

Reducing Local Oscillator Phase Noise Limitations on the Frequency Stability of Passive Frequency Standards: Tests of a New Concept

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Abstract—We report on the experimental test of a new concept for reducing the limitation on short-term frequency stability of passive frequency standards due to local oscillator phase noise. This concept is general and can be applied to many passive frequency standards. Systems that use sinewave modulation to interrogate a stable resonance are limited in short-term frequency stability by phase noise at the second harmonic of the modulation, f_m . This effect limits the fractional frequency stability to approximately $\sigma_y(\tau) = 0.9(f_m/\nu_0) (S_\phi(2f_m))^{1/2} \tau^{-1/2}$, where ν_0 is the carrier frequency and $S_\phi(2f_m)$ is the phase noise at twice the modulation frequency. (Contributions from higher even harmonics of the modulation generally can be neglected). This new concept uses notch filters at $\pm 2f_m$ from the carrier to reduce this effect. Tests on a modified passive rubidium standard demonstrate an improvement of approximately 18 in $\sigma_y(\tau)$. The dual notch filters proved to be feasible and were obtained commercially. Measurements suggest that ultimate performances of less than $2 \times 10^{-14} \tau^{-1/2}$ are possible if the atomic resonance has sufficient quality.

I. INTRODUCTION

THE SHORT-TERM frequency stability of passive standards using sine wave modulation is often limited by phase noise at the second harmonic of the modulation frequency f_m [1]–[4]. When examined in detail, the limit on short-term frequency capability depends on the modulation index, linewidth, and all even harmonics of f_m [4].

$$\sigma_y(\tau) = \left[\sum_{n=1}^{\infty} \left(\frac{P_{2n-1} - P_{2n+1}}{P_1} \right)^2 S_y(2nf_m) \right]^{1/2} / 2\tau^{1/2}, \quad (1)$$

where $\sigma_y(\tau)$ is the Allan deviation, $S_y(f)$ is the spectral density of fractional frequency fluctuations, and P_{2n+1} is the Fourier coefficient of rank $2n + 1$ in the response of the resonance. For most systems the largest contribution is due to the noise at $\pm 2f_m$ from the carrier ($n = 1$). This first term can be rewritten as

$$\sigma_y(\tau) = 0.9 \frac{f_m}{\nu_0} \sqrt{S_\phi(2f_m)} \tau^{-1/2}, \quad (2)$$

where $S_\phi(f)$ is the spectral density of phase noise. For example, consider a system with $f_m = 137$ Hz and $S_\phi(274$

Hz) = $10^{-15.2}$ rad²/Hz at a carrier frequency of 5 MHz. The limit to $\sigma_y(\tau)$ due to $S_\phi(274)$ is approximately $\sigma_y(\tau) = 6.2 \times 10^{-13} \tau^{-1/2}$. This effect is a very serious problem for proposed diode-laser-pumped, passive rubidium standards since a frequency stability of approximately $\sigma_y(\tau) = 1 \times 10^{-14} \tau^{-1/2}$ has been projected, based on signal-to-noise and linewidth [5],[6]. A local oscillator with phase noise at $2f_m$ low enough not to significantly compromise this performance can presently be achieved only with cryogenic techniques [7]–[9].

In this paper we present details on a new approach that significantly reduces the effect of phase noise in the local oscillator on the frequency stability of passive frequency standards such as diode-laser-pumped rubidium [4],[6]. This new room temperature approach makes use of a special filter with a notch at both the upper and lower $2f_m$ sidebands. To minimize the limitation on $\sigma_y(\tau)$, f_m should be small, the carrier frequency at which the filters are applied should be high, and the contribution of the phase noise of the filter and following synthesis chain should be low. As a first choice we have chosen $f_m = 37.5$ Hz and $\nu_0 = 10$ MHz, to avoid power line frequencies, to be consistent with available Q -factors in SC-cut quartz resonators, and to be approximately compatible with the expected linewidths in the new passive rubidium standards. This choice also reduces the contribution from the multiplier chain by 6 dB as compared to filtering at 5 MHz. We show that the available attenuation of the noise around $\pm 2f_m$ approaches 30 dB. From the theory developed by the Laboratoire de l'Horloge Atomique group at Orsay [4], we expect that the limits to the short-term frequency stability due to local oscillator phase noise can be improved by approximately a factor of 15–30 over traditional approaches. Contributions at the 4th and higher even harmonics of the modulation can generally be neglected if the modulation index is not too high [4]. Experimental measurements confirm this view and indicate that short-term frequency stabilities of better than $2 \times 10^{-14} \tau^{-1/2}$ can be expected if the atomic signal has sufficient quality. Further improvements seem likely.

