Results on Limitations in Primary Cesium Standard Operation

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***Abstract*—We report on the most recent design changes in our two primary cesium standards, their current operational use, results obtained, and limitations. NBS-4, the shorter device with an interaction length of $L = 0.52$ m, has been extensively used for many months as a clock. After improvements in the magnetic shielding and microwave feed, we have obtained σ_y (1 week < τ < 2 weeks) = 7×10^{-15} in a 10-Hz bandwidth for its frequency stability. NBS-6, the longer, more accurate device ($L = 3.75$ m), features a linewidth ($\lesssim$30 Hz), which is believed to be the narrowest linewidth ever reported for a cesium device. NBS-6 has been operated to give a short-term stability σ_y (1 s) = 7×10^{-13} in a 10-Hz bandwidth and has capability of easy beam reversal. The current and past rates of the International Atomic Time (TAI) in terms of our primary cesium standards are reported and compared with the results of other laboratories. With NBS-6 we have calibrated the rate of the NBS time scale of an uncertainty of 0.9×10^{-13}.**

INTRODUCTION

IN THE FOLLOWING, we will report design changes and results for the two NBS primary cesium standards, NBS-4 and NBS-6. Particular attention has been paid to improving the long-term stability of NBS-4 and in evaluating the limitations of an absolute frequency determination using NBS-6. Most of the construction details of both standards are reported elsewhere [1], [2] and are not repeated here.

The most recent hardware changes [2] were those completed on NBS-6 (formerly called NBS-5), which included provisions for increased pumping speed and improved beam reversal capability. Larger pumps were installed to reduce beam scattering resulting in pressures < 1.3×10^{-5} N/m^2 (typically 1–3 $\times 10^{-6}$ N/m^2; 1 torr $\simeq$ 133 N/m^2) and observed linewidths as small as 24 Hz. Identical oven-detector assemblies were installed on both ends of NBS-6. These were mounted on movable slides with the provision that the oven and detector on one slide would be totally isolated from each other. For example, if the oven on one end was in use, the detector would be completely isolated from the main vacuum system, hence preventing contamination of the detector system. In addition, the reversal is accomplished without significantly altering the pressure in the main vacuum system. Time required to mechanically move the slides (thereby reversing the beam) is only a matter of minutes. However, typically several hours were used to allow the ovens to cool slowly and prevent clogging of the collimators.

In the following, we report some minor changes made in the NBS-4 system and give the resulting long-term stability. We also report the results of a full accuracy evaluation of NBS-6 and calibration of the NBS time scale. This evaluation has located the important limitations and suggests areas of improvement for further evaluations.

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NBS-4

NBS-4 is a relatively short device [2] with a cavity interaction length of $L = 0.52$ m. It has been used as a clock since July 1975. The main reasons for this are twofold: 1) although a full accuracy evaluation can be performed on NBS-4, it is more difficult than for NBS-6. This is primarily because the confidence in the frequency bias due to cavity phase shift is reduced in NBS-4. Although it is possible to reverse the beam in NBS-4, this can only be done by opening the system to air, physically removing the vacuum bulkheads housing the detector and oven, reinstalling in the opposite orientation (including some rewiring) and pumping down again. In addition to possible systematic errors which are likely to enter, this operation is time consuming and therefore preferably avoided. A power shift [3], [4] evaluation is still possible, but is, however, plagued with systematic shifts of a distributed cavity phase shift effect [4]. (See discussion below for NBS-6.) 2) NBS-4 and NBS-6 are located in the same room, while the ensemble of commercial clocks used in the NBS time scale [5] is located in a different room in environmentally controlled chambers. Therefore, if NBS-4 is included in the time scale, a partial check is provided for effects causing the NBS time scale commercial clock ensemble to change frequency as a whole.

Consequently, it was decided to use NBS-6 strictly to perform the frequency calibrations and to use NBS-4 as part of the time scale ensemble. Before this was done, some minor changes were made on NBS-4 to improve long-term stability. This was motivated by the observation that physical disturbances taking place near NBS-4 caused observable degradation of its stability. For example, a correlation between visits to the standards laboratory and NBS-4 stability was observed (effects in the 10^{-14} range). Therefore, three main changes were effected to improve stability: 1) An additional magnetic shield (0.16-cm thick moly–permalloy) was installed external to the vacuum system. 2) A more rigid microwave feed was installed with the idea of reducing mechanical instabilities associated with the tuner used to provide matching of the input waveguide to the Ramsey cavity. Previously, a commercial E-H tuner provided this function; it has been replaced by a section of guide with three capacitive screws placed $\lambda/4$ apart along the guide. 3) Finally, the entire NBS-4 apparatus was moved to a new location in the standards room and is isolated by more than 5 m from visitors and other disturbances associated with normal laboratory activities.

