was measured with the same technique as described above. The PM noise was measured using the modified three-cornered-hat technique outlined in [4]. The PM noise follows Parker's model with flicker FM, white frequency, and white PM noise [10]. The AM noise falls as f^{-1} and follows the white PM noise of the PM noise at higher frequencies from the carrier.

V. DISCUSSION

The AT-cut 5 MHz oscillator, shown in Fig. 4, fits the following models with minimal variations:

$$\text{PM Model: } \frac{10^{-12.5}}{f^4} + \frac{10^{-13.9}}{f^3} + \frac{10^{-12.3}}{f^2} + 10^{-14.4}. \quad (8)$$

$$\text{AM Model: } \frac{10^{-12.4}}{f^2} + 10^{-14.7}. \quad (9)$$

The AM and PM models differ by 1 dB for the f^{-2} noise and in the f^0 component of the noise by 3 dB. The SC-cut 5 MHz

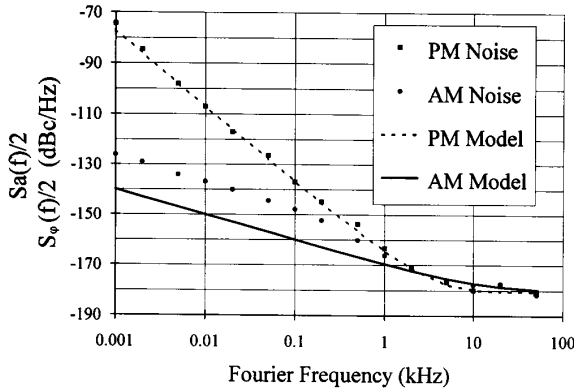


Fig. 8. AM and PM noise with models for SC-cut 100 MHz oscillator.

oscillator in Fig. 5 follows the following models:

$$\text{PM Model: } \frac{10^{-12.1}}{f^4} + \frac{10^{-12.8}}{f^3} + \frac{10^{-14.7}}{f} + 10^{-18.02}. \quad (10)$$

$$\text{AM Model: } \frac{10^{-13.4}}{f^2} + \frac{10^{-15}}{f} + 10^{-18.1}. \quad (11)$$

The f^{-1} noise of the AM and PM models in this case differ by 3 dB and the f^0 component by only eight tenths of a dB. The 5 MHz oscillator in Fig. 6 shows a similar AM and PM model relationship as the previous SC-cut 5 MHz oscillator with only slight variations in the exponents of the model:

$$\text{PM Model: } \frac{10^{-12.65}}{f^3} + \frac{10^{-14.87}}{f} + 10^{-17.9}. \quad (12)$$

$$\text{AM Model: } \frac{10^{-13.65}}{f^2} + \frac{10^{-14.87}}{f} + 10^{-17.9}. \quad (13)$$

An interesting feature in these AM and PM models is that the f^{-1} and f^0 noise components are identical.

The 100 MHz oscillator in Fig. 7 resembles the models from Fig. 8 quite closely except for a slight difference in the f^{-2} component of the PM noise model. The similarities between the AM and PM models of this oscillator are seen in the f^0 component where there is approximately a 3 dB difference between the two models:

$$\text{PM Model: } \frac{10^{-7.46}}{f^3} + \frac{10^{-9.9}}{f^2} + 10^{-17.77}. \quad (14)$$

$$\text{AM Model: } \frac{10^{-13.5}}{f} + 10^{-18}. \quad (15)$$

The 100 MHz oscillator in Fig. 8 matches the following models with slight discrepancies between the model and the close in data. In this case the f^0 noise characteristics are identical:

$$\text{PM Model: } \frac{10^{-7.7}}{f^3} + \frac{10^{-10.8}}{f^2} + 10^{-18.06}. \quad (16)$$

$$\text{AM Model: } \frac{10^{-14}}{f} + 10^{-18.06}. \quad (17)$$

We do not understand the differences between the models for our 100 MHz AM and PM noise measurements. However, the model characteristics of Figs 7 and 8, whose oscillators were

of similar cut and brand, have similar AM and PM models for their f^0 component. The f^0 model differs at most by 3 dB. The f^{-2} component, however, differs by about 9 dB.

VI. CONCLUSION

The model relationships between the various 5 MHz oscillators present strong evidence to suggest that there is a connection between the AM and PM noise for a specific oscillator. f^{-1} and f^0 components in both the AM and PM noise seem to come from the same source. The 100 MHz oscillator relationships are inconclusive in this respect, although they maintain similar model relationships between the two SC-cut oscillators tested. We speculate that the difference in the model and the data in Fig. 8 could be due to compression in the output amplifier of our system. The f^0 component of the AM and PM noise for the 100 MHz oscillators, however, seems to be similar and should give some information about the thermal noise characteristics in our oscillators.

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Fred L. Walls (SM'94) for a photograph and biography, see p. 521 of the July 1994 issue of this TRANSACTIONS.