II. DUAL NOTCH FILTERS

The special dual notch filters proved to be feasible and were obtained commercially. They were assembled using two pairs of third overtone 10 MHz SC-cut resonators with nominal Q factors of 10^6 [10]. The resonators were selected to have

Manuscript received October 25, 1993; revised February 18, 1994; accepted February 27, 1994.

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IEEE Log Number 9401989.

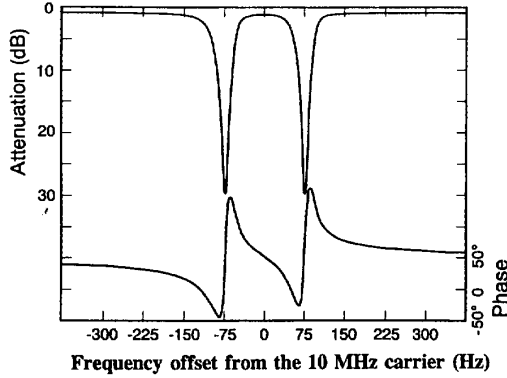


Fig. 1. Transmission loss and phase shift across dual notch filter at 10 MHz.

matched temperature turnover points at approximately 60°C. Fig. 1 shows the insertion loss and the phase shift across the resonator as measured on a 50 Ω network analyzer. The insertion loss through the filter at 10 MHz is about 1 dB while the insertion loss at the bottom of the notch approaches 30 dB. Since the 10 MHz carrier signal is far away from the frequency of the notches, the crystal resonators carry very little power, basically only the noise power. This means that approximately 30 dB more power can be transmitted through the dual notch filter than could be transmitted through a band-pass filter made from 10 MHz resonators of similar quality. High output power is critical in minimizing the noise in the multiplier chain following the filter. The phase variation with frequency at 10 MHz is at least a factor of 30 lower than the phase slope at the center of the notches. This means that the phase of the 10 MHz carrier is much less affected by the frequency variations of the resonators than if a band-pass filter had been used. Some temperature control is needed but not the same thermal regulation that would have been required for a traditional band-pass filter.

Fig. 2 shows the block diagram of the system used to measure the phase noise of the filter. The sensitivity of the mixer and gain of the amplifier were calibrated using the PMCAL approach [11]. The accuracy of the calibration is typically ± 0.5 dB. Two notch filters were used in the measurement to equalize the delays on both sides of the phase bridge. This significantly improves the rejection of the phase noise of the source oscillator resulting in a lower noise floor for the measurements. The phase noise of a single filter (assuming equal contributions) was measured to be flicker phase at a level of $S_{\phi}(f) = 10^{-14}/f$ rad²/Hz (-143 dBc/Hz at 1 Hz) and $S_{\phi}(75 \text{ Hz}) = 10^{-16.4}$ rad²/Hz (-167.4 dBc/Hz) at the center of the notch. These results for the phase noise near the carrier are extraordinarily low and further proof of the advantages of notch filters over band-pass filters.

III. EXPERIMENTAL TEST OF SHORT-TERM FREQUENCY STABILITY

To test the concept of using a dual notch filter for improving the short-term frequency stability of passive standards, we modified the electronics of a small commercial rubidium

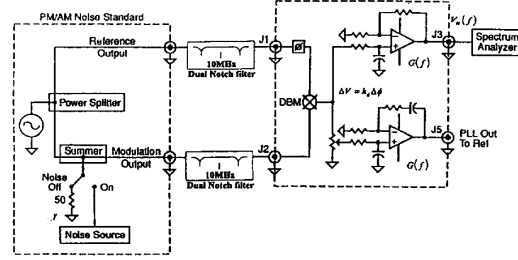


Fig. 2. Block diagram of system used to measure phase noise of reference source and dual notch filter.

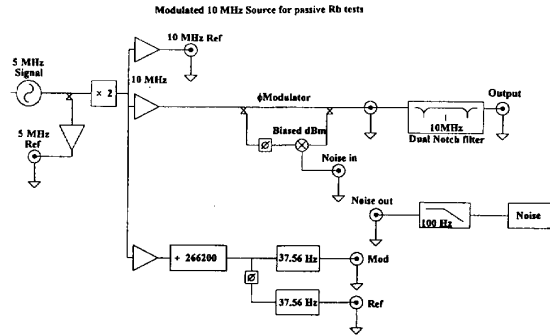


Fig. 3. Block diagram of the external 10 MHz source used to drive the modified commercial passive rubidium frequency standard.

standard to use an external 10 MHz local oscillator and a modulation frequency of 37.5 Hz. The discharge lamp was removed and the cell was optically pumped using a diode laser that was both frequency and amplitude stabilized [12]. The system was adjusted to accommodate the different noise levels in the electronics.