With the idea of prolonging running time, the oven temperature was reduced (to 80°C from 109°C) until the short-term stability was made comparable to that of the other clocks in the ensemble. Operating conditions since December, 1975, are such that $\sigma_y(1\ \text{s}) \cong 7 \times 10^{-12}$ in a 10-Hz bandwidth for a signal current = 1.65 pA (current at center of Ramsey pattern at P_{opt} minus background current), background current = 0.20 pA (current with microwave power off). With these operating conditions we have observed the frequency stability shown in Fig. 1. For times 1 week < τ < 2 weeks, the stability has been improved by approximately a factor of 2.5 from the measurements made previous to the above changes.

NBS-6

NBS-6 [2] has been in operation since March of 1975. Since that time its use has been directed toward making an accuracy evaluation of the NBS time scale. Hardware changes have been minimal since that time, and construction is detailed in [1] and [2]. Linewidths as small as 24 Hz have been observed (L = 3.75 m), but the conditions for maximum signal to noise (coinciding with the condition for geometrical alignment of source, detector, magnets, and

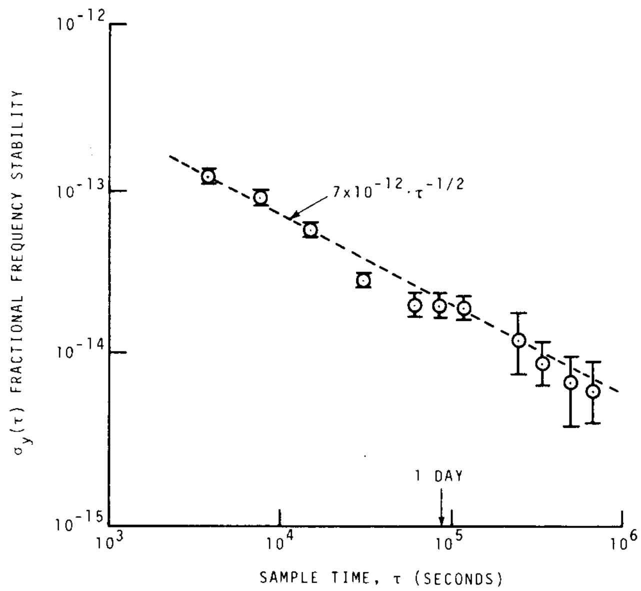

Fig. 1. Measured stability of NBS-4 from December 1975 to January 1976. (bandwidth = 10 Hz).

cavities) yielded linewidths of approximately 28–30 Hz. With relatively high beam current (T_{oven} = 88°C, signal current = 9 pA, background current = 13.5 pA), a frequency stability of σ_y (1 s) = 7×10^{-13} in a 10-Hz bandwidth was observed when compared to a hydrogen maser. (Normal operating conditions were for signal current $\simeq$ 2 pA, background current $\simeq$ 2 pA.)

The systematic frequency offsets that have been investigated are listed in Table I. The procedure adopted here is similar to that of the last complete evaluation which was reported in [1]. Below, each of the (potential) biases is discussed and reference to previous work made when the evaluation procedure was essentially unchanged. Other possible effects should be discussed for complete generality; those that are clearly negligible (e.g., Bloch–Siegert shifts) are not discussed here[1].

Ideally, all biases and potential biases should be checked *in situ*—that is, with the standard in normal operating condition. Then checks would be made by varying one system parameter and observing the frequency change of the standard. This approach is illustrated in the first category of systematic offsets discussed below. This type of approach can be realized in most, but not all, of the categories discussed below. A notable exception is the frequency shift due to magnetic-field inhomogeneity discussed in [2].

1) Servo System Offsets: A frequency bias might occur here, for example, if the first amplifier after the audio demodulator (18.75 Hz) [1], [2] had an associated input offset voltage. Although this type of offset could be detected by directly measuring the offset on the amplifier, it is simpler and more general to vary the ac gain of the

[1] The Millman effect was reported as a possible cause for frequency bias [12]. However, the Millman effect is absent for the $\Delta m_F = 0$ transitions which include the clock transition.

TABLE I

Bias	Bias (Δy)	Uncertainty
1. Servo-System Offsets		
(a) Amplifier offsets	0	0.02×10^{-13}
(b) Second harmonic distortion	0	0.15×10^{-13}
2. Magnetic-Field Effects		
(a) Offset due to finite field	$+536 \times 10^{-13}$ (typical)	0.03×10^{-13}
(b) Magnetic-field inhomogeneity	$+0.02 \times 10^{-13}$	0.02×10^{-13}
(c) Majorana transitions	0	0.03×10^{-13}
3. Pulling by Neighboring Transitions	$+0.4 \times 10^{-13}$	0.20×10^{-13}
4. Cavity Pulling	0	0.01×10^{-13}
5. RF Spectrum	0	0.02×10^{-13}
6. Second-Order Doppler Shift	-3.1×10^{-13} (typical)	0.10×10^{-13}
Cavity Phase Shift (for a particular direction)	$+0.25 \times 10^{-13}$	0.80×10^{-13}
7. Total Error Due to Systematic Frequency Biases		
(a) Root mean square	—	0.85×10^{-13}
(b) Sum of errors	—	1.38×10^{-13}
8. Random Uncertainty	0	0.31×10^{-13}

system, for example, by attenuating the output of the preamplifier on the detector before the audio demodulator.

A more subtle effect is an offset due to second harmonic content at the point where the 18.75-Hz phase modulation is introduced into the RF. This effect could be studied independently by constructing an identical phase modulator and running the two, first in series and then in parallel. Elimination of second harmonic in both cases eliminates second harmonic effect for both phase modulators. The problem with this type of measurement is that the phase modulators are not in their actual operating environment; and unless an extremely high degree of isolation is provided, second harmonic which can be nulled out by the above scheme may creep back into the system when the phase modulator is placed in its actual operating environment. Just such system-dependent effects were observed. Therefore, an *in situ* approach was taken. It is noted that if second harmonic content is present at the phase modulator, its effect on the output frequency of the locked standard should vary with the amplitude of phase modulation. The mathematical details of this effect are outlined in [6]. It is sufficient to say that the magnitude of the frequency bias can be varied by approximately a factor of two, therefore placing an upper limit on the size of the effect in the operating output frequency. Relative frequency shifts ($\lesssim 10^{-14}$) were measured for several different modulation amplitude settings, yielding an upper limit to this effect (see Table I).

2) Magnetic Field Associated Errors: The average field is, of course, easily measured and can be found, for example, by determining $\nu_z \equiv \nu[(4,1) \leftrightarrow (3,1)] - \nu[(4,0) \leftrightarrow (3,0)]$. The uncertainty here is small and is usually due to a slight field drift observed over the time of a measurement of the clock frequency. For our case, $\nu_z \cong 23.8$ kHz, and drift was typically 2 to 3 Hz over the one-day measurement time, resulting in a shift of order 10^{-14}. A possible important systematic offset is that due to magnetic field inhomogeneity [7]. Unfortunately, this offset is not easily determined "*in situ*." A necessary but not sufficient condition is that the Ramsey peaks be well centered on the Rabi pedestals for the first-order field dependent lines. This test plus actual field probe measurements at the time of assembly indicates this effect is small but not totally negligible [1], [2]. Finally, the possibility of Majorana flops may cause a slight frequency pulling [7]. Here we rely on the measurements of spatial field variations at the time of assembly and estimate this effect to be very small [1], [2].

3) Pulling Due to Neighboring Lines: This familiar overlap problem can be eliminated by going to higher magnetic field or somehow equalizing the signal strengths of the symmetrically placed field-dependent lines. The latter solution is difficult to achieve in practice, and unfortunately the first solution has the limitation that magnetic field problems become more severe as the field increases. For example, a given change in field (due to some external perturbation, for instance) shifts the output frequency by an amount proportional to the field strength, therefore degrading long-term stability. We have operated at a low enough field where the overlap pulling is small (but not completely negligible), and the stability is acceptable. For NBS-6, $\nu_z \cong 23.8$ kHz, and the frequency stability flickers out at $\sigma_y(\sim 1 \text{ day}) \cong 2 \times 10^{-14}$, corresponding to the observed daily field variation of approximately $\Delta\nu_z \cong 3$ Hz.