Fig. 3 shows the block diagram of the 10 MHz source used for testing the effectiveness of the dual notch filter to improve the short-term frequency stability of passive standards. An external noise source with white noise from approximately 1 to 100 Hz can be used to drive a dc-coupled phase modulator. This is used to artificially increase the noise of the 10 MHz local oscillator. The output of the oscillator can be taken before or after the dual notch filter. Fig. 4 shows the phase noise after the filter with noise on. The frequency stability of the diode-laser-pumped passive rubidium standard without noise and without notch filter is shown as curve A in Fig. 5. The frequency stability with noise added to the oscillator and no notch filter is shown in curve B, while the frequency stability with noise and with filter is shown in curve C. The frequency stability achieved with the filter and noise on is virtually identical with that achieved with no noise, demonstrating that the dual notch filter is very effective in reducing the effect of the noise at $\pm 2f_m$ on the short-term frequency stability. The frequency stability achieved with the dual notch filter and added noise is not limited by the notch performance but by other details in the small commercial physics package.

The phase noise at $2f_m$ was measured to be $S_{\phi}(2f_m) = 10^{-9.7}$ rad²/Hz with noise modulation and no filter. Using this value in (2) yields a frequency stability of $\sigma_y(\tau) =$

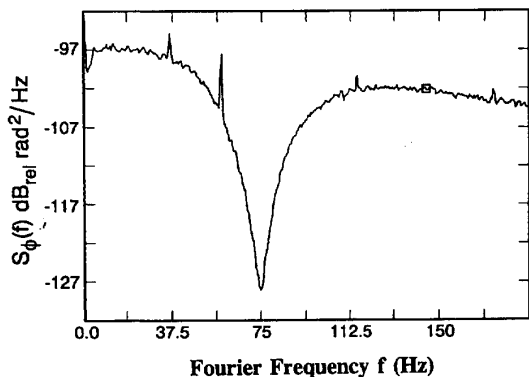


Fig. 4. Phase noise of the local oscillator with noise on measured after the dual notch filter.

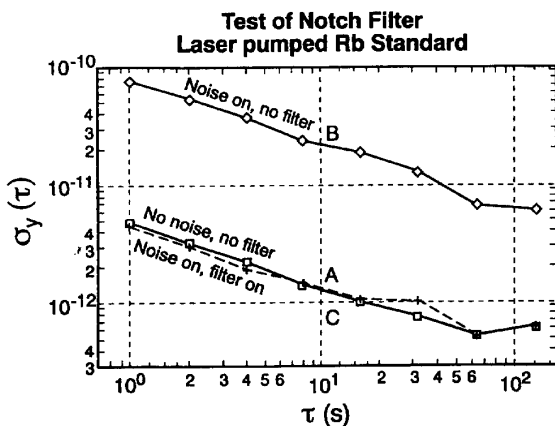


Fig. 5. Short-term frequency stability of modified commercial passive rubidium frequency standard. Curve A shows the performance with no added noise and no filter. Curve B shows the performance with added noise and no filter. Curve C shows the performance with added noise and filter on.

$5 \times 10^{-11} \tau^{-1/2}$, which agrees well with the measured value of $\sigma_y(\tau) = 7.5 \times 10^{-11} \tau^{-1/2}$.

Demonstrating a full electronics system capable of $\sigma_y(\tau) = 1 \times 10^{-14} \tau^{-1/2}$ will have to wait for further work on a rubidium physics package designed to take full advantage of the diode-laser pumping. We have made several 10 to 100 MHz multiplier chains that do not significantly degrade the measured noise floor at the bottom of the notch filter. Using the presently measured performance of the dual notch filter and (2), we project a lower limit to the short-term frequency stability due solely to the electronics of $2 \times 10^{-14} \tau^{-1/2}$. Additional measurements should confirm this. If necessary, additional reductions in the phase noise at the bottom of the notches may also be obtained by selection of lower noise resonators and/or passive components.

IV. CONCLUSION

This work confirms the validity of a new general concept for substantially improving the short-term frequency stability of many passive standards by using notch filters at $\pm 2f_m$ from the carrier and confirms that the theory advanced in

[4] is substantially correct. Further, we have shown that appropriate notch filters for passive rubidium standards can be constructed from available quartz resonators. Such filters have many practical advantages such as less effect on the phase noise of the carrier and higher power handling capability than traditional band-pass filters. Measurements on prototype filters and multiplier chains indicate that this approach could support diode-laser-pumped passive rubidium standards operating at approximately $\sigma_y(\tau) = 2 \times 10^{-14} \tau^{-1/2}$. Additional refinements may reduce this limitation even further.

ACKNOWLEDGMENT

The authors thank Franklin Ascarrunz, Huascar Ascarrunz, and Lisa M. Nelson for assistance in constructing and testing the 10 MHz source, and Roland Barillet for helpful discussions.

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R. E. Drullinger, photograph and biography not available at the time of publication.

A. Novick, photograph and biography not available at the time of publication.