For these conditions the overlap pulling was calculated [7] from the entire microwave spectrum at optimum power to be $\Delta y = +(0.26 \pm 0.15) \times 10^{-13}$. We can also measure the magnitude of the pulling by measuring the output frequency as a function of magnetic field. Without overlap pulling the output frequency should be of the form $\nu(H_z) = \nu_0' + \beta H_z^2$. ν_0' is a frequency which depends on other systematic effects but should be independent of magnetic field. We can now make a comparison of the observed values of $\nu(H_z)$ versus the calculated values based on the above equation; any departure we will attribute to overlap pulling. (For fields large enough that the Rabi pedestals do not overlap, the pulling should be proportional to H_z^{-3}.) In this way, we have attributed a shift $\Delta y = (+0.4 \pm 0.2) \times 10^{-14}$ to overlap pulling at the typical operating field ($\nu_z \cong 23.8$ kHz).

The field extrapolation procedure is actually more general than indicated above; we have been able to explain measured departures on the basis of overlap pulling. However, more extreme departures could be explained by other effects. For example, if the field inhomogeneity were field dependent, gross effects would show a departure as the inhomogeneity shift changed. If Majorana transition pulling was present, it might also be expected to change with field. Therefore, unexplained deviations in the field extrapolation might point out the existence of other systematic offsets in addition to overlap pulling. For NBS-6 at the operating field $\nu_z \cong 23.8$ kHz the deviations could be explained by the pulling from neighboring lines.

4) Cavity Pulling: Cavity pulling [7] is usually not a problem but is not totally negligible if one operates at nonoptimum power. (See discussion on power shift method of determining phase shift.) The cavity is set by maximizing beam output at ½ optimum power; the random uncertainty in our ability to set the cavity to the cesium frequency is determined by the dispersion of center frequency measurements as determined by cavity absorption measurements. The uncertainty in Table I is estimated at optimum power from a combination of the uncertainty in our ability to set the cavity to the cesium frequency and our ability to set microwave power to optimum.

5) RF Spectrum: This was measured directly at X-band. The only measurable spurious sidebands were 120-Hz sidebands, which were below the carrier by at least 50 dB and symmetric to at least 5 percent; therefore, offsets [7] of less than 10^{-14} are expected here.

6) Second-Order Doppler and Cavity Phase Shift: Second-order Doppler shift is calculated from the expression $\Delta y = -\frac{1}{2}\langle (v/c)^2 \rangle$, where the average involves the velocity distribution determined from the Ramsey resonance curves [3], [4]. The error quoted here is an estimate of the ambiguity of this method.

Cavity phase shift is determined two ways: by beam reversal and by the power shift method [3], [4]. Both methods have difficulties if there exists a spatial distribution of phase across the cavity [8] and, therefore, across the beam. This problem is, however, somewhat more tractable with beam reversal, and, therefore, this method is discussed first and is actually used in the evaluation.

Cavity phase shift determination would be quite easy, of course, if beam reversal could be done completely symmetrically and if there were no spatially distributed phase variations across the cavity. Neither assumption is strictly true in practice; however, we have evidence indicating that beam retrace is obtained to a high degree. This is based on the following observations: 1) Maximum signal to noise is obtained in either direction when the source, detector, magnets, and cavities are geometrically aligned to within a precision of ±0.075 cm. Signal to noise versus horizontal position of the detector ribbon is an approximately bell-shaped curve with a half-width of ~0.4 cm, which is equal to the width of the detector ribbon. 2) With this geometrical alignment the velocity distributions in both directions agree to better than 10 percent. Moreover, as the detectors or ovens are moved away from this position of geometrical alignment the agreement between velocity distributions becomes worse.

Consequently, we will assume that exact beam retrace can be obtained with an uncertainty which can be expressed as an uncertainty in horizontal detector (or oven) position of 0.075 cm. We will assume an uncertainty in velocity distribution retrace upon beam reversal of 10 percent. Therefore, since the frequency shift due to cavity phase shift is proportional to velocity, the uncertainty of

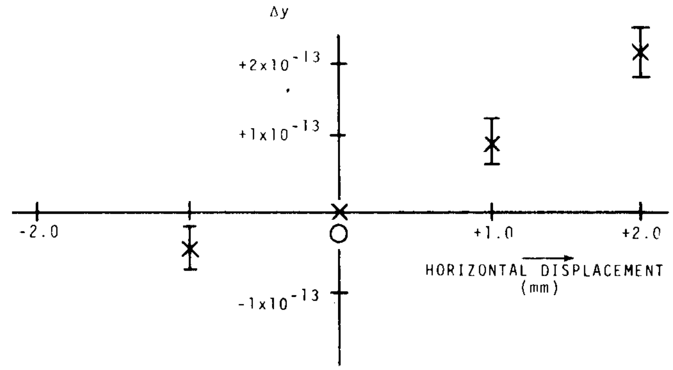

Fig. 2. Plot of relative output frequency versus detector position. On the vertical scale is plotted $\Delta y \equiv$ fractional frequency offset relative to the geometrically aligned position. On the horizontal scale is plotted relative distance that the detector is moved from the geometrically aligned position.

any frequency shift measured between the two directions is at least 10 percent of the value measured.

To obtain a measure of the effect of the distributed cavity phase shift, several frequency measurements were made with the detector moved relative to the geometrically aligned position. Such measurements for the beam in one direction are shown in Fig. 2. All frequency measurements are referred to the geometrically aligned position. All have been corrected for changes in second-order Doppler shift (due to measured changes in velocity distribution), and changes (due to measured velocity changes) in the frequency bias caused by cavity phase shift, assuming the phase shift (measured with the beam geometrically aligned) is constant over the cavity. A similar plot was obtained with the beam in the opposite direction and is consistent with that shown in Fig. 2. Therefore, using Fig. 2 and assuming that we can make the beam retrace with a precision corresponding to ±0.075 cm in the detector position, we assign an uncertainty of $\pm 0.7 \times 10^{-13}$ due to the distributed cavity phase shift. With the beam geometrically aligned and in a particular direction the measured frequency shift (via beam reversal) is $\Delta y = (+0.25 \pm 0.1) \times 10^{-13}$. Therefore, we assume that the error in frequency shift due to cavity phase shift could be as large as $\pm 0.8 \times 10^{-13}$.

As previously noted, the cavity phase shift may also be determined by the power shift method [3], [4]. If the phase shift was constant over the beam, this would be quite straightforward. However, because there is evidence of a rather large distributed effect from the above, the problem is considerably more complicated. One must make assumptions about both the distribution of velocities and distribution of phase across the cavity. In addition, residual cavity mistuning, pulling due to neighboring lines, magnetic field inhomogeneities, and second harmonic content in the phase modulation all give effects which mimic cavity phase shift in a power shift determination. Of these, only the cavity tuning error causes important shifts in NBS-6. However, these can give cavity phase shift uncertainties (as determined by power shift) of up to 2×10^{-13}. These arise because cavity pulling is a function of microwave power [7]. Frequency shift due to cavity phase shift is also a function of microwave power [3], [4]; therefore, a power shift measurement using only two different powers cannot distinguish between cavity pulling and phase-shift pulling. In practice, the phase shift determinations by the power shift method gave results which were several parts in 10^{13} different from those given by beam reversal; however, in view of the above difficulties, these results were not used in the evaluation.

7) *Random Uncertainty:* This is the random uncorrelated uncertainty and is taken as the standard deviation of five frequency calibrations of the NBS time scale (see below).

The uncertainties in the systematic frequency biases can be handled in two ways. The first is to assume that these biases are uncorrelated. This assumption seems to be justified with the possible exception of the second order Doppler shift and the cavity phase shift. We have assumed these also are uncorrelated, and therefore the total error is the root mean square error 0.85×10^{-13}. A more conservative estimate is given by summing the errors (the case if all of the errors are correlated) which gives 1.38×10^{-13} In the next section we have assumed that the error is 0.85×10^{-13}.

As estimate of the NBS-6 frequency stability has been obtained by comparing NBS-6 to the most stable clock in the time scale. These measurements indicate that the flicker floor of NBS-6 is 1 to 2 parts in 10^{14}, which corresponds to the instability of the measured magnetic field. Reducing the field further actually reduces stability, largely because the frequency pulling due to neighboring lines increases significantly and instabilities which alter this pulling reduce the stability of the output frequency.

Estimate of TAI Frequency

As previously reported [4], NBS-4 and NBS-5 have been used via an NBS accuracy algorithm [9] to control the rate of AT (NBS) and also to estimate the length of the TAI second, or the rate (frequency) of International Atomic Time (TAI). The last best estimate (April 1975) using these two standards is plotted in Fig. 3 and gives the rate of TAI to be too high (the TAI second is too short) by (7.8 ± 1.5) parts in 10^{13}. This value and the new NBS-6 value both make allowance for a 1.8 part in 10^{13} gravitational blue shift at the location of NBS relative to sea level. In Fig. 3 we also plot estimates of TAI given by Physikalisch-Technische Bundesanstalt (PTB), Germany, and National Research Council (NRC) [10]. Five sequential calibrations of the UTC(NBS) time scale were made during the period 15 April to 1 June, 1976. The UTC (NBS) scale was found to be high in frequency by $+8.3 \times 10^{-13}$ with a standard deviation on the five calibrations of 0.3×10^{-13}. This standard deviation is then taken as the random uncertainty of the NBS-6 inaccuracy (Table I). An estimate of the uncertainties associated with the systematic bias corrections as outlined above is 0.85×10^{-13}, giving an overall accuracy for the NBS-6 calibration of 0.9×10^{-13}. Therefore, $y_{\mathrm{UTC(NBS)}} - y_{\mathrm{NBS\text{-}6}} = +(8.3 \pm 0.9) \times 10^{-13}$.

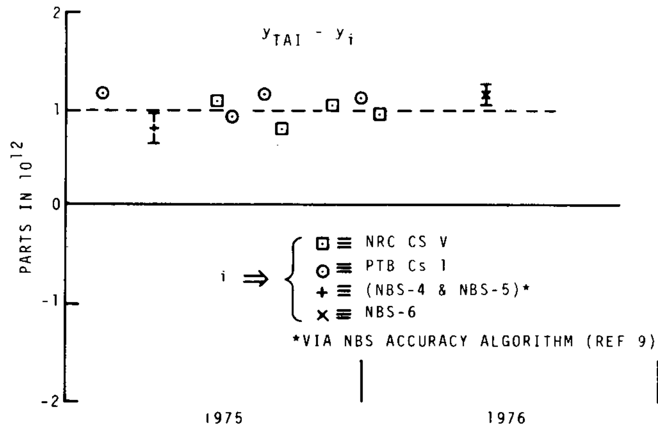

Fig. 3. Estimates of the frequency of International Atomic Time (TAI) relative to some primary standards.

The frequency y'_{TAI} of TAI as observed at NBS (*not* including any gravitational correction) during the time of these calibrations was measured at NBS using (a) Loran-C and NBS/U.S. Naval Observatory portable clock data:

$$y'_{\mathrm{TAI}} - y_{\mathrm{UTC(NBS)}} = (+1.8 \pm 0.8) \times 10^{-13}$$

(b) NBS portable clock trips to the BIH:

$$y'_{\mathrm{TAI}} - y_{\mathrm{UTC(NBS)}} = (+1.5 \pm 0.4) \times 10^{-13}.$$

Using a weighted average of (a) and (b) and after inclusion of the gravitational redshift of TAI as seen from Boulder, $y_{\mathrm{TAI}} - y'_{\mathrm{TAI}} = +1.8 \times 10^{-13}$, we obtain

$$y_{\mathrm{TAI}} - y_{\mathrm{NBS\text{-}6}} = (+11.7 \pm 1.0) \times 10^{-13}.$$

This value as compared to our 1975 value may lead to the conclusion that there is a small upward drift in the frequency of TAI. We feel that the measurements do not fully support this conclusion, since the 1975 value was obtained with only limited accuracy evaluation techniques,[2] and the full-scale evaluation of October–December 1974, yielded $9 \pm 2 \times 10^{-13}$, which is consistent with no drift in TAI. However, we feel that the present value is a better estimate than all past values because of the actual improvement by a factor of 2 in the accuracy obtained with NBS-6 as compared to previous NBS primary standard evaluations.

We intend to use NBS-6 as a continually operating clock for extended periods (months) only interrupted by experiments aimed at further improvements and new evaluations. NBS-6 thus would contribute to the preservation of frequency accuracy and would join NBS-4, which already has been used in this manner during the past year.

Future Work

The most important area for future work centers on the measurement of cavity phase shift. Perhaps independent

[2] Checks of microwave power, Ramsey type microwave spectrum, and magnetic field were carried out, but no direct measurements of the cavity phase shift, the distributed phase shift, or the servo loop were done.

measurements of phase shift can be obtained by studying the distortion of the Ramsey profile [11], and this will be pursued in the future. Signal to noise could be improved by approximately a factor of 2 if we use a squarewave digital servo; moreover, servo system systematic shifts are then much more easily accounted for than with the sine wave modulation. Such a servo is at present under construction.

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